Portfolio Assessment

Name: Brian Pember

Expected Date of Graduation: ________

Mathematics
Secondary Option

	Semester Completed
Math 151 – Calculus I	Spring 08
Math 152 – Calculus II	
Math 225 – Introduction to Abstract and Discrete Mathematics	
Math 231 – Linear Algebra and Differential Equations	
Math 241 – Probability and Statistics I	

Brian Pember

12/4/08

During my time here at Keene State College I have taken two math courses and am currently enrolled in two others. I plan on continuing taking math courses on my way to becoming a math teacher. I find math to be challenging and interesting because somehow all my classes end up being connected in the material. Throughout my mathematical career I have used and memorized many different algorithms and formulas. Such as in calculus two where we learned how to take the integral of polynomials using different techniques. The two main ways that we did this was through U-substitution and another way called integration by parts. One, and sometimes both of these formulas needs to be used to integrate. In multiple math courses that I have taken I have been asked to research a historical mathematician. In calculus two this year I did a research paper and presentation on Leonhard Euler. I learned about his lifestyle and his discoveries that changed mathematics. Also one of my assignments in this portfolio involved me reading about the discovery of Pascal's Triangle and the history behind why it is called that. It is amazing to me how hundreds of years ago people were able to be intelligent enough to figure out how to use mathematics and apply it to the real world.

As part of my general education requirements I took an economics course. This is where my math classes were able to help me because during the course I had to understand how to determine the present value of an item if I knew the rate at which the price was increasing or decreasing. I had learned the formula during my experiences with other math classes and I was fortunate enough to have an understanding for why the formula worked and when to use it. Another important aspect of mathematics that I learned this semester was how to verbalize math and present it in a way that is understood at multiple levels of math experience. I learned this in my Abstract and Discrete math course where we proved how two sets are equal and other important parts of math such as the zero product property. Another part of my abstract and discrete math class was reading proofs and problems and being able to figure out what needed to be done to solve the equation or what needed to be proved.

Some of the courses which I thought would never be able to be linked to math such as my computer programming class, which I have taken this semester, ended up using math more than I thought possible. Almost every assignment we needed to calculate some number such as changing from degrees Fahrenheit to degrees Celsius.

I look forward to taking more math related courses and challenging myself even more here at Keene State College. I have found that even though I struggle sometimes I am able to pick myself back up and learn from my mistakes in a constructive way.

Math 141 (Introductory Statistics) Artifacts

Applicable NCATE Middle School Standards

6.1 Use knowledge of mathematics to select and use appropriate technological tools.

14.1 Design investigations, collect data, and use a variety of ways to display data and interpret data representations.

14.2 Draw conclusions involving uncertainty by using hands-on and computer-based simulation for estimating probabilities and gathering data to make inferences and decisions.

14.3 Identify misuses of statistics and invalid conclusions from probability.

14.4 Use appropriate statistical methods and technological tools to describe shape and analyze spread and center.

14.5 Investigate, interpret, and construct representations for conditional probability, geometric probability, and for bivariate data.

14.6 Demonstrate knowledge of the historical development of statistics and probability including contributions from diverse cultures.

Math 151 (Calculus I) Artifacts

Applicable NCATE Secondary Standards

10.1 Analyze patterns, relations, and functions of one and two variables.

12.1 Demonstrate a conceptual understanding of and procedural facility with basic calculus concepts.

12.2 Apply concepts of function, geometry, and trigonometry in solving problems involving calculus.

12.3 Use the concepts of calculus and mathematical modeling to represent and solve problems taken from real-world contexts.

12.5 Demonstrate knowledge of the historical development of calculus including contributions from diverse cultures.

Applicable NCATE Middle School Standards

10.5 Analyze change in various contexts.

12.1 Demonstrate a conceptual understanding of and procedural facility with basic calculus concepts.

12.2 Demonstrate knowledge of the historical development of calculus including contributions from diverse cultures.

Brian Pember

MATH 151 Stanish
Spring 2008
Application Project

due: Wednesday, April 30, 2008

To complete this project you must complete each of the four problems below. Each problem is worth **25** points for a total of **100** possible points on this project. You may work individually or in groups of 2 or 3 people. While you should only submit one write-up, each member of the group should work on all of the problems and understand what is submitted for each problem. In completing the this project, you may consult your textbook and your class notes, and you are strongly encouraged to ask me questions. You may NOT, however, discuss this project with anyone other than your group members or me. In particular, you may NOT discuss this project with classmates outside of your group, other students, the PCA, or other professors. In addition, you may NOT utilize other textbooks, the internet, or any other outside source.

1. Read Section 4.11 *The Shroud of Turin* on p. 203 of your textbook. Then find the age of the Shroud of Turin.

2. Read Section 4.15 *Gravity* on p. 215-218 of your textbook. Then complete Exercises # 2, 4, 6, 8, 10 on p. 218-219.

3. Read Section 6.2 *Newton's Method* on p. 301-306 of your textbook. Make sure you understand the Example of p. 305-306 in which we approximate $\sqrt{3}$. Then use Excel to contruct a spreadsheet that calcuates this approximation of $\sqrt{3}$. One group member should email this spreadsheet to your instructor with the subject line "**Newton spreadsheet**". In the body of the email please include **all group members names**. Your instructor will evaluate the correctness of your spreadsheet by changing the initial guess of 1 to a different number to see if your spreadsheet will still approximate $\sqrt{3}$.

4. Read Section 6.12 *Making the Most Money* on p. 377 of your textbook. Then complete Assignment # 1-4 on p. 377.

Extra Credit: If you hand-in 2 copies of your project write-up per group, everyone in the group will received 5 points of extra credit.

Curt Guild
Brian Pember
Spencer Roux

The Shroud of Turin p. 203

We're looking for age here. If the shroud of Turin is legitimate it should produce an age close to 1994 years.

given: There was 92% of Carbon 14 isotope in the sample when measured

$$C(age) = 0.92\, C(0)$$

given: The equation must be in the format $C(age) = Ae^{-kt}$

given: $k = 0.0001245$

so $C(0) = Ae^{k(0)} = Ae^{0} = A(1) = A$ so $C(0) = A$

we found earlier that $C(age) = 0.92\, C(0)$, so if $C(0) = A$, this

$$C(age) = 0.92\, A$$

earlier as well, we were given $k\ (0.0001245)$ and the format of $C(age) = Ae^{-kt}$, so if that's the case, we can set $0.92\, A$ and Ae^{-kt} equal to one another. (Both are functions of age)

$$0.92A = Ae^{-0.0001245t}$$

we immediately factor out A to get

$$0.92 = e^{-0.0001245t}$$

to remove the e, we apply a natural log (ln) to both sides.

$$\ln(0.92) = \ln(e^{-0.0001245t})$$

the log and e cancel, then it is simply a matter of isolating t, the age of the shroud of Turin.

$$\frac{\ln(0.92)}{0.0001245} = \frac{0.0001245t}{0.0001245}$$

$$-669.7317987 = t$$

As 669.7 years is old, this does not place the shroud of Turin in existance at the assumed time of Christ. The fabric is simply not old enough.

Page 218-219 #2-10

2. An object is dropped from a building 100 m tall. When is it halfway down?

$v(0) = 0$ m/s

$h(0) = 100$ m

$h(x) = -4.9t^2 + h(0)$ height equation

$50 = -4.9t^2 + 100$ set to 50

$\frac{-50}{-4.9} = \frac{-4.9t^2}{-4.9}$

$10.204 = t^2$

$3.194 = t$

The object is halfway down the building at approx. 3.20 seconds after release.

4. Standing on the ground, I toss an object upward with a velocity of 15 meters/second. When does it reach its highest point?

$v(0) = 15$ m/s $h(0) = 0$

$h(x) = -4.9t^2 + 15t$ height equation

$h'(t) = -9.8t + 15$ velocity

At its highest point the velo
of the object will be zero

$0 = -9.8t + 15$

$-15 = -9.8t$

$1.530 = t$

The object will reach its apex at $t = 1.530$

6. Standing on a building 100 m tall, I toss an object upwards with a velocity of 15 m/s. What is the height after 3 seconds?

$h(0) = 100 \quad v(0) = 15 \quad h(3) = ?$

$-4.9t^2 + 15t + 100 = h(t)$ height equation

$-4.9(3)^2 + 15(3) + 100 = h(3)$ plug in 3 seconds

$-44.1 + 45 + 100 = h(3)$

$.9 + 100 = h(3)$

$100.9 = h(3)$

8. Standing on a building 100 m tall I toss an object upward with a velocity of 15 m/s. When does it hit the ground?

$h(0) = 100 \quad v(0) = 15 \quad h(t) = 0$

$h(t) = -4.9t^2 + 15t + 100$ height equation

Since $h(t) = 0$, we set the equation equal to zero. Since there are multiple occurrences of t, we must use the quadratic equation.

equation: $\dfrac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2a}$

coefficients: $a = -4.9$

$b = 15$

$c = 100$

setup: $\dfrac{-15 \pm \sqrt{15^2 - 4(-4.9)(100)}}{2(-4.9)}$

Using the equation will produce two answers.

Answer 1: $t = \dfrac{-15 + \sqrt{2185}}{-9.8} = \sim -3.289$

Answer 2: $t = \dfrac{-15 - \sqrt{2185}}{-9.8} = \sim 6.300$

check: $-4.9(6.3)^2 + 15(6.5) + 100 = 0$

Since one answer is negative, (time is positive), we determine the object hit the ground at 6.3 seconds.

10. Standing on a building 100 meters tall, I toss an object downward at 15 m/s. How fast is it going when it hits the ground?

$h(0) = 100 \qquad v(0) = -15 \qquad v(h(0)) = ?$

$h(t) = -4.9t^2 - 15t + 100$ height equation

$0 = -4.9t^2 - 15t + 100$ set to zero

Since there are multiple t's and the correct form is present, we use the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 0 \qquad a = -4.9 \quad b = -15 \quad c = 100$$

$$\frac{15 \pm \sqrt{15^2 - (4(-4.9(100)))}}{2a} = (-6.300, 3.239)$$

Since this is negative, we assume the second is our zero point.

To determine velocity, we must first differentiate the height equation.

$h'(t) = v(t) = -9.8t - 15$

Since we know the object struck the ground at 3.239 seconds, we can determine the velocity at time of impact by solving $v(t)$ for $t = 3.239$

$v(3.239) = -9.8(3.239) - 15$

$v(3.239) = -46.74$

The object was travelling at -46 m/s when it struck the ground.

Pg. 377 questions 1-4

1. To find revenues, we must first find the value of n. The function is given to us as

$$n = 360,000 - 20,000p.$$

Revenues equal the number of units sold times the price, thus

$$R(p) = np$$

As we know the function of n, we can deduce that

$$R(p) = p(360,000 - 20,000p)$$

And distributed, our final revenue function is

$$R(p) = 360,000p - 20,000p^2$$

2. To find cost, we use the function $C(p) = 2n$ (Given: the cost to produce one unit is \$2.) By plugging in our known function for n, our final cost function is

$$C(p) = 2(360,000 - 20,000p) = 720,000 - 40,000p$$

3. To find total profits, we use the function $R(p) - C(p) = P(p)$ Thus we combine our final results from 1 and 2 to form our profit function.

$$(360,000p - 20,000p^2) - (720,000 - 40,000p) = P(p)$$

4. To find our maximum profit margin, we must first find the derivative of our profit function.

$$P'(p) = (360,000 - 40,000p) + 40,000$$

The maximum price will have a derivative of 0, so we set $P'(p)$ to 0.

$$0 = (360{,}000 - 40{,}000p) + (40{,}000)$$

$$-40{,}000 = (360{,}000 - 40{,}000p)$$

$$\frac{-400{,}000}{-40{,}000} = \frac{-40{,}000p}{-40{,}000}$$

$$10 = p$$

The optimum price to set for each game to maximize profits is \$10.00/unit.

Brian Pember

Calculus 1

There are many people instrumental in the discovery of Calculus. There are great names such as Isaac Newton who invented Calculus or Archimedes who discovered area and volume. These men take most of the glory for their discoveries but they are not the only ones who deserve credit for their hard work. Isaac Barrow was another mathematician who discovered that differentiation and integration are inverse operations. Without his discovery calculus and mathematics would not be the same as it is today.

Isaac Barrow was born in London, England during October of the year 1630. He was a troubled child during his early years of life. It was said that he was such a turbulent child that his father prayed to God if he was going to take one of his children it be Isaac. Isaac attended Trinity College in Cambridge where he studied arithmetic, geometry and optics. Students during this time period were encouraged not to specialize in a subject such as mathematics before graduating. Once he graduated from Cambridge he went on to be a college professor at the same school he got his degree from, Cambridge. In 1663 he was made the first Lucasian professor of mathematics at Cambridge where he eventually resigned to let Isaac Newton, his former student, take over. Barrow had taken an oath to study divinity while he was at Cambridge which led him to the study of astronomy. After astronomy came the study of geometry which he self taught himself.

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Calculus 1

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Throughout Isaac's career of teaching at Cambridge he devoted himself to the three series of Lectiones which is where the majority of his fame lies, they were the speeches he gave in the years 1664, 1665, and 1666. His earliest work was a complete edition of the Elements of Euclid which was published in Latin. He is also known for his work in optics where he developed the Euclidean law of reflection and the sine law of refraction. He was not the original creator of these ideas because he did take some of his information from Alhazen, Kepler, Scheiner, and Decartes. Barrow is also well known for his contact with Isaac Newton however the details of their friendship are unknown. Some say that they never even knew each other and their relation is a myth.

Unfotunatly Isaac Barrow's life was cut short by an apparent drug overdose in May 4, 1677. However, even though his life was cut short he was instrumental in discovering how the sine law of refraction works. Also how it affected his discovery that differentiation and integration are inverse operations. Without his discoveries the mathematics world would not operate the same.

References

Gillispie, Charles Coulston. Dictionary of Scientific Biography. Charles Scribner's Sons. New York. 1970.

Cousin, John William. A Short Biographical Dictionary of English Literature. London, J.M. Dent & sons; New York, 1910.

MATH 151 Stanish
May 7, 2008
Final Exam

64

Name: Brian [illegible]

For problems 1)-3) translate the sentences into mathematical notation.

1. (4 pts) a) My net worth (call it W) right now ($t = 0$) is \$55,000.

$W(0) = 55,000$

4

b) Thanks to some good investments, my net worth right now is growing at the rate of \$6,000 per year.

$W'(0) = 6,000$

2. (2 pts) From 1998 to 1999, the number of serious crimes that were reported in New York City (call it S) fell by 23,627 reports.

2

$\int_{1998}^{1999} S'(t) = -23,627$

3. (2 pts) The number of people with O-negative blood (call it O) in this country is directly proportional to the total population (call it P).

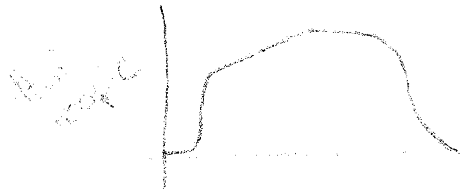
2

$O = PK$

4. (2 pts) Translate the following passage into a curve on a plane with appropriately labeled axes. In brackets you are given the quantity that should appear on the vertical axis.

An illness hit a small town. At first it spread rapidly, but as medicine became available it began to spread more slowly. It finally stopped spreading. Then people began to get better and the number of infected people started dropping more and more quickly until everyone had recovered. [number of ill people]

2



5. (4 pts) Let $f(x) = x^3 - 1$. Write an equation to the line tangent to the graph of f at $x = -2$

$F'(a)(x-a) + F(a)$ $F' = 3x^2$

4

$F'(-2) = 3(-2)^2 = 12$

$F(-2) = -2^3 - 1 =$
$-8 - 1 = -9$

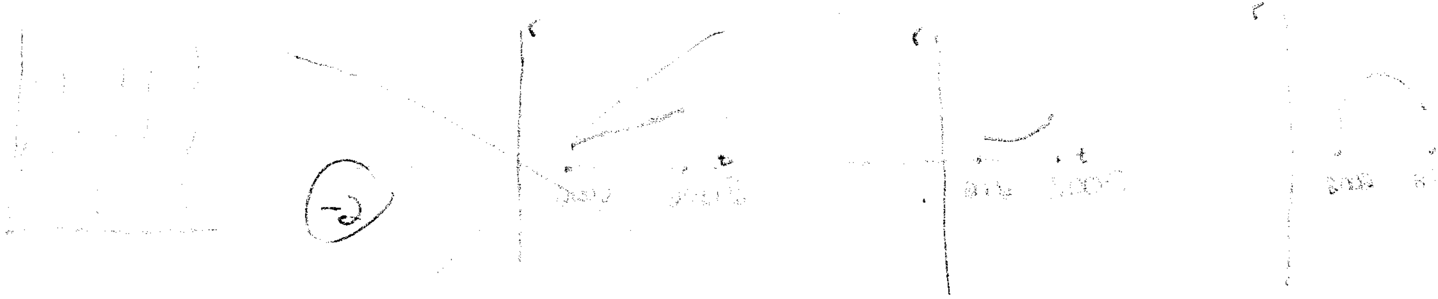
$y = 12(x+2) - 9$

6. (8 pts) From 2002 to 2005, my retirement account grew at a rate of 2% per year. Let $t = 0$ represent 2002, and let $r(t)$ be the value of my retirement account in year t.

a) Translate the above statement into a differential equation.

$r'(t) = 0.2r(t) + 50\cancel{,000}$ -1

b) Draw slope field for f given by this equation and sketch three possible graphs that could represent the value of my retirement.



-2

2

c) Given that the value of my retirement account was \$50,000 in 2002, find the solution to the differential equation you found in part a) that gives the value of my retirement account in year t.

$r(0) = 50,000$

$r(t) = 0.\cancel{0}2t(50,000) + 50,000$

-2

d) Find the value of my retirement account in 2005.

$r(3) = 0.02(3)(50,000) + 50,000 = 53,000$

-1

7. (6 pts) Let $f(x) = x^2 + 3x - 1$. Find the **difference quotient** for this function. Then show that the derivative is $f'(x) = 2x + 3$ by taking the **limit of the difference quotient** as Δx goes to 0.

$$\frac{F(x+\Delta x) - F(x)}{\Delta x} = x^2 + 3x - 1$$

1

$$\frac{F(x+\Delta x) - (x^2+3x-1)}{\Delta x}$$

$$\cancel{\lim_{\Delta x \to 0} (2x + 3)} = 3$$

8. **(6 pts)** Find each of the following limits (if it exits):

a) $\lim_{x\to-1}(x^2+3x-4)$

x^2+3x-4

$(-1)^2+3(-1)-4 = 1+(-3)-4 = -6$ $\lim_{x\to-1}(x^2+3x-4) = -6$

b) $\lim_{x\to2}\frac{x-2}{x^2-4}$ $x \neq 2$

6

$\frac{x-2}{(x-2)(x+2)}\ \frac{1}{(x+2)} = \frac{1}{4}$ $\lim_{x\to2}\left(\frac{1}{x+2}\right) = 1/4,\ x \neq 2$

c) $\lim_{x\to0}\frac{x+1}{x} =$

DNE

4 9. **(4 pts)** Determine the intervals on which the function $f(x) = \frac{3}{x+1}$ is continuous.

continuity $(-\infty,-1)\cup(-1,\infty)$

10. **(10 pts)** Differentiate each of the following functions.

a) $f(x) = x^5 + 3x^2 - 2x + 1$

$F'(x) = 5x^4 + 6x - 2$

b) $f(x) = x^3\sqrt{x-2}$ $= x^3(x-2)^{1/2}$

$F'(x) = 3x^2(x-2)^{1/2} + x^3(1/2(x-2)^{-1/2})$

$\boxed{-1}$ $\cancel{(\sqrt{x-2})^2}$

8

c) $f(x) = \frac{\sin(x^2)}{x+1}$

$F'(x) = \frac{\cos(x^2)2x(x+1) - \sin(x^2)(1)}{(x+1)^2}$

d) $f(x) = \ln(x^2+3) + e^{2x}$

$F'(x)\ \frac{1}{x^2+3}(2x) + e^{(2x)}(2)$

$\boxed{-1}$

e) $f(x) = -3\tan(x)$ $\sec^2$

$F'(x) = -3\sec(x)^2$

+C

11. (10 pts) Find the following indefinite integrals.

a) $\int (x^4 + 2x - 1)\, dx$

$F(x) = 1/5\, x^5 + x^2 - x + C$

b) $\int \left(\frac{1}{x^3} + \sqrt{x} \right) dx$ $= \int (1/x^3) dx \int (\sqrt{x}) dx =$

$\int x^{-3} dx + \int x^{1/2} dx = -1/4\, x^{-4} \cancel{+} 1/2\, x^{-1/2} + C$

-2

6

c) $\int (\sin(3x) + \sec(x)\tan(x))\, dx$

$\tan \to \sec^2$

$\sec \to \tan \sec$

$F(x) = -1/3 \cos(3x) + \sec(x) + C$

d) $\int 0.2e^{2x}\, dx$

$F(x) = e^{2x}(.1)$

e) $\int \frac{3x^2 - 2}{x}\, dx$ $= \int 3x^{2} - 2\, dx + \int \frac{1}{x} dx$

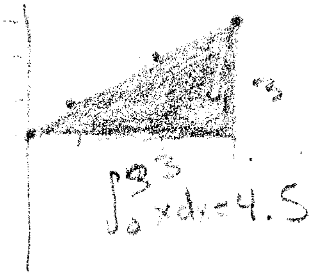
$F(x) = x^3 - 2x \cancel{+} \ln|x| + C$

-2

12. (6 pts) Consider the definite integral $\int_0^3 x\,dx$

a) Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

x	y
0	0
1	1
2	2
3	3



$\int_0^3 x\,dx = 4.5$

$\frac{1}{2}(3)(3) = 4.5$

b) Use the Fundamental Theorem of Calculus to evaluate the definite integral.

3

13. (6 pts) Consider the definite integral $\int_0^3 e^{2x}\,dx$

a) Use the Method of Rectangles to approximate the value of this integral.

$.5\left[e^{2(0)} + e^{2(.5)} + e^{2(1)} + e^{2(1.5)} + e^{2(2)} + e^{2(2.5)}\right]$

3

117.102

b) Use the Fundamental Theorem of Calculus to evaluate the definite integral exactly.

14. (6 pts) For a dosage of x cubic centimeters of a certain drug, the resulting blood pressure B is approximated by

$$B(x) = 0.05x^2 - 0.3x^3, \quad 0 \le x \le 0.16$$

Find the maximum blood pressure and the dosage at which it occurs.

$B(x) = 0.05x^2 - 0.3x^3$

$B'(x) = .1x - .9x^2$

4

$.1x - .9x^2 = 0$

$\frac{.1x}{.9} = \frac{.9x^2}{.9}$

~~$x = 1/3$~~ max

~~1/9 cubic cm~~

~~.085~~

endpoints?

-2

15. (10 pts) Let $f(x) = x^3 + 6x^2 + 9x$

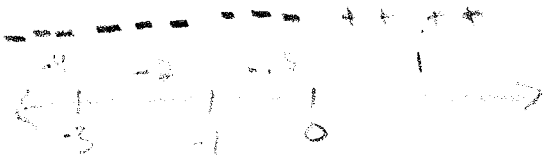
a) On what intervals is $f(x)$ increasing/decreasing?

$f'(x) = 3x^2 + 12x + 9$

$3(x^2 + 4x + 3)$

$(x+3)(x+1) = 0$

$x = -3$ or -1



~~increasing $(0, \infty)$~~
~~decreasing $(-\infty, 0)$~~

-7

b) Find the relative extrema of $f(x)$.

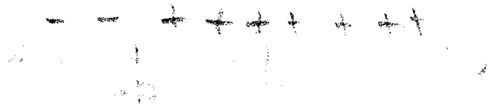
relative max ~~$x = 0$~~
relative min ~~$x = -3$~~

-2

c) On what intervals is $f(x)$ concave up/concave down?

$f''(x) = 6x + 12$

$x = 2$



concave up $(-2, \infty)$
concave down $(-\infty, -2)$

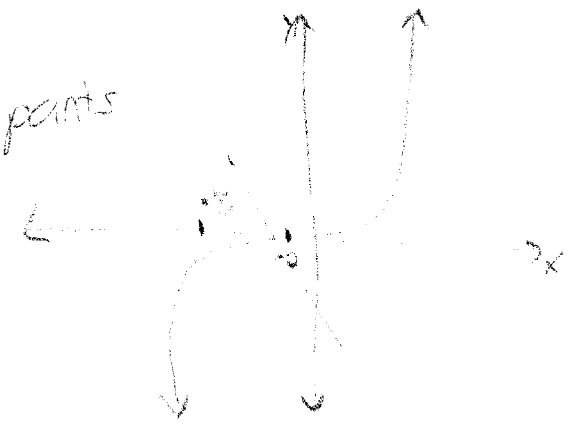
d) Find the inflection points of $f(x)$.

$x = -2$

e) Using the information in parts a)-d), sketch the graph of $f(x)$.

plot points

-2



16. **(8 pts)** Solve the following differential equations for y as a function of t.

a) $y' = ty^2$

$y' = \int t\,dt \int y^2\,dy$

$y = \frac{1}{2}t^2 \; \frac{1}{3}y^2$

0

b) $y' = t^2y$

$y = \frac{1}{3}t^3 \; \frac{1}{2}y^2$

17. **(6 pts)** Calculate the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ for each of the following functions.

a) $f(x,y) = 4x^5 + 3x^2y - 2xy + y^3$

$F_{\cancel{x}}(x,y) = 3x^2 - 2x + 3y^2$

$\overset{y}{F}_{\cancel{x}}(x,y) = 20x^4 + 6xy - 2y$

x

$\boxed{-1}$

5

b) $f(x,y) = \cos(xy)$

$F_x(x,y) = -\sin(xy)\,y$

$F_y(x,y) = -\sin(xy)\,x$

Extra Credit (3 pts each):

a) Differentiate $f(x) = \ln(\ln(x\cos(5x-1)))$

$F'(x) = \left(\frac{1}{\ln(x\cos(5x-1))}\right)\left(\frac{1}{x\cos(5x-1)}\right)(\sin(5x-1))(5)$

3

b) Evaluate $\int 2xe^{x^2}\,dx$ $f(x) = e^{x^2}$ ✓

Math 225 (Intro to Abstract/Discrete Math) Artifacts

Applicable NCATE Secondary Standards

2.1 Recognize reasoning and proof as fundamental aspects of mathematics.

2.2 Make and investigate mathematical conjectures.

2.3 Develop and evaluate mathematical arguments and proofs.

2.4 Select and use various types of reasoning and methods of proof.

9.5 Apply the fundamental ideas of number theory.

13.1 Demonstrate knowledge of basic elements of discrete mathematics.

13.2 Apply the fundamental ideas of discrete mathematics in the formulation and solution of problems arising form real-world situations.

13.3 Use technological tools to solve problems involving the use of discrete structures and the application of algorithms.

13.4 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

Applicable NCATE Middle School Standards

2.1 Recognize reasoning and proof as fundamental aspects of mathematics.

2.2 Make and investigate mathematical conjectures.

2.3 Develop and evaluate mathematical arguments and proofs.

2.4 Select and use various types of reasoning and methods of proof.

9.5 Apply the fundamental ideas of number theory.

13.1 Demonstrate a conceptual understanding of the fundamental ideas of discrete mathematics such as finite graphs, trees, and combinatorics.

13.2 Use technological tools to apply the fundamental ideas of discrete mathematics.

13.3 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

4R. A Mathematical Research Situation: Investigating the Relationship Between Two Sets

For any sets A, B, and C, we can form the sets $(A \cup B) - C$ and $A \cup (B - C)$. As budding mathematicians, we might then become interested in the relationship between these two sets. For instance, we might ask

- are the two sets $(A \cup B) - C$ and $A \cup (B - C)$ always equal?
- if they are not equal, is one always a subset of the other?
- if one of them is not a subset of the other, is there a condition on the sets A, B, and C which *would* guarantee a subset relationship?

By carrying out an investigation of the two sets $(A \cup B) - C$ and $A \cup (B - C)$ we might hope to formulate conjectures that propose answers to these questions, and perhaps others as well. We could then use what we have learned about proof writing to attempt to develop and write proofs for our conjectures.

> *Remark:* What we have just described illustrates the mathematical research process. We start with a mathematical issue of interest; pose questions related to the issue that we would like to answer; carry out an investigation leading to the formulation of conjectures that, if proved, will provide answers to our questions; and, finally, work toward producing proofs for these conjectures.

1. In attempting to answer the questions posed above, we might use Venn diagrams to illustrate the sets $(A \cup B) - C$ and $A \cup (B - C)$. Draw two Venn diagrams, one depicting $(A \cup B) - C$ and the other depicting $A \cup (B - C)$. Then try using the pictures you have drawn to answer the following questions:

   a. Must the sets $(A \cup B) - C$ and $A \cup (B - C)$ be equal?

   b. Does it appear that one of the sets $(A \cup B) - C$ or $A \cup (B - C)$ is always a subset of the other?

   c. If you believe that one of the sets $(A \cup B) - C$ or $A \cup (B - C)$ is not always a subset of the other, can you find a condition on the sets A, B, and C which you think *would* guarantee a subset relationship?

Use your answers to these questions to formulate several conjectures concerning the sets $(A \cup B) - C$ and $A \cup (B - C)$ that we could then attempt to prove.

2. For each of the conjectures you formulated in the investigation, either prove it or give evidence showing why it is not true.

3. Write up your work from (1) and (2) as a mathematical paper. Your paper should include:

- an introduction in which you describe the focus of your investigation;

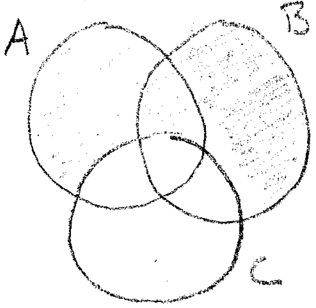
Brian Pember

10/21/08

There are a few different focuses for this paper based on the two sets (AuB)-C and Au(B-C). ✓ As a mathematician there are always questions to prove and ask, this paper will involve three different questions that were shown to either be true, false, or depending on the conditions could change. By looking at the two sets (AuB)-C and Au(B-C) questions come to mind such as "Are the two sets always equal?", or "If they are not equal, is one always a subset of the other?", and then "If one of them is not a subset of the other, is there a condition on the sets A, B, C which would guarantee a subset relationship?". All of those are the focus of this paper and investigation. ✓

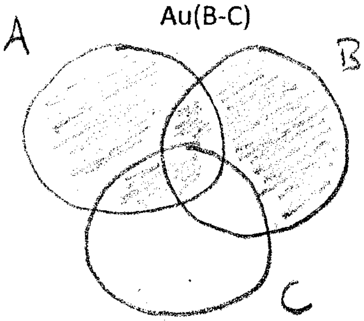
The first part of proving the three questions is to have an understanding of what the sets are. In order to have this understanding it is a good idea to look at Venn Diagrams to show the members of the sets. ✓

(AuB)-C



✓

Au(B-C)



✓

After drawing the Venn Diagrams the next step is to look at the possible members and try to answer the questions that were originally asked or some variation of those questions.

Must the two sets be equal?

The answer is no because Au(B-C) includes AnC and (AuB)-C does not contain these members, making them uneven. O.K.

Does one appear to always be a subset of the other?

Yes one is a subset of the other. (AuB)-C is always subset of Au(B-C) because the set Au(B-C) will always contain the members of (AuB)-C however, Au(B-C) cannot, under these circumstances, be a subset of (AuB)-C because Au(B-C) contains more members. ✓

If you believe that one of the sets is not always a subset of the other can you find a condition on the sets A, B, and C which you think would guarantee a subset relationship?

Yes there is a condition which guarantees a subset relationship between the two sets. If ~~X is a member of Au(B-C) and~~ there are no members of AnC then under this condition Au(B-C) is a subset of (AuB)-C. ✓

Once those questions were answered then it is time to move on to proving those answers. We shall start with the first question. Are the two sets always equal? The way that this question is phrased and the fact we know the two sets are not equal from the Venn Diagrams it becomes apparent that the following proof will only need to come up with one instance where the two sets are not equal. Thus proving that the two sets are not always equal.

Claim: For some sets A, B, and C (AuB)-C does not equal Au(B-C).

Proof: Let A = {1,2,3}, B = {1,6,9}, and C = {3,4,5,6}. Then (AuB) = {1,2,3,6,9} so (AuB)-C = {1,2,9}. Also B-C = {1,9} so Au(B-C) = {1,2,3,9}. Since 3 is not a member of (AuB)-C and 3 is a member of Au(B-C) we may conclude that the two sets are not equal.•

After proving the first question that the two sets in fact are not equal we need to prove the second question which is: Is one always a subset of the other?

Claim: For any sets A, B, and C (AuB)-C is a subset of Au(B-C).

Proof: Consider any sets A, B, and C. We must show that (AuB)-C is a subset of Au(B-C). Assume X is a member of (AuB)-C. By definition of union X is a member of A or X is a member of B. Also by the definition of set difference X is not a member of C. Assume X is not a member of A. If so then X must be a member of B. Since X is a member of B and X is not a member of C it is a ~~subset~~ member of Au(B-C). Assume that X is not a member of B. Then X is a member of A and X is not a member of C. Since X is a member of A by the definition of union ~~(AuB)-C is a subset~~ X is a member of of Au(B-C).•

Our last question that needs to be proved is: If one of them is not a subset of the other, is there a condition on the sets A, B, C which would guarantee a subset relationship?

Claim: If AnC is the empty set then Au(B-C) is a subset of (AuB)-C.

Proof: Suppose AnC is the empty set. ~~Consider any sets A, B, and C.~~ We will show that Au(B-C) is a subset of (AuB)-C. Assume X is a member of Au(B-C). By definition of union X is a member of A or X is a member of B-C. By the definition of set difference X is a member of B and X is not a member of C. Since AnC is the empty set there are no members. Therefore if X is a member of A, then X is not a member of C. Since X is a member of A or X is a member of B and X cannot be a member of C. Au(B-C) is a subset of (AuB)-C.•

At first glance when looking at the two sets Au(B-C) and (AuB)-C there are questions that come to mind if they are equal and when they are equal that we were able to answer. Making Venn Diagrams and going through proofs is a good way to problem solve. Visualizing is always a helpful ability and drawing and planning proofs out is the best option.

True if $X \in B-C$ ... need to set up cases for the two possibilities $X \in A$ or $X \in B-C$
You've essentially done this, but you should make this clearer.

Assessment of Math 225 Graded Assignment

Cullinane

The assessment of this graded assignment has taken into account all of the following:

- the correctness of your work;
- the validity of your reasoning;
- your ability to communicate your conclusions and your reasoning;
- the quality of your writing;
- the organization of your work;
- the degree to which you have completed the assignment.

Specific Areas of Concern

General

_____ incomplete assignment

_____ directions not followed

_____ handwriting not legible or difficult to read

_____ not following requirement of sentences organized into paragraphs

_____ writing incoherent or difficult to follow

_____ spelling, grammar, punctuation, or capitalization errors

_____ overall presentation of work not at an appropriately professional level

Mathematical (proofs or other types of problems)

_____ incorrect mathematics

_____ incorrect or inappropriate use of mathematical terminology or notation

_____ invalid reasoning

_____ evidence provided not specific enough, conclusions not always justified, or incorrect justification

_____ interpretations not always accurate

Mathematical (proof-specific)

_____ appropriate labels (*Claim*, *Proof*, etc.) not always provided

_____ informal explanation given instead of proof

_____ not following standard proof formats or strategies

_____ incorrect or inappropriate application of a proof strategy

_____ confusing what is known and what needs to be shown

_____ sequencing of deductions incorrect

Grade A– Very nice overall!

6C. Alternative Methods for Proving $n^2 + n$ Is Even

1. In Example 6.8 in §6.2 we defined what it means for an integer to be even. Create a similar definition for the notion of a natural number being *odd*.

2. Formulate conjectures concerning the sum of two even integers, the sum of two odd integers, the square of an even integer, and the square of an odd integer. Try to prove your conjectures.

3. Use your conclusions in (2) to prove, without using induction, that $n^2 + n$ is even for every positive integer n.

4. Formulate a conjecture concerning the product of an even integer and an odd integer. Try to prove your conjecture.

5. Use factoring and your conclusion in (4) to construct another proof, one that does not use induction and which is different from the one you developed in (3), that $n^2 + n$ is even for every positive integer n.

6M. Counting Problems

7. Twelve runners compete in a race for which prizes are awarded for first place, second place, and third place finishers. In how many different ways could the prizes be assigned?

In addition to solving this problem, do both of the following:

a. Solving this problem may be viewed as determining the numerical value of $P(n, k)$ for specific numerical values of n and k. Explain why and identify the relevant values of n and k.

b. Most graphing calculators can compute $P(n, k)$ directly, as can a computer algebra system such as *Maple*. Figure out how either your graphing calculator or the *Maple* computer algebra system could be used to directly compute $P(n, k)$ for given values of n and k. Write a short paragraph describing the process as accurately and clearly as possible.

6P. Pascal's Triangle

4. A copy of David Burton's book, *The History of mathematics: An Introduction*, has been put on reserve in the library (there is a 3-hour limit to the time for which the book can be checked out and it cannot be removed from the library). Read the passage, "Pascal's Arithmetic Triangle" (pp. 429-440) and then answer the following questions.

a. Where and when did Blaise Pascal live?

b. Discuss two appearances of the triangular arrangement of binomial coefficients within the mathematics of non-European cultures before the time of Pascal.

c. Though Pascal cannot be credited with inventing the triangle that bears his name, what did he accomplish in his work Triangle Arithmetique that makes it reasonable to refer to the triangle as Pascal's Triangle? (You don't need to go into a lot of detail or specifics here; just give the gist of Pascal's accomplishment in this paper.)

A - Nice!

P

[illegible]

1. [illegible] compete in a race [illegible] awarded for first place, second place, and third place finishes. In [illegible] ways can [illegible] be assigned?

P(10,3) $\frac{10!}{(10-3)!}$ = [illegible] $\frac{}{7!}$ = [illegible] ✓

[illegible] the [illegible] may be viewed as determining the number [illegible] specific [illegible] n and k.

P(n,k) is the number of ways [illegible] choose [illegible] ✓

Why does this apply to this situation?

[illegible] graphing calculator [illegible] compute P(n,k) [illegible]

On my graphing calculator I was able to calculate the P(n,K) by hitting the Math button and then going to the probability and then #2, nPr. ✓ The number of total objects goes before this nPr and the amount that goes in order goes after. My answer was the same as #

6P(4)

a. Where and when did Blaise Pascal live?

Pascal lived during the mid 1600s in Europe. ✓

b. One other triangle approximated the value of a square root $\sqrt{a^n+r}$ by $r/(a+1)^n-r$ where the denominator was calculated by the binomial expansion. Another triangle was called The Precious Mirror of the Four elements and it was binomial coefficients throughthe eighth power. Written in rod numerals and a round zero sign. ✓

C. Pascal listed 19 properties of the binomial coefficients that could be discovered from the arithmetic triangle. Such as that each number in the triangle is the sum of the entries preceding it. The triangle allows people to computate $(x+y)^n$ for small values of n quickly. ✓

A it was for the first part of the question.