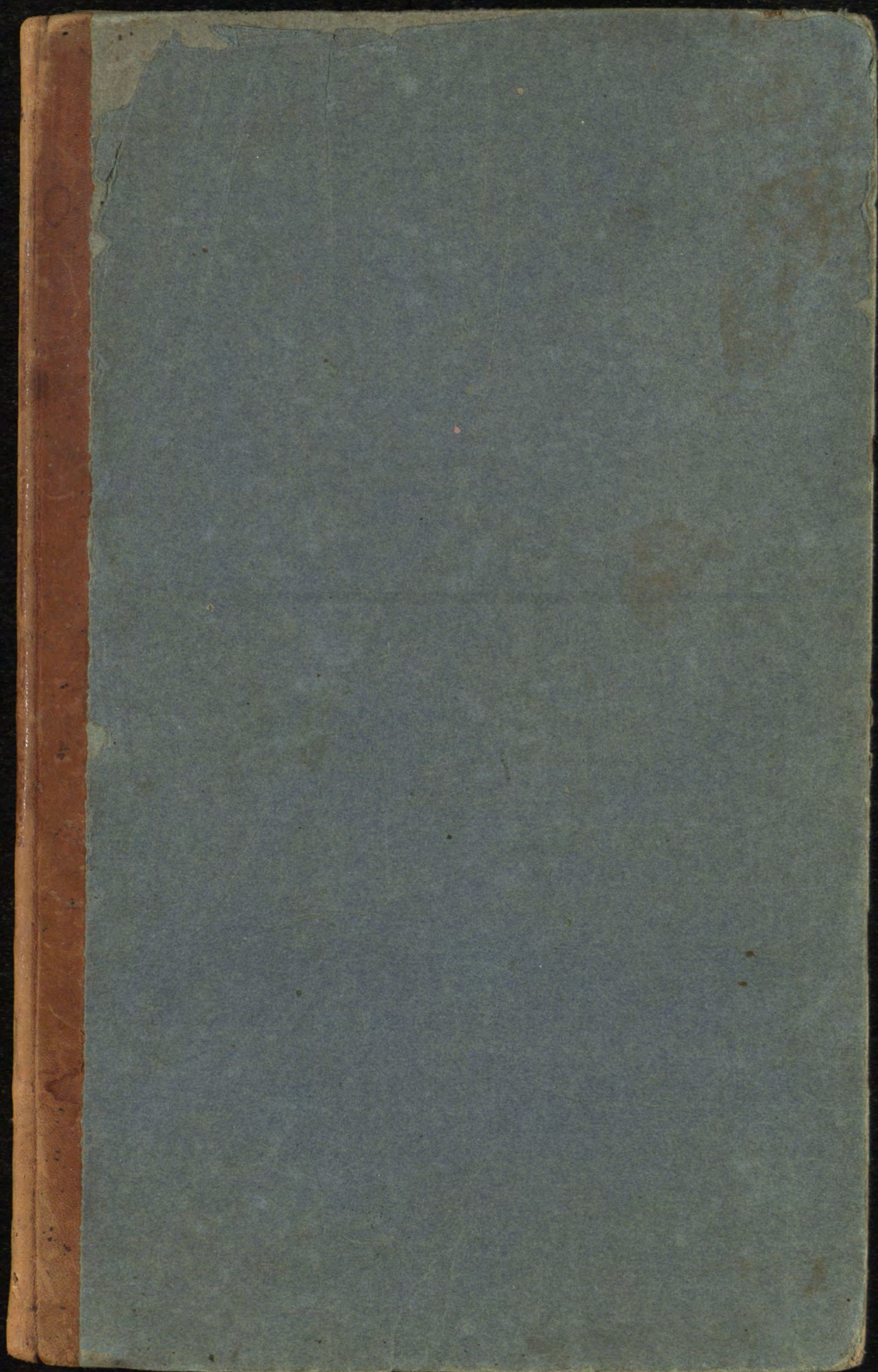
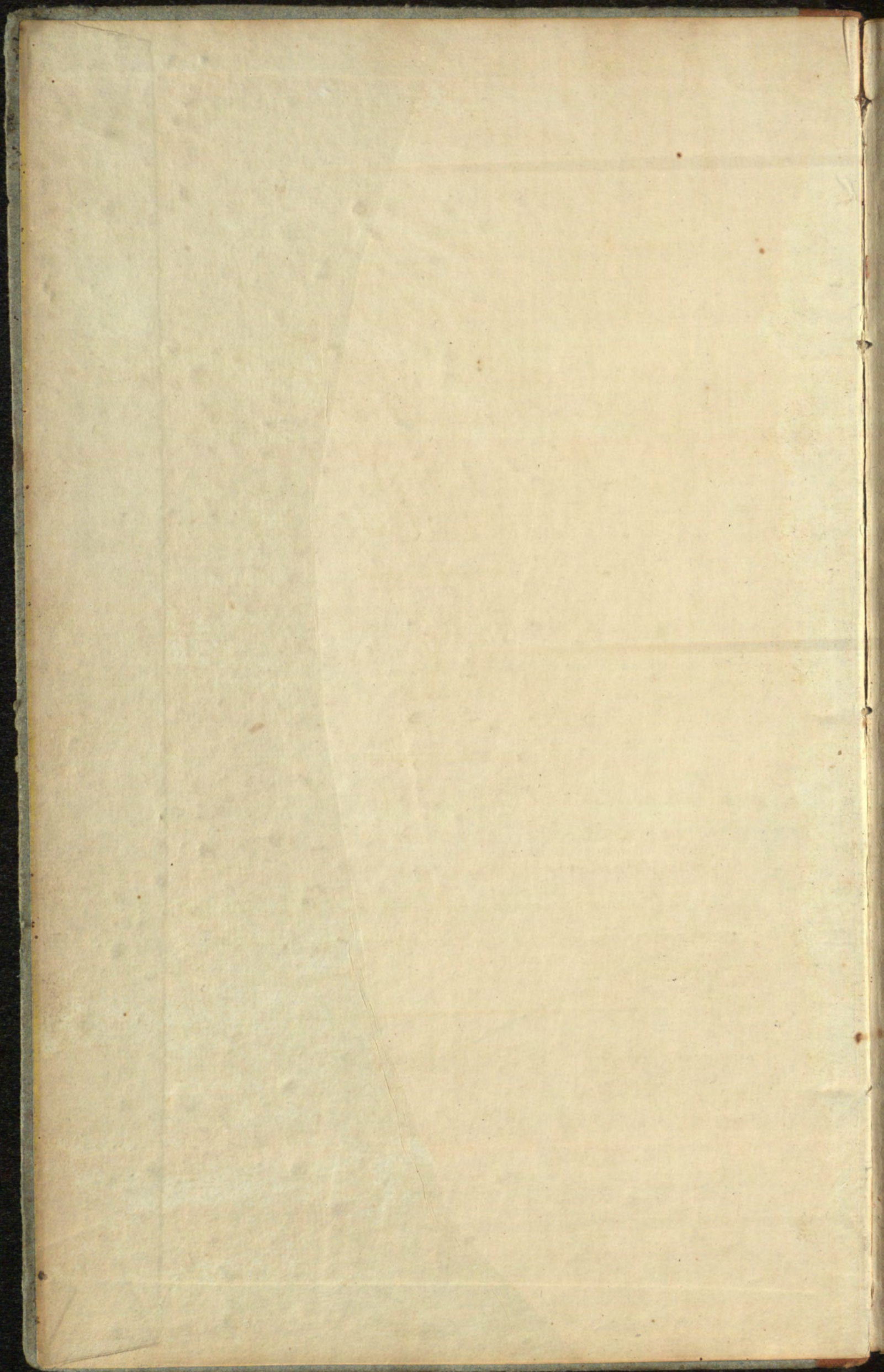






The
Gentlemen and Ladies'
ARITHMETICK,
Made Plain and Easy.
Being

A New and Improved System
Of that Science
For the Use of Schools and
Academies in the
United States.

By Dudley Leavitt.
Teacher of Mathematics, &c.

Intellectual or mental Arithmetick
is useful in reckoning small Num-
bers, but no one can be a good arith-
metician who does not learn to cipher
by rule as well as reason.

Published, &c.

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Preface.

Scholars learn to cipher as they learn to talk: the beginning, in both, is naturally simple and without rules. As learners improve in the knowledge of numbers by study, practice, and observation, some regular and uniform method of computation is found to be needed. The first steps in arithmetick, are to add, subtract, multiply, and divide small numbers in the mind, without the use of pen or pencil: this the scholar learns to do by degrees, without hardly knowing how, and such operations may be called natural arithmetick. The mind next begins to operate on large numbers in the same manner as on small ones, but here the memory and other intellectual faculties begin to give way, their powers being too limited for such comprehensive and complicated investigations, and it becomes necessary to write down the numbers:— Hence the origin of notation and numeration. Under this view of the subject the student also perceives that it is difficult to add, subtract, multiply, or divide, large numbers, without some rules by which the required result or answer can be obtained by finding a part at a time, instead of the whole at once. This consideration first suggested the idea of inventing the rules of arithmetick. As soon as the scholar has acquired a knowledge of the fundamental rules of the science, the operations on any numbers, however large, may be commenced, and it requires but little observation and reflection to show that the plainest and easiest rules should be studied first. For this reason whole numbers are treated of before fractions, and for the same reason decimal fractions should be learned ^{before} vulgar, except a general view of them.

In composing this treatise most of the different arithmeticks extant have been consulted and compared; from some of which ^{whose} copy rights have expired, the author has made several extracts, but the necessary alterations and amendments were so numerous and

Preface

various that the whole may be considered as new, both as to matter and form.

Should any ask why the author should publish another arithmetick since there are so many different ones already in use, he answers, it is because that no one now commonly used in schools contains so many rules as most students of arithmetick seem to wish, and the ^{reason of the} rules which are given is either not explained at all, or not so clearly as to be easily understood. In this treatise the author has inserted more rules than are contained in any other arithmetick in common use, and endeavoured to explain the reason of them. A sufficient number of easy questions are given at the beginning of each rule to explain the nature of it, with the method of solving questions mentally. Examples are given both in sterling or lawful, and Federal Money, but principally in the latter. The scholar will likewise find a variety of useful and curious miscellaneous questions with rules for solving them; also rules for measuring planes and solids. Chronological and astronomical problems, and so forth. The arrangement of the rules is such as appears to be adapted to the natural progress of the mind from one rule to another, but may be varied as the teacher or student may find it convenient.

Within a few years some authors and teachers have attempted to revive the ancient methods of acquiring and imparting knowledge. As instances of this, the late Lancastrian system of education is one, and the inductive or intellectual method of learning arithmetick without rules, is another. These methods, though generally considered as new, appear to have had their origin from an old writing and arithmetical machine called by the ancient Greeks the Abax; by the Romans Abacus; and by the Chinese the swan-pan, or computing table. In the old Grecian schools boys learned to write by strewing the Abax or smooth piece of board with green sand, (the pulvis cridatus of classic authors) and tracing letters on it with a small rod or wooden pin. They learned to cipher by it by forming rows of counters of small pebbles, pieces of bone, or ivory,* and counting or moving them different ways, on the same principle that some of our modern authors would teach arith-

* See Curiosities for the Ingenious. Art. Palpable Arithmetic.

Preface.

metick by using "beans, nuts, &c.", instead of figures! — [^{Mr.} Wade Colburn's Arithmetic.] These methods, in remote ages, were better than none, and are still so to the Chinese, and others who have not the capacity of learning any other way, but in the present enlightened age they ought to be laid aside by all who possess better means of acquiring knowledge. — Some of these new-fashioned modes of learning may please children, and no wonder, for they are generally pleased with confusion. The task of solving all questions in the Rule of Three, Position, &c., by general principles instead of by rule, is like a farmer's breaking up his ground with a hoe, when he could do it much easier and sooner with a good plough.

It has been supposed by some, that arithmetick is a proper study for children five or six years of age, but in my opinion all the arithmetick necessary for such scholars is contained in the Spelling-Book, in the table of numbers, which will inform them how to know the chapters and verses in the Bible; the pages of a book; the day of the month; and the date of the year. I would not, however, be understood to discourage any attempt for the advancement of the knowledge of arithmetick, in any form whatever, as almost every treatise on the subject contains something new and useful.

Meredith, March 31, 1831.

The Author.

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Explanation of Arithmetical Characters.

$=$ Is the sign of equality: thus, 2 and 3 = 5; 12 pence = 1 shilling.

$+$ Is the sign of addition: thus, $5 + 4 = 9$, signifies that 5 added to 4 are equal to 9.

$-$ Is the sign of subtraction: thus, $9 - 5 = 4$, signifies that 9 less 5, is equal to 4; or, that if 5 is subtracted from 9, the remainder is 4.

$\times$ Is the sign of multiplication: thus, $4 \times 5 = 20$, signifies that 4 multiplied by 5, is equal to 20; or, if 4 is multiplied by 5, the product is 20.

$\div$ Is the sign of division: thus, $20 \div 4 = 5$, signifies that 20 divided by 4, is equal to 5; or, that if 20 is divided by 4, the quotient, or number of times that 4 is contained in 20, is 5. Also,

$4)20($ shows that 20 is to be divided by 4.

$\frac{20}{4}$ likewise signifies the same as $4)20($ or $20 \div 4$; the upper number shows it to be the dividend, or number to be divided, and the lower number signifies the divisor, or number to divide by.

$:$ Is the sign of proportion: thus, $3:4::6:8$, signifies as 3 is to 4, so is 6 to 8.

$\divdiv$ Is the sign of arithmetical progression.

$\div\div\div$ Is the sign of geometrical progression.

$7 - 2 + 4 = 9$, shows that the difference between 7 and 2, added to 4, is equal to 9, which by signs, thus; 7 minus 2 plus 4 is equal to 9. Minus signifies less, and plus signifies more. The line drawn over the 7 and 2, is called a vinculum.

$8 - 2 + 5 = 1$, shows that the sum of 2 and 5 subtracted from 8, leaves or is equal to 1.

7^2 signifies the square, or second power.

7^3 signifies the cube, or third power.

7^m signifies either the square or cube, or any other power.

$\sqrt{\quad}$ or $7^{\frac{1}{2}}$ signifies the square root.

$\sqrt[3]{\quad}$ or $7^{\frac{1}{3}}$ signifies the cube root.

$\sqrt[m]{\quad}$ or $7^{\frac{1}{m}}$ signifies some root, that is any root whatever.

When several letters are set together, they are considered to be multiplied into each other: thus,

$a b c d$ signify the same as $a \times b \times c \times d$. If a be a root, then a^2 or $a a$ ($= a \times a$) is the square of a and a^3 or $a a a$ ($= a \times a \times a$) is the cube of a , &c. The figure above an a or any other letter is called the index of the power.

$4/6$ or $4/6$ signifies 4 shillings and six pence. $13/8$ is 13s. 8d., &c.

$\$4$ signifies 4 dollars; $\$4, 25$ is 4 dollars and 25 cents; $\$4, 25, 4$ is 4 dollars and 25 cents and 4 mills. When there are figures after mills, they are decimals of a mill. All this should be well understood.

ARITHMETICK

Made Plain and Easy.

Arithmetick* is the art or science of computing by numbers†, and is so necessary in business that scarcely any thing in the common affairs of life, and nothing of a commercial nature can be transacted without it. Arithmetick consists in theory and practice. The theory or science comprises the knowledge of the principles and rules. The practice or art consists in the application of the principles and rules to the operation of computing by numbers, and in the solution of arithmetical questions. Arithmetick comprises five principal or fundamental rules, viz. Numeration, Addition, Subtraction, Multiplication, and Division. Numeration or Notation teaches to read and write numbers. Addition teaches how to collect several numbers into one sum total. Subtraction teaches how to find the difference between two numbers. Multiplication teaches how to multiply or increase one number by another. Division teaches to find how often one number is contained in another. All the operations of arithmetick are performed by the means of the ten following characters, either separate or combined.

One, two, three, four, five, six, seven, eight, nine, cipher.
1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The first nine characters are called digits, probably from the Lat. in digitus, a finger, because numbers were formerly reckoned by the fingers. The cipher takes its name from the Hebrew word keper, a number, or pertaining to number; whence all the figures or characters were formerly called ciphers, and the art of computing by numbers was called ciphering.

* From the Greek arithmos, number.

† A number is either 1 (a unit) or more than 1, and by the continual addition of 1 to itself, all numbers are generated.

N U M E R A T I O N.

When the value of a quantity is nothing, it is expressed by a cipher. Any number not less than one, without any parts belonging to it, is called a whole number.^s A quantity less than one, is properly called a fraction.^t Common or vulgar fractions are expressed in the following manner.

one fourth. $\frac{1}{4}$	one third. $\frac{1}{3}$	one half. $\frac{1}{2}$	two thirds. $\frac{2}{3}$	three fourths. $\frac{3}{4}$	four fifths. $\frac{4}{5}$	seven eighths. $\frac{7}{8}$
$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{7}{8}$

$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}, &c.$

N U M E R A T I O N.

Numeration teaches to read and write numbers according to their proper values. In whole numbers, a cipher with no other figure to the left hand of it, signifies nothing, but at the right hand of any number it makes that number ten times more; thus 1 signifies one, and 10 signifies ten. Each of the nine digits, separately considered, signifies such a number as its name expresses; thus, 1 is one; 2 are two, &c. But when the value of two or more figures is read as one number, the value of any figure to the left hand of another, is ten times greater than if it were placed alone; thus, 9 signifies nine, but if any other figure is placed to the right hand of 9, then 9 signifies ten times 9, or ninety; and so of any other figure.

In order to read the proper or separate value of figures, nothing more is necessary than to learn their names: but to read the combined value of many figures taken collectively, as that of each increases in a tenfold proportion towards

^s Whole numbers are also called integers, from the Latin integer, whole, or unbroken.

^t Fractions take their name from the Latin frango, to break.

NUMERATION

the left hand, an arrangement of figures in a form called a Numeration Table is necessary to assist the learner.

NUMERATION TABLE.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Hundreds.	Tens.	Units.	
							1	= One.
						2	1	= Twenty-one.
						3	2	1 = Three hundred and twenty-one.
						4	3	2 1 = Four thousand 321.
						5	4	3 2 1 = Fifty-four thousand 321.
						6	5	4 3 2 1 = Six hundred and 54 thousand 321.
						7	6	5 4 3 2 1 = Seven million 654 thousand 321.
						8	7	6 5 4 3 2 1 = 87 million 654 thousand 321.
						9	8	7 6 5 4 3 2 1 = 987 million 654 thousand 321.

Read as follows.

Explanation of the Table:—The right hand figure is units, the next towards the left hand is tens; the 3rd is hundreds; the 4th is thousands; the 5th is tens of thousands; the 6th is hundreds of thousands; the 7th is millions; the 8th is tens of millions; and the 9th is hundreds of millions. The places of units, tens, &c., are always the same, let the figures in the table be what they may, as in the following more extensive table.

Hundreds of thousands of trillions.	Tens of thousands of trillions.	Thousands of trillions.	Hundreds of trillions.	Tens of trillions.	Trillions.	Hundreds of thousands of billions.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of billions.	Billions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
5	8	7	6	5	4	3	2	1	3	4	7	6	9	8	7	5	4	3	4	5	1	2	3

Six figures is called a period, and three figures half a period.

N U M E R A T I O N.

To numerate (or enumerate) the figures in the last table, begin, as in the first, at the units, and give each figure its relative name, as tens, hundreds, &c., to the last; then the whole value will be read thus; 587654 trillion, 321347 billion, 698754 million, 345 thousand, 123. — From these examples the learner will soon be able to enumerate all numbers to any necessary extent. The first table is sufficiently extensive for common use.

The Reason of the Method of enumerating.

The reason why the operation of enumerating is performed as it is, is because that the established law of simple numbers requires that the value of any figure expressed by its name, shall increase ten times by being removed one place to the left of units, and because 10 ones are 10; 10 tens 100, &c.

R O M A N N O T A T I O N.

1 - I.	11 - XI.	30 - XXX.
2 - II.	12 - XII.	40 - XL.
3 - III.	13 - XIII.	50 - L.
4 - IV.	14 - XIV.	60 - LX.
5 - V.	15 - XV.	70 - LXX.
6 - VI.	16 - XVI.	80 - LXXX.
7 - VII.	17 - XVII.	90 - XC.
8 - VIII.	18 - XVIII.	100 - C.
9 - IX.	19 - XIX.	500 - D.
10 - X.	20 - XX.	1000 - M.

Note 1st. The value of any character is increased as often as the character is repeated; as for example, II. represents 2. III. represents 3. — IIII. represents 4. — XXX. represents three tens or 30. — CC. two hundred, &c.

2d. A less character before a greater decreases the value of the greater; thus, IV. stands for one less than five, i. e. 4. — IX. stands for one less than ten, i. e. 9. — XC. stands for ten less than 100, i. e. 90.

3d. Every C affixed to I (500) increases the value ten times; thus, IC are equal to 5000.

4th. Every C and I put one at each end of CI (1000) increases the value ten times; thus, CCIC = 10000; and CCCI = 100000.

5th. A line over any number increases its value a thousand times; thus, V. (5) by putting a line over it $\overline{V}$ becomes 5000, and $\overline{X}$ = 10,000, and $\overline{MM}$ = 2000,000, &c.

The date of the present year, (1831) stands thus, according to the Roman Notation.

M D C C C X X X I.

To read or enumerate Numbers.*

Rule. To the simple value of each figure, join

S I M P L E A D D I T I O N.

the name of its place, as hundreds, thousands, &c., beginning at the left hand, and reading towards the right.

Examples.

1. How is 347 enumerated? Ans. Three hundred and forty seven.

2. How is 6758 enumerated? Ans. Six thousand and seven hundred and fifty eight.

3. How is 957605 enumerated? Ans. Nine hundred and fifty seven thousand, six hundred and five.

To write Numbers.*

Rule. Write the figures in the same order as they are read.

Examples.

1. How should ninety four be written? Ans. 94.

2. How should three hundred and twenty five be written? Ans. 325.

Questions for Examination.

1. What is Arithmetick?

2. What is the theory or science of arithmetick?

3. What is the art or practice of arithmetick?

4. Which are the principal or fundamental rules of arithmetick?

6. What does Numeration teach?

7. What do Addition, Subtraction, Multiplication and Division teach?

8. How do you read or enumerate numbers?

9. How do you write numbers?

Ans. By the questions for examination, the instructor may examine the scholars, or the scholars may examine themselves, by way of review. Both are useful and necessary.

S I M P L E A D D I T I O N.

** Properly speaking, reading numbers is Numeration, and writing them is Notation.

S I M P L E A D D I T I O N .

Simple Addition teaches to collect two or more numbers of the same denomination into one number, called the sum.

Questions explaining the Nature of Addition, to be performed mentally, by Reason only.

1. How many are 5, 3, 7, and 9? Answer, 24. Because 5 and 3 are 8, and 7 are 15, and 9 are 24, the whole sum.

2. Three boys went a fishing; one caught 10, another 15, and another 20. How many did they all catch? Answer, 45. Because 10 and 15 are 25, and 25 and 20 are 45.

3. Four scholars bought some paper; one bought 6 sheets, another 10, another 18, and another 24. How many sheets did they all buy? Answer, 58. Because, since 18 are 10 and 8, and 24 are two tens and 4, say 6 and 10 are 16, and 10 are 26, and 8 are 34, and 10 are 44, and 10 are 54, and 4 are 58.

Rule.

1. Place the numbers under each other so that units may stand under units, tens under tens, &c., and draw a line under them.

2. Add the figures in the column of units, and find how many tens are contained in their sum.

3. If their sum is any even number of tens set down 0 under the column, and carry as many as there ^{are} tens to the next column, but when the sum is not an even number of tens, set down all there are more than ten or the number of tens, and carry as before, and proceed in the same manner with every column, but set down the whole amount of the last column.

Proof. Draw a line under the upper line of figures, and add all the others again. Then add the last sum to the upper line, and if this third

SIMPLE ADDITION.

sum is the same as the first, the work is right.*

Reason of the Rule.—The reason why 1 must be carried for every 10, is because 10 units or ones make 10; 10 tens 100, &c. And the reason why the whole sum of the last column must be set down, is because there is no column to carry to, from the last.

Examples.

1.

$$\begin{array}{r} 4789 \\ 8658 \\ 1341 \\ 7864 \\ 2135 \end{array}$$

24787 Sum.

19998

24787 Proof.

Explanation.

In adding up these numbers, I begin with the unit column, and say 5 and 4 are 9, and 1 are 10, and 8 are 18, and 9 are 27, which is 7 more than 2 tens, so I set down 7, and carry 2 to the next column, and say 2 and 3 are 5, and 6 are

11, and 4 are 15, and 5 are 20, and 8 are 28, which is 8 more than 2 tens, therefore I set down the 8, and carrying 2, say 2 and 1 are 3, and 8 are 11, and 3 are 14, and 6 are 20, and 7 are 27, so setting down 7 and carrying 2, I say 2 and 2 are 4, and 7 are 11, and 1 are 12, and 8 are 20, and 4 are 24, which, as it is the sum of the last column, I set down the whole 24. In the same manner I add all but the upper line, and the sum is 19998, which added to the upper line makes the same sum as the first sum, which proves that the work is right. — All other sums are added up and proved in the same manner.

Remark.—The upper line is not cut off till after the first sum is found.

* This is the method of proof commonly used, but there is another, simple and very ingenious method which was first given by Dr. Wallis, in his *Arithmetick* published A. D. 1657. The method is by casting out the nines, thus.

1. Cast the nines out of each line of figures added, and set the excesses against the respective lines, and find the

SIMPLE ADDITION.

2.
 8 7 9 4 3
 6 5 8 5 7
 3 4 1 4 2
 9 7 8 4 6
 0 2 1 5 3
 8 6 7 5 4
1 3 2 4 5

3.
 3 7 6 9 5
 8 4 7 6 3
 1 5 2 3 6
 8 4 7 9 7
 1 5 2 0 2
 7 3 4 7 5
2 6 5 2 4

4.
 2 7 5 8 7
 6 4 7 5 3
 3 5 2 4 6
 8 4 3 7 5
 1 5 6 2 4
 3 8 7 6 5
6 1 2 3 4

5.
 3 9 8 7 6
 5 4 3 6 7
 4 5 6 3 2
 8 3 2 4 6
 1 6 7 5 3
 3 7 5 4 6
6 2 4 5 3

6.
 5 7 8 9 4
 2 8 7 5 6
 7 1 2 4 3
 8 7 9 5 6
 1 2 0 4 3
 7 5 3 8 7
2 4 6 1 2

7.
 8 7 6 4 7
 3 8 7 5 6
 6 1 2 4 3
 8 3 7 6 5
 1 6 2 3 4
 5 6 7 5 8
4 3 2 4 1

Questions For Exercise.

1. How many are 536, 387, 612, 875, 124, 632, and 367, all added together?

Ans. 3573.

sum of all the excesses. Then cast the nines out of the sum of the excesses, and also out of the sum of the given numbers, and if the two last remainders are equal, the work is right, if all the numbers are accurately set down. Thus, in this example which is the same as the first, I begin with the upper line, and say 4 and 7 are 11, which, casting away the 9, leaves 2; then 2 and 8 are 10, which, less 9 is 1, which place against the first line, and

Example.

4	7	8	9		1
8	6	5	8		0
7	3	4	1		0
7	8	6	4		7
2	1	3	5		2
<u>24787</u>					1

S I M P L E A D D I T I O N .

2. Eight men pay taxes as follows. A pays \$5, B \$7, C \$9, D \$15, E \$24, F \$3, G \$8, and H \$21. How much is the sum of all their taxes? Ans. \$92.

3. Supposing the distance from Portland, in the State of Maine, to Portsmouth, in New-Hampshire, to be 60 miles; from Portsmouth to Boston, 65 miles; from Boston to New-Haven, 162 miles; from New-Haven to Philadelphia, 95 miles; from Philadelphia to Baltimore, 102 miles; from Baltimore to Richmond in Virginia, 176 miles; from Richmond to Charleston in South Carolina, 540 miles; and from Charleston to Savannah, 119 miles, what is the whole distance from Portland to Savannah? Ans. 1407 miles.

4. A man borrowed of his neighbour, \$15 at one time; \$28 at another; \$105 at another, and \$58 at another time. How much did he borrow in the whole?

Ans. \$196.

5. Write the twelve months under each other, with the number of days in each month; and find how many days are contained in the year, allowing thirty days each for the months April, June, September and November, twenty eight days for February; and thirty one days for each of the other months. Ans. 365 days.

6. Celestial observations were first made at Babylon two thousand two hundred and thirty four years before Christ; and Sir Isaac Newton, the famous English Astronomer died Anno Domini, one thousand seven hundred and twenty seven, what number of years elapsed between those two periods? Ans. 3964.

7. Homer, the famous Greek Epic Poet flourished nine hundred and seven years before Christ, and Alexander Pope his elegant English translator died Anno Domini one thousand seven hundred and forty four; tell the distance of time. Answer 2651 years.

8. Virgil the prince of Latin poets died nineteen years before Christ and Dryden his famous English translator Anno Domini, one thousand seven hundred and one; tell the distance of time. Answer 1720 years.

9. The Roman emperor Julius Cæsar was killed by Brutus in the senate house forty four years before Christ, and Louis the 16th, King of France, was beheaded Anno Domini one thousand seven hundred and ninety three; what distance of time between the two events? Answer 1837 years.

in the same manner proceed with all the other lines, and the excesses are 1, 0, 0, 7, 2, the sum of which is 10, which, after rejecting 9, leaves 1 for the remainder. Then casting the nines out of the sum total, as out of the five given lines, the remainder is also 1, and as the two last remainders are equal, the work is proved to be right.

Reason. The reason why this method proves the work, is because if any number is divided by 9, it leaves the same remainder as the sum of its digits or figures divided by 9. This property belongs only to 9 and 3.

SIMPLE SUBTRACTION.

SIMPLE SUBTRACTION

Teaches to take a less number from a greater of the same denomination, and thereby to find the difference. The less number is called the subtrahend, and the greater number is called the minuend. The difference of the two numbers is called the remainder.

Questions explaining the Nature of Subtraction, to be solved mentally.

1. What is the difference between 10 and 15?

Ans. 5.

2. What is the difference between the ages of two persons both born on the first day of January, but one in the year 1810, and the other in 1820?

Ans. 10 years.

3. John drew \$5 in a lottery, William \$15, and James \$50. How many dollars did James draw more than John and William both?

Ans. \$30.

Rule.

1. Place the less number under the greater, so that units may stand under units, tens under tens, &c., and draw a line under them.

2. Begin at the right hand, and take each figure in the subtrahend from the figure which is over it, and set the remainder under the line. If the lower figure is greater than the one over it, add 10 to the upper figure, and then subtract, set down the remainder, and carry one to the next figure in the lower line, and proceed as before till the whole is subtracted.

Proof.—Add the remainder and less number together, and if the sum be equal to the greater number, the work is right.

Reason of the Rule.—The reason of the operation is plain at first view, when the lower figure is less than the upper. The reason why the upper figure is increased by 10, when it is less than the lower figure, is because 1 is borrowed from the next upper figure on

S I M P L E S U B T R A C T I O N.

The left hand, which is equal to 10 in the column you are subtracting in, because 1 ten is equal to 10 units, &c. And the reason why 1 must be carried to the next lower figure every time you borrow one, or increase the upper figure by 10, is because ^{that} after one is borrowed or taken from the upper figure, it is one less than it appears to be; therefore the figure under it must be made or supposed to be one more than it really is, so as to keep the balance right between the upper and lower figures. Thus, if you would subtract 19 from 21, as you cannot take 9 from 1, you must borrow 1 from the 2 in 21, which one ten is 10 units, which add to the 1 over the 9 and the sum is 11, from which subtract 9, and 2 remains. Now since you took one from the 2 in 21, the 2 is in reality but 1 in value, though it appears the same as before one was borrowed from it, therefore the one borrowed from 2 in 21 must be carried to the 1 in 19, to make that appear as much more than it is as 2 in 21 appears more than it is after one is taken from it.

Examples.

$$\begin{array}{r}
 \text{From } 347568 \\
 \text{take } 124704 \\
 \hline
 \text{Diff. } 222864 \\
 \text{Proof } 347568
 \end{array}$$

Explanation.
Beginning at the units' place, I say 4 from 8, and 4 remains, which is set down; then 0 from 6 leaves 6; then

as 7 cannot be subtracted from 5, add 10 to 5 and it makes 15, from which take 7, and 8 remains, which is set down, and as 5 was increased by 10, one is carried to 4, the next left hand figure, which makes it 5, which subtracted from 7 leaves 2. Also, 2 from 4, and 1 from 3, leaves 2 each; and as the whole difference added to the least number is equal to the greater number, the work is proved to be right. — See the remark after the last example.

S I M P L E S U B T R A C T I O N.

2.
From 98745679
take 45837453

3.
From 54876984
take 23587643

4.
From 34768947
take 10894725

5.
From 98769845
take 37894521

6. The Independence of the United States was acknowledged in 1783, and the Bastille was demolished in France in 1789, what distance of time between those two events?

Answer 6 years.

7. General Washington was born in 1732, how old is he this present year being 1797?

Answer 65.

8. The Mariners compass was invented in one thousand three hundred and two, and America was discovered in one thousand four hundred and ninety two.---How many years between the discoveries?

Answer 190.

9. A owes B four hundred and eighteen thousand nine hundred and sixty five dollars. B. owes A. twenty seven thousand five hundred and sixteen dollars. What balance is due to B?

Answer 391449 dollars.

10. The population of the United States is estimated at about four million two hundred thousand inhabitants the population of France at about twenty four millions five hundred thousand, what is their difference?

Answer 20,300,000.

11. The distance of the Earth from the Sun is about ninety four millions four hundred and seventy four miles, the distance of the Herschel (the newly discovered planet) is about one thousand seven hundred and eighty three millions six hundred and ninety eight thousand two hundred and forty one miles.---How much does Herschel's distance exceed that of the earth?

Answer 1,689,697,767 miles.

12. The diameter of the Sun is computed at eight hundred & seventy seven thousand five hundred & forty seven miles, the diameters of the seven primary planets as follows---MERCURY three thousand one hundred & eighty nine miles---VENUS, seven thousand six hundred & nine; the EARTH, seven thousand nine hundred & twenty eight; MARS, five thousand, one hundred & ninety five; JUPITER, Ninety two thousand four hundred & fourteen, SATURN, seventy eight thousand two hundred & thirty six; HERSCHEL, thirty three thousand, nine hundred & fifty four---How much does the Sun's diameter exceed the diameters of all the planets put together? Answer 649,022 miles.

Remark.--- Besides the method of proving Subtraction by adding the least of the two numbers and their difference together, if the mines be cast out of them all, the excess of the greater will be equal to the excess of the sum of the excesses of the other two, if the work be right.

S I M P L E M U L T I P L I C A T I O N.
Simple Multiplication teaches to increase or multiply one number by another as often as there are units in either of the numbers, and is a short method of performing many additions.

1911

The number to be multiplied, is called the multi-
licand.

The number to multiply by, is called the multi-
plier.

The answer or number found by the opera-
tion, is called the product.

The multiplicand and multiplier are, in
general, called factors.

Before much can be done at multiplication,
either by rule or reason, the learner must
commit to memory the following

Multiplication and Division Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn the table for multiplication:

Begin with the figure 2. The second figure in the
left hand column, and multiply every fig-
ure in the upper line by it. Thus twice 2 are 4,
twice 3 are 6, and so to the rest. Then begin with 3,
and say 3 times 2 are 6, 3 times 3 are 9, and
proceed in the same manner with every
multiplier from 2 to 12 in the left hand
column.

Use of the Table in Division

Find the Divisor in the left hand column, and
the Dividend in the same line, then over the
figure in the upper line will be the quo-

EXAMPLE MULTIPLY AND DIVIDE.

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2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn the Table for Multiplication,
Begin with the figure 2, the second from the top
in the left hand column, and multiply every fig-
ure in the upper line by it, thus; twice 2 are 4,
twice 3 are 6, &c., to the last. Then begin with 3,
and say 3 times 2 are 6, 3 times 3 are 9, &c., and
proceed in the same manner with every
multiplier from 2 to 12 in the left hand
column.

Use of the Table in Division.

Find the Divisor in the left hand column, and
the dividend in the same line; then over the
dividend, in the upper line, will be the quo-
tient.

SIMPLE MULTIPLICATION.

To multiply one number by another.

Rule.

1. Place the multiplier under the multiplicand, so that units may stand under units, &c. Then,

2. If the multiplier does not exceed 12, after drawing a line under the numbers, begin at the right hand, and multiply every figure of the multiplicand by the multiplier, setting down and carrying the same as in addition.

3. When the multiplier exceeds 12, multiply by one figure at a time, placing the first figure of each line under the figures you multiply by, and after multiplying the whole, add all the several products together, and the sum will be the total product required.

Proof.—Multiply the multiplier by the multiplicand, and if the second product is the same as the first, the work is supposed to be right. *

Reason of the Rule.—The reason why multiplying one number by another will produce another number as many greater than the multiplicand as the multiplier is greater than 1, is because multiplying by any number is the same in effect, as setting or writing down ^{are} the multiplicand as many times as there are units in the multiplier, and adding all the lines so repeated together. Thus, the product of 5 multiplied by 6 is 30, which is the

* Multiplication may also be proved by casting out the nines, thus; first cast them out of the multiplicand, and then out of the multiplier, and multiply the two excesses together, and cast the nines, if any, out of their product, and reserve the excess. Then cast the nines out of the total product or answer, and if the excess is equal to the reserved excess, the work will be right if the figures are properly set down.

SIMPLE MULTIPLICATION.

same as the sum of five sixes set down under one another and added together. By this it is plain that when the multiplier is small, addition will answer the purpose of multiplication very well, but when the multiplier is a very high number, the operation by addition would be a tedious and almost endless process, whereas by multiplication, the product of the highest numbers is obtained with great ease and expedition.

The reason why the partial products fall one place to the left hand, one after another, is because that when you multiply by a figure in the place of tens, the product is 10 times the value as if you multiplied by the same figure in the place of units, and it will be recollected that the place of tens is one figure to the left of units, which explains the whole operation.⁺

Examples.

(1)

Multiply 3785
by 2

Product = 7570

$$\begin{array}{r} 2 \\ 3785 \\ \hline 10 \\ 16 \\ 14 \\ \hline \end{array}$$

Proof. 7570

Explanation.

First, twice 5 are 10, so 0 is set down, and 1 carried; then twice 8 are 16 and 1 carried is 17, of which the 7 is set down and the 1 ten is carried; then twice 7 are 14 and 1 to carry is 15, of which the 5 is set down, and 1 is carried as before; lastly, twice 3 are 6, and 1 carried is 7. Then the multiplier 2, is multiplied by the multiplicand 3785; thus, 5 times 2 are

⁺ If any wish to see this explained by an example, the following may suffice. Let any number, suppose 325 be multiplied by any other number, suppose 111; then the work will stand as here prefixed. Here 325 is first multiplied by 1 unit, and the product is 325; the same number is next multiplied by 1 ten, and the product is 10 times 325, or 3250; then 325 is multiplied by 1 hundred, and the product 100 times 325, or 32500, and the sum of all three numbers is 36075, the total product. It is not usual to annex a cipher to a partial product when we multiply by a ten, or two ciphers when we multiply by a hundred, &c., but only to suppose them annexed.

$$\begin{array}{r} 325 \\ 111 \\ \hline 325 \\ 3250 \\ 32500 \\ \hline 36075 \end{array}$$

SIMPLE MULTIPLICATION

10, and 8 times 2 are 16, and 7 times 2 are 14, and 3 times 2 are 6, and all these placed and added up according to rule, make 7570, which is equal to the first product, and proves the work to be right. The proof by casting out the nines, is as follows.

First, cast the nines out of the multiplicand, thus; 3 and 7 are 10, out of which cast the 9, and 1 remains; then 1 and 8 are 9 which cast away leaves 0, ~~so~~ the next figure 5 is the excess of the nines. Secondly, as the multiplier 2 is less than 9, there is none to cast away, therefore if the excess of nines in the multiplicand, which in this case is 5, is multiplied by 2, the product is 10, which reserve. Then cast the nines out of the product, (7570) and the excess is 4, the same as that reserved, which proves that the work is right. This last is the shortest way of proving multiplication. After learning Division it is an accurate method of proving multiplication by dividing the product by the multiplicand, and if the work is right the quotient will be the multiplier.

(2)

Multiply 32745654473
by 234

130982617892
98236963419
65491308946

Product 7662483146682

Explanation.

Here the multiplicand is multiplied by 4, and carrying 1 for every 10 as in addition, and setting down all more

than the tens. In the same manner the multiplicand is multiplied by the other two figures, one at a time, because the multiplier exceeds 12. The first figure of each of the three partial products is placed under the figure multiplied by, according to rule, and the sum of all three is the total product. To prove this work by casting out the nines, the excess of nines in the multiplicand is 5, and that of the multiplier is 0, and 5 times 0 is 0; the excess of nines in the product is also 0, therefore the work is right.

3. Multiply 84356745674 by 5. Product 421783728370.

4. Multiply 3274656461 by 12. Prod. 39295877532.

5. Multiply 273580961 by 23. Prod. 6292362103.

S I M P L E D I V I S I O N.

6. Multiply 527535 by 15728. Prod. 8297070480.
7. Multiply 24393 by 402. Prod. 9805986.
8. Multiply 7864371 by 23604. Prod. 185630613084.

Contractions in Multiplication.

1. When the Multiplicand or Multiplier, either or both, have ciphers annexed, they may be neglected in multiplying, and annexed to the product.

2. When the multiplier is such a number that if two less numbers be multiplied together, their product will make the whole multiplier, if the multiplicand be first multiplied by one of the numbers, and the product of that by the other, the last product will be the true answer; the reason of which is plain, after multiplying.

Examples.

$$\begin{array}{r}
 1. \text{ Multiply } 1234500 \\
 \text{by } 7500 \\
 \hline
 61725 \\
 86415 \\
 \hline
 \text{Product } 9258750000
 \end{array}$$

$$\begin{array}{r}
 2. \text{ Multiply } 3748 \text{ by } 24. \\
 4 \times 6 = 24. \\
 \hline
 14992 \\
 6 \\
 \hline
 \text{Prod. } 89952
 \end{array}$$

Explanation. In the first example the operation of multiplying is performed the same as if the factors had no ciphers annexed, and as many ciphers as are annexed to both are annexed to the product.

In the second example 4 multiplied by 6 makes 24, the whole multiplier, therefore multiplying by 4 produces 4 times 3748, and 14992 multiplied by 6, makes 6 times 4 times or 24 times 3748.

Questions further explaining the Nature and Use of Multiplication.

1. How much will 51 pounds of sugar come to at 15 cents a pound, 100 cents being a dollar? Answer. 765 cents, or 7 dollars and 65 cents, which is found by multiplying 51 and 15 together.
2. If a man travel 50 days, at the rate of 20 miles a day, how many miles will he travel in the whole? Answer. 1000 miles.
3. A clock strikes 156 times in 24 hours; how many times does it strike in a common year of 365 days? Answer. 56940 times.

S I M P L E D I V I S I O N.

Simple Division teaches to find how often

SIMPLE DIVISION

one number is contained in another of the same denomination. The number ^{to be} divided is called the dividend. The number ^{to divide by} is called the divisor. The number of times the divisor is contained in the dividend, is called the quotient; and the number left after dividing is called the remainder, which is always less than the divisor, when the work is right.

Easy Questions explaining the Nature and Use of Division.

1. How many fours are there in 20? Ans. 5.
2. How many ones are there in 15 fifths? Ans. 3.
3. How many times 5 are there in 40 eighths? Ans. one 5.

Remark.—It is an easy matter to answer questions in division mentally, when the numbers concerned are very small, but when the numbers run high it is difficult and perplexing, if not impossible, to find, at one step, either of the mind or pencil, how many times one number is contained in another. A method called a rule has therefore been invented by which the quotient, though ever so large, may be found with the greatest ease, by finding a part at a time.

Rule.

1. Draw a curved line on the right and left hand of the dividend, and set the divisor on the left.
2. Find how many times the divisor will go in the least number of the left hand figures of the dividend that will contain the divisor ^{once or more}, and place the number of times on the right hand of the dividend, for the first figure of the quotient.
3. Multiply the divisor by the quotient figure, and set the product under the figures of the dividend used.
4. Subtract the product from that part of the dividend under which it stands, and on the right hand of the remainder bring down the next figure of the dividend, which

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for calculating the ~~south~~ things,
times of the southing, rising,
& setting of the O, D, & S, & Mon-
ets; & containing also many other
things pertaining to practical
Astronomy, which tho' often
bought for by the ingenious
& inquisitive student, are
not to be found in any ~~other~~ ^{work}
~~book~~ on astronomy com^y used
in America.

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$$\begin{array}{r} 552 \\ \hline 353 \\ \hline 363 \\ \hline 373 \\ \hline 383 \\ \hline 393 \\ \hline 403 \\ \hline 413 \\ \hline 423 \\ \hline 433 \\ \hline 443 \\ \hline 453 \\ \hline 463 \\ \hline 473 \\ \hline 483 \\ \hline 493 \\ \hline 503 \end{array}$$

Common Method. Beginning at the right.

$$\begin{array}{r} 24 \overline{) 345} \\ \underline{24} \\ 105 \\ \underline{96} \\ 9 \end{array}$$

Beginning at the right hand in this example, I say how many times 24 in 45, and the quotient figure is 1, and 24 remains, to the left of which the 3 is brought down, and 24 will go 9 times in 324, and 105 remains, and 4 times 24 is 96, which subtracted from 105 leaves 9 for the final remainder; and $1 + 9 + 4 = 14$ the true total quotient. It is easy to see that the divisor might be placed on the right, and the quotient on the left of the dividend, without altering the result.

10. How can Division be proved by casting out the nines?

bring down the next figure of the dividend, which

LONG DIVISION

number divide as before, and proceed in the same manner till the whole dividend is brought down and divided. — This method of dividing is called Long Division.

The best way to find how often the divisor, when large, is contained in a sufficient number of figures of the dividend, is to find how often the first figure of the divisor is contained in the first or two first figures of the dividend, making allowance as nearly as can be judged, for what there will be to carry. If the remainder ever exceeds the divisor, the divisor will go one more than the quotient figure shows.

Proof. — Multiply the quotient by the divisor, and add the remainder, if any, to the product, and the number so found will be equal to the dividend, if the work is right.*

Reason of the Rule. — The reason ^{why} division is performed as it is, is because the dividend is supposed to be divided into parts, and it is found by trial how often the divisor is contained in each part. The reason of beginning at the left hand instead of the right, as in other rules, is because that if the division should begin at the unit's place, the last quotient figure would be found first, and the remainder would be left at the first operation instead of the last, which would be a very awkward way of working division.

Examples.

1. Divide 3479 by 5.

$$\begin{array}{r} 5 \overline{) 3479} \quad (695 \\ \underline{30} \\ 47 \\ \underline{45} \\ 29 \\ \underline{25} \\ 4 \end{array}$$

Explanation.

Here a curved line is drawn on each side of the dividend, 3479, and the divisor 5 is placed on the left. Then as 5 is not contained in 3, the first figure of the dividend, I say

how many times 5 in 34, and the number is 6; then the divisor 5 is multiplied by the quotient figure 6, and

* Division is easily proved by casting out the nines, in the following manner: Multiply the excess of the nines in the divisor and quotient together, and cast the nines out of the product, and reserve the excess. Cast the nines out of the dividend diminished by the remainder, and if the excess is the same as the reserved excess, the work is right if no figures are misplaced. — There are other methods of proof, but these are sufficient.

LONG DIVISION

the product 30 is placed under 34, and subtracted from it, and the remainder is 4, to the right hand of which the next figure 7, of the dividend is brought down. Then I say how many times 5 in 47, and the answer is 9 times, therefore 9 is placed in the quotient to the right hand of 6: then the divisor 5 is multiplied by the last quotient figure 9, and the product 45 is placed under 47, and being subtracted from it leaves a remainder of 2, to the right of which the last figure, 9, of the dividend is brought down, which makes the last part of the dividend to be divided 29. Then the inquiry is how many times 5 may be had in 29, and the number of times is 5, which place in the quotient as before. Then the divisor 5 multiplied by the last quotient figure 5, gives 25, which placed under 29, and subtracted from it leaves a remainder of 4, which completes the operation.

Proof by
Multiplication.

Quotient	6	9	5
Divisor			5
	3	4	7
Remr. add			4
Dividend	3	4	7

Explanation.

Here the quotient 695 is multiplied by the divisor 5, and the remainder 4 is added to the product 3475, and the sum is 3479, which, as it is equal to the dividend, proves the work to be right.

The proof by casting out the nines is thus. First, as the divisor is less than 9, there is no 9 to cast out of that, therefore the divisor itself, 5, is taken for the excess. The excess of the nines in the quotient, is 2, and 2 multiplied by 5 gives 10, and casting out 9 leaves 1 for the reserved excess. Then subtract the remainder 4 from the dividend and the remainder is 3475, and casting the nines out it leaves the excess of 1, which is the same as the reserved excess, therefore the work is right.

2. Divide 71685, by 72.

72)	7	1	6	8	5	(	9	9	5
	6	4	8						
			6	8	8				
				6	4	8			
					4	0	5		
						3	6	0	

45 remains.

The next operations were to multiply, subtract, and bring down another figure, &c., as in the first example.

In working this sum the first step was to find how often 72 could be had in 716, which is 9 times.

The next operations were

I 9 M P L E D 9 U 9 I 9 O N.

3. Divide 5130652 by 132. Quot. 38868. Remn. 76.
4. Divide 1406373 by 108. Quot. 13021. Remn. 105.
5. Divide 123456789 by 365. Quot. 338237. Remn. 284.
6. Divide 293839455936 by 8405. Quot. 34960078. Remn. 346.
7. Divide 4637064283 by 57606. Quot. 80496. Remn. 11707.
8. Divide 5793759959 by 1512. Quot. 3831851. Remn. 1247.

Contractions and Abbreviations in Division.*

Case I. When the Divisor does not exceed 12, Division is performed by a method called Short Division.

Rule. Set down the divisor and dividend as in Long Division, and draw a line under the dividend. Then find how many times the divisor is contained in the first, or two first left hand figures of the dividend, and set the quotient under the line; next, carry and add as many tens to the next figure as there are ones remaining after dividing the first, and divide as before, and proceed in the same manner till the whole is divided.

Examples.

(1)

$$\begin{array}{r} 2 \overline{) 87654375} \\ 43827187 \dots 1 \text{ remn.} \end{array}$$

Explanation.

First, the twos in 8 are 4; 2d, the twos in 7 are 3 times, and 1 over, which is equal to 10 in the next right hand place, therefore 10 is added to 6, which makes 16, and the twos in 16 are 8; next, the twos in 5 are 2 times, the twos in 14 are 7 times; 2 in 3 once; 2 in 17, 8 times; 2 in 15, 7 times, and 1 remains.

Short Division is proved in the same manner as Long Division.

2. Divide 13545728 by 5. Quot. 2709145. Remn. 3.
3. Divide 123456789 by 7. Quot. 17636684. Remn. 1.
4. Divide 1357975313 by 9. Quot. 150886145. Remn. 3.

* The reason why Long Division is set before the short methods, is because Long Division answers the purpose of all the other methods.

SIMPLE DIVISION

5. Divide 72646206 by 11. Quot. 6604200. Rem. 6.
6. Divide 123456789034 by 12. Quot. 10288065752. Rem. 10.

Case II. To divide when the divisor ends with ^{one} or more ciphers.

Rule. Cut off the ciphers from the divisor, and also as many figures or places from the right hand of the dividend, as are cut off from the divisor, and divide the remainder of the dividend by the remainder of the divisor, and place what remains to the left hand of what is cut off from the dividend, and the whole will be the true remainder.

Examples.

1. Divide 310869 by 7100.

$$\begin{array}{r} 71 \overline{) 310869} \\ \underline{284} \\ 268 \\ \underline{213} \\ 5569 \text{ rem.} \end{array}$$

Explanation.

Here the two ciphers are cut off from the divisor 7100, and two figures from the dividend, and 3108 is divided by 71, and the quotient is 43, and to 55 which remained, is annexed the 69 which was cut off from the dividend, and 5569 is what remains after dividing 310869 by 7100.

2. Divide 7380964 by 23000. Quot. 320. Rem. 20964.
3. Divide 5873126 by 8000. Quot. 734. Rem. 1126.
4. Divide 29628754963 by 35000. Quot. 846535. Rem. 20963.

Case III. When the divisor is the product of two or more numbers.

Rule. Divide by those numbers one after another, instead of the whole of the number at once.

Examples.

1. Divide 31046835 by 56. Now as 7 times ⁸ are 56, first divide the dividend by 7, and the quotient of that by 8, and the last quotient will be the answer. Thus,

$$\begin{array}{r} 7 \overline{) 31046835} \\ \underline{217} \\ 83435262 \dots 1 \text{ rem.} \\ \underline{554407} \dots 6 \text{ rem.} \end{array}$$

In this example the quotient required is 554307, but as something remained after each division, the true remainder of the whole is found as follows. Multiply the first divisor by the last remainder, and to the product add the first remainder; the sum is the true remainder. Here the first divisor 7, multiplied by the last remainder 6, produces 42, to which add the first remainder 1, and the sum 43 is the true remainder, supposing you divided 31046835 by 56, by Long Division. This is the method of proof when there are only two divisors.* When there are more than two divisors, the best method of proof is by the help of fractions, after fractions are understood.

2. Divide 7014596 by 72. Quot. 97425. Rem. 68.

In this example divide first by 8, and then by 9, because 8 times 9 are 72. The remainder is found as directed to be by the last rule, without the help of fractions.

3. Divide 87564 by 48, or 6×8 . Quot. 1824. Rem. 12.

4. Divide 76587 by 96, or 12×8 . Quot. 797. Rem. 75.

* When there are three or more divisors, the true remainder may be found by the following Rule.—Find the true remainder for the two first divisors as before taught; then multiply together the two first divisors and 3rd remainder, and add in the true remainder last found, which will give the true remainder for three divisors. If there are four divisors, multiply together continually the three first divisors and the fourth remainder, and add the true remainder last found, and the sum will be the true remainder four divisors, &c.

Reason of this rule of proof.—It is because that what remains after the first division, is always so many units, and what remains after the second division is always as many more than so many units as the first divisor is more than 1. For example; let 44 be divided by 15, according to the method of Long Division, and the remainder is 14. Now divide 44 by 5, and 4 will remain, which is evidently 4 units or ones of the dividend. Then divide 8 (the quotient of $44 \div 5$) by 3, and 2 will remain, and as the first quotient 8 is so many times 5, the 2 that remains of 8 is twice 5 or 10, which added to 4 the first remainder, makes 14, the true remainder, as by Long Division, which explains the whole principle.

To divide a number when the Divisor has a fraction belonging to it.

Rule. Reduce the whole divisor to a fraction, and likewise the dividend to the same denomination, and then divide as usual.

Examples.

1. Divide 1478 by $24\frac{1}{2}$.

$$\frac{24\frac{1}{2}}{2} = \frac{49}{2} = \text{the divisor.}$$

$$\frac{1478}{2} = \frac{2956}{2} = \text{the dividend.}$$

$$49 \overline{) 2956} \begin{array}{r} 60 \\ 294 \\ \hline 16 \text{ halves} = 8 \text{ rem.} \end{array}$$

Explanation. As the divisor is $24\frac{1}{2}$, reduce it all to halves, and it makes 49, and since the divisor is reduced to halves, the dividend must be too, otherwise the quotient would be too small by one half. As the remainder 16 is halves, it must be divided by 2, and the true remainder is 8.

2. Divide 65734 by $51\frac{3}{5}$. Quot. 1273. Rem. 236 fifths.

Questions further explaining the Nature and Use of Division.

1. An army of 3750 men have for their whole number 42000 pounds of flour to subsist on. If it be equally divided among them how much will each have for his share? Ans. 112 lb.

To solve this question it is not difficult to perceive that the whole number of pounds must be divided by the whole number of men.

2. Supposing the Atlantic ocean to be 15850000 feet wide, how many times the length of a vessel 100 feet long, must it sail to sail across the ocean? Ans. 158500.

3. If $13\frac{1}{2}$ lb. of powder will load a great gun once, how many times will $99\frac{3}{4}$ lb. load the same gun? Ans. 57 times.

I Questions for Examination.

1. What does simple Division teach?
2. What is the number to be divided called?
3. What is the number called which you divide by?
4. What is the quotient?
5. What is the remainder?
6. What is Long Division?
7. What is Short Division?
8. How is Long Division performed?
9. How is Short Division performed?
10. How do you divide when the divisor contains ciphers on the right hand?

COMPOUND ADDITION.

11. How do you divide when the divisor is a composite number, or the product of two or more smaller numbers?
12. How do you prove division?

COMPOUND ADDITION.

Compound Addition teaches to add several numbers of different denominations into one sum.

Rule. 1. Set down the numbers so that the same denominations may stand directly under one another, and draw a line under them.

2. Add the figures in the lowest denomination, and find how many of the next higher are contained in it. Set down the remainder, and carry as many as there are ones to the next denomination, and so proceed till the whole is added, and under the last column set down the whole sum of it.

Proof. The method of proof is the same as in simple addition.

Federal Money.

The coin of the United States is called Federal Money, and was established by Congress in 1786. The Gold coins are three; the Eagle, Half Eagle, and Quarter Eagle. The Silver coins are five; the Dollar, Half Dollar, Quarter Dollar, Dime, and Half Dime. The Copper coins are two; the Cent and Half Cent. Mill is an imaginary coin, there being none of that denomination.

10 Mills (<u>m.</u>)	} make one {	Cent, <u>c.</u>
10 Cents		Dime, <u>d.</u>
10 Dimes		Dollar, <u>D.</u> or <u>℥.</u>
10 Dollars		Eagle, <u>E.</u>

The Eagle	} should weigh {	^{pure.} 247½ grains	^{standard.} 270 grs.
The Dollar		371¼	416.
The Cent		208	208

COMPOUND ADDITION.

Examples.

(1.)					(2.)				
<u>£.</u>	<u>£.</u>	<u>d.</u>	<u>c.</u>	<u>m.</u>	<u>£.</u>	<u>£.</u>	<u>d.</u>	<u>c.</u>	<u>m.</u>
75	3	4	8	3	37	4	5	7	3
98	7	6	9	8	83	5	7	6	4
01	2	3	0	1	16	4	2	3	5
47	6	5	8	7	87	5	4	6	7
52	3	4	1	2	12	4	5	3	2
76	7	5	8	7	43	8	7	6	5
23	2	4	1	2	56	1	2	3	4
Sum	375	34	80		Sum	337	45	73	
	299	99	97			299	99	97	
Proof	375	34	80		Proof	337	45	73	

Remark.—Though the money of the United States, that is, the Federal money, consists of five denominations, real and imaginary, yet, in practice, computations are made, and accounts are kept in dollars and cents; one dollar being equal to 100 cents. But, although 100 cents make a dollar, it is carried for every 10 in adding cents as well as dollars, but you must remember, to prevent mistakes, that when the number of cents is less than 10, a cipher must (3.) be placed to the left hand of it.

<u>Dols.</u>	<u>cts.</u>
375	64
876	95
437	63
586	47
847	69
586	74
Sum	3711
	1529

<u>Dols.</u>	<u>cts.</u>
476	84
865	43
654	87
576	94
398	76
947	68
Sum	4768
	074

5. A trader has accounts against five different persons as follows. One owes him \$75, 89; another \$63, 44; another \$36, 55; another \$87, 32; and another \$12, 67; how much do they all owe him? Ans. \$275, 67.

English Money.

4 Farthings (qrs.)	} make one {	Penny, <u>d.</u>
12 Pence		Shilling, <u>s.</u>
20 Shillings		Pound, <u>£.</u>

COMPOUND ADDITION.

Remark. Instead of farthings, it is now usual to set down fractions of a penny; $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$; so that in adding, 4 fourths, or 2 halves, make a penny, &c.

Examples.

(1.)			(2.)		
£.	s.	d.	£.	s.	d.
7 ¹⁴	18	5 $\frac{1}{4}$	376	19	7 $\frac{1}{4}$
385	12	9 $\frac{1}{2}$	641	17	3 $\frac{1}{2}$
541	14	7 $\frac{1}{4}$	729	15	6 $\frac{1}{2}$
725	15	8 $\frac{1}{4}$	841	16	4 $\frac{1}{4}$
236	19	3 $\frac{1}{2}$	576	14	7 $\frac{1}{2}$
314	17	8 $\frac{1}{4}$	427	16	3 $\frac{1}{4}$
Sum	2919	18 7	3594	19 8 $\frac{1}{4}$	
	2205	0 1 $\frac{1}{4}$	3218	0 1	
Proof	2919	18 7	3594	19 8 $\frac{1}{4}$	

(3.)			(4.)			(5.)		
£.	s.	d.	£.	s.	d.	£.	s.	d.
537	13	9 $\frac{1}{2}$	672	14	8 $\frac{1}{4}$	847	19	5 $\frac{1}{2}$
219	15	7 $\frac{1}{4}$	847	16	3 $\frac{1}{4}$	176	13	9 $\frac{1}{2}$
742	14	6 $\frac{1}{2}$	219	18	7 $\frac{1}{2}$	642	14	7 $\frac{1}{4}$
637	19	5 $\frac{1}{4}$	752	16	4 $\frac{1}{2}$	736	17	8 $\frac{1}{4}$
351	13	7 $\frac{1}{2}$	819	13	6 $\frac{1}{4}$	284	19	3 $\frac{1}{2}$
217	18	5 $\frac{1}{4}$	741	14	7 $\frac{1}{4}$	837	15	7 $\frac{1}{4}$

Troy weight.

24 Grains	} make one {	Penny-weight,	grs.	pwt.
20 Penny-weights		Ounce,		oz.
12 Ounces		Pound,		<u>℔</u> or <u>lb.</u>

Grains 24 = 1 Penny-weight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

This Table is used for weighing gold, silver, jewels, amber, electuaries and liquors. The pound Troy is lighter than the pound Avoirdupois by 2 ounces, 11 dwts, 15 $\frac{1}{2}$ grains; but the ounce Troy is heavier than the Avoirdupois by 32 grains: wherefore 80 ounces Avoirdupois are equal to 73 ounces Troy, and 17 lb Troy are equal to 14 lb Avoirdupois.

Examples.

(1.)				(2.)				(3.)			
lb.	oz.	dwts.	grs.	lb.	oz.	dwts.	grs.	lb.	oz.	dwts.	grs.
75	7	14	21	37	6	14	20	71	9	17	14
56	8	17	23	42	7	17	14	94	3	12	19
82	10	13	16	71	8	10	22	62	5	18	17
39	10	19	20	49	3	16	23	87	6	12	22
57	6	13	12	81	1	7	15	52	9	19	18
64	3	17	20	37	11	15	19	67	7	14	19

*

- 1 Farthing (or one fourth of a penny)
- 2 Farthings
- 3 Farthings (or three fourths of a penny)
- 1 Half-penny.

Also,
 $\frac{grs.}{4} = \frac{d.}{1}$
 $48 = 12 = 1$
 $960 = 240 = 20 = 1 \text{ L.}$

COMPOUND ADDITION Avoirdupois Weight.

16 Drams	} make one {	Ounce, marked dr. oz.
16 Ounces		Pound, lb.
28 Pounds		quarter of a hund. weight qr.
4 Quarters		hund. weight or 112 pounds
20 Hund. wt.		Ton. T. [Cwt.]

* By Avoirdupois are weighed all coarse and grossy goods, grocery and chandlery wares; bread, and all metals, except Gold and Silver.

A Barrel of Pork weighs 220lb — a Barrel of Beef, 220lb — 1 Quintal of Fish, 1 Cwt. avoird — 12 particular things make 1 dozen; 12 doz. 1 gross, and 144 dozen 1 great gross. — 20 particular things make 1 score.

A Quintal in the West Indies	100 lbs.
— in England	112
— called Stannary for weighing Tin, &c. to the King's farmers.	120

Raw silk from Persia and Turkey is sold by the lb. of 24 ounces = $1\frac{1}{2}$ lb. Avoirdupois.

Freighting Ships.

A Ton in weight is 2000 lbs. so that a ship of 100 Tons can carry 200 000 lbs. burthen. A Ton in measure, for Wines, Brandys, &c. is 2 Pipes or Butts; but for Bale Goods, Boxes, Cases, &c. 40 cubic feet make a Ton. In Holland and other northern Countries, they use the term *Loft*, which in freighting ships is equal to 2 Tons or 4000 lbs.

Examples.

(1.)				(2.)			
Tons.	Cwts.	Qrs.	lb.	Qrs.	lb.	3.	drs.
183	14	2	25	756	27	14	13
547	18	1	21	273	22	15	12
325	19	2	27	384	28	11	10
638	17	3	21	519	16	14	11
726	13	1	14	237	13	15	13
854	18	3	26	647	19	12	9
263	14	1	25	911	27	13	15

Apothecaries' Weight.

20 Grains (grs.)	} make one {	Scruple, ℥.
3 Scruples		Dram, ʒ.
8 Drams		Ounce, ℥.
12 Ounces		Pound, lb.

Apothecaries and physicians compound or mix their medicines by this weight, but they buy and sell their drugs by Avoirdupois.

16 Drams	1 Ounce.
256 = 16 = 1 pound.	
7168 = 448 = 28 = 1 quarter.	
28672 = 1792 = 112 = 4 = 1 hund. weight.	
573440 = 35840 = 2240 = 80 = 20 = 1 Ton.	

+ Grains
 $20 = 1$ Scruple.
 $60 = 3 = 1$ Dram.
 $480 = 24 = 8 = 1$ Ounce.
 $5760 = 288 = 96 = 12 = 1$ Pound.

COMPOUND ADDITION.

Examples.

(1.)

(2.)

lb.	3.	3.	D.	grs.	lb.	3.	3.	D.	grs.
473	7	4	1	14	735	9	4	2	15
342	5	7	2	19	642	5	6	1	17
726	1	4	2	13	817	10	7	0	16
297	8	6	1	17	425	7	3	2	18
525	9	4	2	18	484	9	2	1	15
678	4	5	1	18	532	11	7	0	14
472	6	3	2	19	876	10	2	1	12

Cloth Measure.

2 $\frac{1}{4}$ Inches (in.)	} make one	Nail, na.
4 Nails		Quarter of a yard, qr.
4 Quarters		Yard, yd.
3 Quarters		Ell Flemish, E. Fl.
5 Quarters		Ell English, E. E.
6 Quarters		Ell French, E. Fr.
37 inches		Ell Scotch, E. Sc.
3 $\frac{2}{3}$ Quarters		Spanish Var.

All Scotch and Irish linens are bought by the English or American yard, which are the same; and all Dutch linens by the Ell Flemish; but they are all sold in America by the American yard. — The Scotch allow yard in every 20 yards.

Examples.

(1.)

(2.)

(3.)

E. Fr.	qrs.	n.	E. E.	qrs.	n.	Yds.	qrs.	n.
375	3	2	576	2	3	475	2	3
584	2	1	375	3	2	547	3	2
875	3	2	483	2	1	354	2	3
658	2	1	536	3	2	576	3	2
876	3	2	478	2	3	897	2	3
483	2	3	567	3	2	143	3	2
578	3	2	476	2	1	576	3	3

Long Measure.

COMPOUND ADDITION.

3 Barley corns (<u>bar.</u>)	} make one	Inch, <u>in.</u>
12 Inches		Foot, <u>ft.</u>
3 Feet		Yard, <u>yd.</u>
5 $\frac{1}{2}$ Yards, or 16 $\frac{1}{2}$ feet		Rod, Pole or Perch, <u>pol.</u>
40 Rods or poles		Furlong, <u>fur.</u>
8 Furlongs		Mile, <u>mile.</u>
3 Miles		League, <u>lea.</u>
60 Geometrical miles, ...		Degree, of a great circle of the Earth, <u>deg.</u>
69 $\frac{1}{2}$ Statute miles, <u>nearly</u>		Circle, <u>cir.</u>
360 Degrees		

Long Measure is used for measuring distances only; without ~~without~~ regard to breadth or thickness.*

Examples.

<u>Deg.</u>	<u>Stat. m.</u>	<u>fur.</u>	<u>pol.</u>	(1.) <u>ft.</u>	<u>in.</u>	<u>bar.</u>
120	31	5	18	14	9	2
39	14	5	37	15	10	1
45	17	5	19	10	11	2
8	62	6	38	12	7	1

<u>Deg.</u>	<u>G. mil.</u>	<u>fur.</u>	<u>pol.</u>	<u>ft.</u>	<u>in.</u>	<u>bar.</u>
150	57	18	38	10	11	2
76	13	34	16	15	9	1
87	56	27	35	14	10	2
17	35	30	19	11	11	1

* In measuring Land,

7 $\frac{1}{2}$ Inches	} make one	Link
25 Links		Pole
100 Links		Chain
10 Chains		Furlong
8 Furlongs		Mile
B. corns. 3 =	1 Inch.	
36 =	12 =	1 Foot.
108 =	36 =	3 = 1 Yard.
594 =	108 =	16 $\frac{1}{2}$ = 5 $\frac{1}{2}$ = 1 Pole.
23760 =	7960 =	660 = 220 = 40 = 1 Fur.
190080 =	63360 =	5280 = 1760 = 320 = 8 = 1 mile
Inches.	Links.	
7 $\frac{1}{2}$ =	1 Pole or Perch.	
198 =	25 =	1 Chain.
792 =	100 =	4 = 1 Furlong.
7920 =	1000 =	40 = 10 = 1 Mile.
63360 =	8000 =	320 = 80 = 8 = 1

Sally.

Land or Square Measure

144 Square Feet	1 Acre
9 Feet	1 Yard
30 1/2 Yards, or 272 Feet	1 Rod or Pole
4 Rods or Poles	1 Furlong
10 Rods or 160 Poles	1 Mile
100 Acres	1 Square Mile

Measures of Length

12 Inches	1 Foot
3 Feet	1 Yard
5 1/2 Yards	1 Rod or Pole
4 Rods	1 Furlong
10 Rods	1 Mile

Measures of Capacity

1728 Cubic Feet	1 Cubic Yard
27 Feet	1 Cubic Yard
10 Feet	1 Cubic Yard
1/2 or 1/4	1 Bushel
1/2 or 1/4	1 Bushel
1/2 or 1/4	1 Bushel

Any Solid Measure of things are measured that have length, breadth, and height.

Examples

144 Square Feet	1 Acre
9 Feet	1 Yard
30 1/2 Yards, or 272 Feet	1 Rod or Pole
4 Rods or Poles	1 Furlong
10 Rods or 160 Poles	1 Mile
100 Acres	1 Square Mile

Inch, ... in
 foot, ... ft.
 yard, ... yd.
 Rod, Pole or perch, ...
 mile long, ...
 mile, ...
 league, ...
 a great ...
 of the earth, ...
 ...

long ... for ... distance
 only, without ... or thick-
 ness

Examples.

ft.	in.	yd.	rod	perch	mile	league
12	1	1	1	1	1	1
36	3	1	1	1	1	1
108	9	3	1	1	1	1
216	18	6	2	2	1	1
432	36	12	4	4	2	1

ft.	in.	yd.	rod	perch	mile	league
12	1	1	1	1	1	1
36	3	1	1	1	1	1
108	9	3	1	1	1	1
216	18	6	2	2	1	1
432	36	12	4	4	2	1

COMPOUND ADDITION Land, or Square Measure.[†]

144 Square Inches (<u>sq. in.</u>)	} make one	Square Foot,	<u>ft.</u>
9 Feet		2 yard,	<u>yd.</u>
30 $\frac{1}{2}$ yards, or 27 $\frac{1}{2}$ feet		Rod or Pole,	<u>r.</u>
40 Rods or Poles		Rood,	<u>roo.</u>
4 Roods or 160 Rods		Acre,	<u>ac.</u>
640 Acres		Mile,	<u>mil.</u>

Examples.

(1.)

(2.)

Poles.	Yds.	Ft.	In.	Acres.	Roo.	Po.	Yds.
975	29	7	127	635	2	37	25
428	17	4	95	172	1	26	27
673	23	8	114	763	3	38	21
527	27	3	108	287	1	22	23
716	21	6	87	125	2	36	19
457	19	0	136	287	1	12	24
873	25	7	75	738	3	27	15

Solid Measure.[‡]

1728 Inches (<u>in.</u>)	} make one	Foot,	<u>ft.</u>
27 Feet		Yard,	<u>yd.</u>
50 Feet of round timber, or 50 feet of hewn timber,		Ton,	<u>t.</u>
128 solid feet		Cord of wood,	<u>cor.</u>

By Solid Measure all things are measured that have length, breadth, and thickness.

Examples.

†	Inches	144 =	1 Foot.
		1296 =	9 = 1 Yard.
		39204 =	272 $\frac{1}{2}$ = 30 $\frac{1}{2}$ = 1 Pole.
		1568160 =	10890 = 1210 = 40 = 1 Rood.
		6272640 =	43560 = 4840 = 160 = 4 = 1 Acre.
		4014489600 =	27878400 = 30976000 = 102400 = 2560 = 640 = 1 mile.

COMPOUND ADDITION

(1.)			(2.)		
Ton.	ht.	in.	Cord.	ht.	in.
3478	39	1574	574	120	1214
4769	24	1357	754	57	1615
6545	38	1547	347	104	1376
7438	27	1143	854	112	1475
5764	34	1472	387	114	3794
3875	14	1357	576	98	1457

Wine Measure.*

2 Pints	} make	Quart marked	pts. qts.
4 Quarts		Gallon	gal.
10 Gallons		Anchor of Brandy	anc.
18 Gallons		Runlet	run.
31½ Gallons		Half an hoghead	½ hhd.
42 Gallons		Tierce	tier.
63 Gallons		Hoghead	hhd.
2 Hogheads		Pipe or Butt	P. or B.
2 Pipes		Tun	Tun.

* All Brandies, Spirits, Perry, Cyder, Mead, Vinegar, Honey, and Oil are measured by Wine-measure: Honey is commonly sold by the pound Avoirdupois.

Wine measure is deduced from Troy weight. 12 oz. Troy made into a concave measure is a Wine pint. 8 pints make 1 Gallon containing 231 cubic inches. A Ton, 252 Gallons weighs 2016 lbs. Troy or 1890 lbs. Avoirdupois.

Examples.

(1.)				(2.)			
Sier.	gal.	qts.	pts.	Hhd.	gal.	qts.	pts.
27	40	3	1	175	50	3	1
35	34	2	0	398	47	2	0
76	38	3	1	476	49	3	1
57	16	1	1	847	51	2	1
76	28	2	0	374	50	1	1
54	39	1	1	476	57	3	1

*

Cubic Inches	
288	= 1 Pint.
576	= 2 = 1 Quart.
231	= 8 = 4 = 1 Gallon.
9702	= 336 = 168 = 42 = 1 Tierce.
14553	= 504 = 252 = 63 = 1½ = 1 Hoghead.
19404	= 672 = 336 = 84 = 2 = 1½ = 1 Puncheon.
20106	= 1008 = 504 = 126 = 3 = 2 = 1½ = 1 Pipe.
58212	= 2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

8 lbs. Troy is 1 Gallon; 16 lbs. a Peck; and 64 lbs. Bushel.

COMPOUND ADDITION. Ale or Beer Measure.

2 Pints	} make one	Quart marked pints.	qts.
4 Quarts		Gallon	gal.
8 Gallons		Firkin of ale in London	A fir
8½ Gallons		Firkin of Ale or Beer.	
9 Gallons		Firkin of beer in Lon.	B. fir.
2 Firkins		Kilderkin	Kil.
2 Kilderkins		Barrel	Bar.
1½ Barrel, or 54 Gallons		Hogshead of Beer	hhd.
2 Barrels		Punchon	Pun.
3 Barrels, or 2 Hogsheads		Butt	Butt.
Cubic Inches.			

In this measure 282 cubic inches make 1 gallon; 10152 inches 1 barrel; and 15288 inches 1 hogshead. — Milk is sold by the beer quart.

By a law in Massachusetts, a barrel of mackerel or other barrelled fish is to contain not less than 30 gallons; In Connecticut and New-York, the shad and salmon barrel must contain 200 pounds weight.

Examples.

(1.)			(2.)			(3.)		
A. B.	fir.	gal.	B. B.	fir.	gal.	Hhd.	gal.	qts.
83	3	5	54	2	8	37	50	3
17	1	6	37	3	6	55	37	2
83	2	4	54	2	6	87	14	1
67	1	5	39	1	7	63	49	2
85	3	7	65	3	5	84	17	1
64	2	6	47	2	8	39	14	3

DRY MEASURE.

2 Pints	} make one	Quart, marked	pits.	qts.
2 Quarts		Pottle	pot.	
2 Bottles		Gallon	gal.	
2 Gallons		Peck	pk.	
4 Pecks		Bushel	bu.	
2 Bushels		Strike	str.	
2 Strikes		Coorn	com.	
2 Coorns, or 8 bushels		Quarter	qr.	
4 Quarters		Chaldron	chl.	
4½ Quarters		Chaldron in London.		
5 Quarters		Wey	wey.	
2 Weys		Last	last.	

268½ cubic inches = 1 gallon; and 2150½ cubic inches = 1 bushel, strict measure.

COMPOUND ADDITION.

Dry measure is applied to all dry goods, as corn, seed, fruit, roots, salt, sand, oysters, coal, &c.

A Winchester bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep. In measuring potatoes, apples, &c., about three strit quarts ought to be heaped on every strit half bushel. — In measuring coals, heap a little more.

Examples.

(1.)				(2.)				(3.)			
chd.*	bu.	pkts.	qts.	Drs.	bu.	pkts.	qts.	bu.	pkts.	qts.	pt.
54	20	2	5	87	4	2	1	65	3	7	1
39	14	3	7	94	7	3	5	39	2	5	0
65	34	1	6	76	3	2	4	76	3	2	1
74	27	3	2	98	6	3	2	58	2	6	0
57	16	2	4	57	7	2	1	45	1	4	1
35	28	3	5	45	3	1	4	37	3	2	0

Time. †

60 Seconds	} make	Minute, marked s. m.
60 Minutes		Hour h.
24 Hours		Day d.
7 Days		Week w.
4 Weeks [6 hours]		Month mo.
12 Months, 1 day & 13		Julian year yr.

Seconds 60 = 1 Minute,
 3600 = 60 = 1 Hour.
 86400 = 1440 = 24 = 1 Day.
 604800 = 10080 = 168 = 7 = 1 Week.
 2419200 = 40320 = 672 = 28 = 4 = 1 month.

$$\begin{aligned}
 \frac{d.}{365} \frac{h.}{6} &= 52 \frac{1}{6} = 1 \text{ Julian year. } \parallel \\
 \frac{d.}{365} \frac{h.}{6} \frac{m.}{9} \frac{s.}{11\frac{1}{2}} &= 1 \text{ Siderial year. } \S \\
 \frac{d.}{365} \frac{h.}{5} \frac{m.}{48} \frac{s.}{48} &= 1 \text{ Tropical year. } +
 \end{aligned}$$

† By the Calendar, the year is divided in the following manner.
 THIRTY Days hath September, April, June and November;
 FEBRUARY, twenty-eight alone, & all the rest have thirty-one.
 When you can divide the year by 4, without any remainder,
 it is then Bisextile, or Leap-Year, in which February has 29 days,

generally.

(48)

‡ The civil Solar year of 365 days being short of the true by 5h. 48m. occasioned the beginning of the year to run forwards through the seasons nearly one day in four years, on this account JULIUS CAESAR ordained that one day should be added to February, every fourth year, by causing the 24th day to be reckoned twice; and because this 24th day was the sixth (sextilis) before the kalends of March, there were, in this year, two of these sextiles,

name of Bisextile to this year, which, being thus corrected, was, from thence, called the Julian year.

§ A siderial year is the time in which the sun appears to revolve from any fixed star, to the same star again.

+ A tropical year is the time in which the sun revolves from either of the equinoctial points to the same again; or from either of the tropicks to the same again.

* A common chaldron is 32 bushels. A chaldron in London is 36 bushels.

done is 36 bushels.

John Richardson in Lon-

COMPOUND ADDITION

Examples.

Yrs.*	mo.	w.	d.	h.	m.	s.
87	3	2	5	10	20	10
54	11	3	6	14	17	50
89	10	2	4	20	39	47
63	8	1	5	14	13	57
94	11	3	6	11	48	39
66	9	2	5	23	59	48

Yrs.*	mo.	w.	d.	h.	m.
5	10	3	4	10	38
19	7	2	6	14	40
32	9	3	5	20	39
57	6	1	5	19	13
89	10	2	3	22	49
63	7	3	5	18	57

Circular Motion.

60 Seconds	} make one {	Prime Minute, marked "4'.
60 Minutes		Degree
30 Degrees		Sign
12 Signs or		{ the whole circle of the Ecliptic.
360 Degrees		

Examples.

(1.)	(2.)	(3.)
8 ^h 20° 14' 37"	5 ^h 14° 20' 18"	4 ^h 18° 27' 48"
5 14 17 49	3 17 16 14	3 15 39 27
3 29 50 58	6 18 17 15	6 24 48 56
4 18 17 16	7 24 48 56	5 25 39 48

Remark. As 12 signs make a circle, reject or cast out all the twelves, after adding the signs, and set down the remainder.

I Questions for Examination.

1. What is Compound Addition, or what does it teach?
2. How must compound numbers be set down, so as to be added?
3. How do you add and carry, in compound Addition?
4. How is compound addition proved?

* The year, in these examples is reckoned 13 months only, or 1d. 6h. short of the Julian year.

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION

Teaches to find the difference between two numbers of different denominations.

Rule.—First, set down the greater number, or that to be subtracted from. Secondly, set the less number under the greater, each denomination under its kind. Thirdly, begin with the least denomination, and subtract the lower from the upper number, and set down the difference; but if the lower number be greater than the one over it, add as many to the upper number as it takes to make one of the next greater denomination, and then subtract; remembering to carry one to the next number you are to subtract, every time you borrow or add to the upper number.

The Method of Proof, and Reason of the Rule, are the same as in Simple Subtraction.

EXAMPLES.

Federal Money.

	¢	¢.	m.
From	57	38	4
Take	36	84	3
Rem.	20	54	1
Proof	57	38	4

	¢	¢.	m.
Lent	110	34	7
Received	34	63	5
Due	75	71	2
Proof	110	34	7

	¢	¢.	m.
From	76	57	8
Take	34	19	9
Rem.	42	37	9
Proof			

	¢	¢.	m.
From	57	67	4
Take	48	58	5
Rem.	9	8	9
Proof			

	¢	¢.	m.
From	65	15	6
Take	38	75	4
Rem.			
Proof			

	¢	¢.	m.
From	875	57	3
Take	147	64	2
Rem.			
Proof			

COMPOUND SUBTRACTION. English Money.

£. s. d.	£. s. d.	£. s. d.
742 12 6 $\frac{1}{4}$	284 13 0	713 16 5 $\frac{1}{2}$
194 17 8 $\frac{1}{2}$	157 18 4 $\frac{1}{4}$	257 19 5 $\frac{1}{2}$
<u>547 14 9 $\frac{1}{4}$</u>		
£. s. d.	£. s. d.	£. s. d.
742 13 6 $\frac{1}{2}$	825 10 7	614 15 2 $\frac{1}{2}$
187 16 8 $\frac{1}{4}$	155 14 7 $\frac{1}{2}$	376 15 2 $\frac{1}{2}$
£. s. d.	£. s. d.	£. s. d.
310 17 4 $\frac{1}{2}$	294 0 0	721 0 0
9 18 7 $\frac{1}{4}$	1 0 0 $\frac{1}{4}$	7 13 8 $\frac{1}{2}$

Troy Weight.

lb. oz. dwts. grs.	lb. oz. dwts. grs.
471 7 12 18	627 8 13 20
184 8 17 21	149 10 17 21
<u>286 10 14 21</u>	

Avoirdupois Weight.

lb. oz. dr.	lb. oz. dr.
10 14 9	8 14 3
7 12 13	5 14 2
	18 10 15
	14 14

Apothecaries' Weight.

lb. ʒ. ʒ. ʒ. grs.	lb. ʒ. ʒ. ʒ. grs.
7 7 4 1	6 8 5 2 17
3 8 5 2	4 9 6 1 18

Cloth Measure.

yds. qrs. na.	ells. qrs. na. in.
87 2 2	37 3 2 1
45 3 3	15 4 3 2

COMPOUND SUBTRACTION.

Long Measure.

Dez.	mi.	fur.	pol.	yds.	ft.	in.	bar.
77	37	3	27	4	2	10	1
37	40	5	26	5	1	11	2

Land or Square Measure.

A.	R.	pol.	pol.	ft.	in.
34	2	39	27	100	120
27	2	38	15	120	142

Solid Measure.

Tons.	h.ft.	in.	Cords.	ft.	in.
44	38	107	327	120	327
38	41	275	134	121	1251

Wine Measure.

P.	hhd.	gal.	qts.	P.	hhd.	gal.	qts.
325	0	27	1	127	1	42	0
197	1	43	2	88	1	59	1

Hhd.	gal.	qts.	pts.	Hhd.	gal.	qts.	pts.
612	23	1	0	521	14	2	1
75	37	1	1	256	25	3	0

Ale & Beer Measure.†

Bar.	fir.	gal.	qts.	Bar.	fir.	gal.	qts.
845	2	1	0	710	1	4	1
158	2	5	1	276	2	4	2

Butt.	hhd.	kild.	fir.	gal.
729	1	0	1	3
259	3	1	1	7

Butt.	hhd.	kild.	fir.	gal.
527	2	1	0	4
148	2	1	1	5

Time.

Yrs.	mo.	w.	d.	h.	m.	s.
27	10	2	4	18	39	47
13	7	2	5	20	41	57

† In these examples 9 gallons may make a firkin, and 4 firkins a barrel; 3 kilderkins a hogshead, and 2 hogsheads a butt.

Circular Motion.

$$\begin{array}{r} 8^{\circ} \quad 20' \quad 47'' \quad 34''' \\ 3 \quad 21 \quad 34 \quad 50 \\ \hline \end{array}$$

$$\begin{array}{r} 9^{\circ} \quad 17' \quad 41'' \quad 57''' \\ 4 \quad 28 \quad 39 \quad 58 \\ \hline \end{array}$$

Observation.

Circular motion in signs, degrees, minutes, and seconds, chiefly respects the planets: the orbit of each being supposed to be divided into 12 equal parts called signs, and each sign into 30 equal parts called degrees, &c.

I Questions for Examination.

1. What is Compound Subtraction, or, what does it teach?
2. After setting down the two given numbers, the less under the greater, as in Simple Subtraction, how do you proceed to subtract the different denominations or numbers of different names?
3. Is there any difference in the methods of proving Simple and Compound Subtraction, as commonly practised?

SUPPLEMENTARY EXERCISES

The following Supplementary Exercises in the primary rules of arithmetick, will explain more fully their nature and use.

ADDISON.

1. The Old and New Testament together, are called the Bible, and there

Books in the <u>Old</u> Testament, - - - - -	39
Chapters, - - - - -	929
Verses, - - - - -	23214
Words, - - - - -	592439
Letters, - - - - -	2728100
Books in the <u>New</u> Testament, - - - - -	27
Chapters, - - - - -	260
Verses, - - - - -	7959
Words, - - - - -	181253
Letters, - - - - -	838380.

How many books, chapters, verses, words, and letters, does the whole Bible contain?

Ans. 66 books; 1189 chapters; 31173 verses; 773692 words; and 3566480 letters.

Let the scholar set down the numbers and find the answers.

2. The common annual amount of college charges at Cambridge college, has been estimated as follows.

Steward's fees, \$10. Board in commons 38 weeks of term time at about three dollars per week, \$114. Room rent \$12. Instruction, two first years, forty six dollars each year; 3d year, sixty four dollars; 4th year seventy four dollars; averaging \$57, 50. Librarian's fees, repairs, lecture rooms, catalogues, and contingencies, \$8. Wood \$16. Books used in classes, \$15. What is the whole expense for one year?

Ans. \$232, 50.

SHUTRACK ADDISON.

1. Columbus discovered America in the year 1492; how many years from that time to the year 1831?

Ans. 339.

2. The revolutionary war between America

SUPPLEMENTARY EXERCISES.

and Great Britain commenced April 19th, 1775, and the last war between the two countries was declared June 18th, 1812. How long was the time between?

Ans. 37 y. 1 mo. 29 d.

In solving this question and others of the same nature, where 30 days are reckoned a month, and 12 months a year, set down the years as usual, and the number of the months from January, reckoning January 1, February 2, &c. and set down the days belonging to each. The following will explain the operation in finding the answer to the second question.

y.	mo.	d.
1812	6	18
1775	4	19
37	1	29

MULTIPLY 321 BY 100.

The captain of a vessel having 321 men on board took a prize of such value that besides the captain's half each sailor had \$100 for his share. What was the worth of the whole prize?*

Ans. \$64200.

DIVIDE 154 BY 4.

1. Two boys had 142 chestnuts between them, but one had 28 more than the other; how many had each?

Ans. One had 57, and the other 85.

To solve this question, first subtract 28 from 142, and the remainder divided by 2 will give the least number.

2. If \$154 be divided among 4 persons in such a manner that B shall have \$1 more than A; C \$2 more than A; and D \$3 more than A, how much will each have?

Ans. A \$12, B \$13, C \$14, & D \$15.

This question is solved thus. First subtract \$6 from \$154, viz., 1 for B, 2 for C, and 3 for D; then divide the remainder, \$148, by 4, and the quotient will be A's share, from which the other shares are easily found.

* To solve this question the learner will soon perceive that all there is to do is to multiply 321 by 100, and that product by 2.

† The learner will observe that more than one rule is often used in solving one question.

REDUCTION.

REDUCTION.

Reduction teaches to change the form or denomination of numbers without altering their value. Thus, if \$1 is reduced to cents, the value will still be the same, but the form and denomination will be different; before being reduced the value of a dollar is a dollar, and the form or expression is 1; reduced to cents, \$1 is equal to 100 of them, so that though the value is the same, the form and denomination are different, the one being \$1, and the other 100 cents; and so of any other denomination.

Reduction is of two kinds, descending and ascending. Reduction descending is the reducing of numbers of high denominations to lower; as, dollars to cents, pounds to shillings or pence, &c. Reduction Descending is performed by Simple Multiplication.†

Reduction Ascending is the reducing of low denominations to higher; as, cents to dollars, pence to pounds, &c. Reduction Ascending is performed by Simple Division.‡

REDUCTION DESCENDING.

Rule.—Multiply the highest denomination given by such a number as it takes of the next less denomination to make one of the greater, and thus proceed to reduce from one denomination to the next less until you have brought it down to the denomination required: remembering, as you proceed, to add in the numbers of the less denominations than the highest in the given number, when the given number consists of more denominations than one.

Proof.—Divide by the same numbers you multiply by, only change the order, and divide the lowest denomination by the number you multiplied by last, and the quotient by the last multiplier but one, and so on.

† As Reduction is performed by simple multiplication and simple division, it ought to be learned before compound multiplication and compound division.

R E D U C T I O N.

Examples. English Money.

1. In $\underline{\pounds} 28$ $13s.$ $5d.$ $3qrs.$ how many farthings?

The Operation and Proof.

$$\begin{array}{r} \pounds 28 \ 13s. \ 5d. \ 3qrs. \\ \underline{20} \\ 573 \text{ shillings.} \\ \underline{12} \\ 6881 \text{ pence.} \\ \underline{4} \end{array}$$

Ans. 27527 farthings.

$$\begin{array}{r} 4 \overline{) 27527} \\ 12 \overline{) 6881} \dots 3qrs. \\ 2 \overline{) 0573} \dots 5d. \end{array}$$

Proof. $\pounds 28$ $13s.$ $5d.$ $3qrs.$

Explanation.

In the first place I multiply the pounds by 20, and add in the $13s.$ because 20 shillings make a pound. I next multiply the $573s.$ by 12, because 12 pence make a shilling, and, adding in the $5d.$, the

whole number of pence is 6881, which are multiplied by 4, because 4 farthings make a penny, and, adding in the $3qrs.$ of the given number, the whole number of farthings in $\pounds 28$ $13s.$ $5d.$ $3qrs.$ is found to be 27527, which is the answer required.

To prove the work, the number of farthings is divided by 4, the last multiplier, and the quotient is 6881, and 3 qrs. remain. Then the 6881 pence are divided by 12, the last multiplier but one, and the quotient is 573s. and 5d. remain, which proves the second multiplication to be right. Lastly, the 573s. is divided by the first multiplier 20, after cutting off the cipher from the divisor, and also the right hand figure of the dividend, and the quotient is $\pounds 28$, and 13s. remain; and as the last quotient and the three remainders read together make $\pounds 28$ $13s.$ $5d.$ $3qrs.$, the whole given sum, it proves all the work to be right. On the same principle proceed in all other cases, let the denominations be what they may.

* Instead of adding in the $13s.$ when multiplying, which is the best way, they might have been added to the product of 28 by 20, afterwards, and so of the other numbers.

REDUCTION

2. In £ 47 13s. 8d. how many pence and farthings?
Ans. 11444 pence, & 45776 farthings
3. In 45 Eagles, 5 dollars, 8 dimes, and 7 cents, how many cents?
Ans. 45587.
4. In 56 cwt. 2 qrs. 16 lb. how many pounds?
Ans. 6344.
5. In 365 days and 6 hours (a Julian year) how many seconds?
Ans. 31557600.

REDUCTION ASCENDING

Rule.—Divide the lowest denomination given by such a number as it takes of that denomination to make one of the next higher, and proceed in the same manner with every denomination to the highest required.

Proof.—Reduction Descending, and Reduction Ascending, like Multiplication and Division, prove each other; therefore, to prove Reduction Ascending, multiply first by what you divided by last, and so on: adding in the remainders as you proceed, if there are any.

Remark.—The four examples next following, prove the four last in Reduction Descending; that is, the operations and answers of the one, prove those of the other.

Examples.

1. In 45776 farthings, how many pence, shillings, and pounds?

$$\text{Farthings in a penny} = 4 \overline{) 45776}$$

$$\text{Pence in a shilling} = 12 \overline{) 11444} \text{ Pence.}$$

$$\text{Shillings in a pound} = 20 \overline{) 953} \text{ 3s. 8d.}$$

Value in the highest terms = £ 47 13s. 8d. Or, the answer in other terms is 11444 pence; greatest number of shillings = 953; and greatest number of pounds = 47.

To prove the work,

+ Eagles, dollars, dimes, cents, and mills, are reduced to mills by writing them all down in their order; thus, 4 E. 30. 6d. 7c. and 8m., are 43678 mills; and 4 E. = 40000 mills.

REDUCTION

£ 47 13s. 8d.

20
953 shillings.

12
11444 pence.

Proof. 45776 farthings.

Explanation.

Here in multiplying by 20, the 13s. were taken in; ^{and} in multiplying by 12, the 8d. were taken in

2. In 45587 cents, how many dimes, dollars, and eagles?

Ans. 45 E. 5 D. 8d. 7c.

Note.—Any number of cents or mills are most readily reduced to higher denominations by cutting off one figure at a time for mills, cents, dimes, dollars, and eagles; thus, 78476 mills will be expressed

E. D. d. c. m.
7, 8, 4, 7, 6.

To reduce mills to dollars, cents, and mills, cut off one figure for mills, and two for cents, and the remaining left hand figures will be dollars. Thus, 47698 mills =

47, 69, 8
\$ cts. m.

The reason for cutting off is because cutting off one figure is the same as dividing by 10, and cutting off two figures is dividing by 100.

3. In 6344 lb. how many quarters and hundred?

Ans. 226 qrs. 56 cwt., and 16 lb. remain.

4. In 31557600 seconds, how many minutes, hours, and days?

Ans. 525960 m., 8766 h., 365 d. and 6 h.

REDUCTION DESCENDING AND ASCENDING.

Examples. English Money.

1. In £ 3271 17s. 4d., how many pence?

Ans. 785248.

2. In 1712805 pence, how many pounds, &c.?

Ans. £ 7136 13s. 9d.

REDUCTIONS.

Federal Money.

1. In 31271 cents, how many dollars?
Ans. \$312. 71.
2. In 85132 dollars, how many cents? Ans. 85132.
3. In 576948 mills, how many dollars?
Ans. \$576. 94. 8.

Troy Weight.

1. In 437 lb. 10 oz. 19 pwts. 16 grs. how many grains?
Ans. 2522392.
2. In 675793 pennyweights, how many pounds?
Ans. 1982 lb. 5 oz. 13 pwts.

Avoirdupois Weight.

1. In 45 tons 14 cwt. 2 qrs. 14 lb., how many ounces?
Ans. 102438.
2. In 8099850 ounces, how many tons?
Ans. 226.

Apothecaries' Weight.

1. In 984 lb. 10 oz. 4 dr. 0 ss. 12 grs., how many grains?
Ans. 5672892.
2. In 115200 grains, how many pounds?
Ans. 20 lb.

Cloth Measure.

1. In 58 yds. 3 qrs., how many nails?
Ans. 740 na.
2. In 1507 nails, how many English Ells?
Ans. 75 El. 1 qr. 3 na.

Long Measure.

1. In 100 miles, how many barley corns?
Ans. 19008000 bar.
2. How many barley corns will reach round the world, it being 360 degrees, and 69 $\frac{1}{2}$ miles being reckoned a degree? [See the following remark.]
Ans. 47555801600.
3. Reduce 5424194160 barley corns to degrees.
Ans. 410 deg. 41 mil. 3 fur.

REDUCTION

Remark.—To multiply any number by a fraction, as $\frac{1}{4}$, $\frac{1}{2}$, &c., take $\frac{1}{4}$, $\frac{1}{2}$, &c. of the number. Thus, to reduce 4 deg. 10 mil. 3 fur. to feet, the operation is as follows.

$$\begin{array}{r} \text{deg. mil. fur.} \\ 4 \quad 10 \quad 3 \\ 69\frac{1}{2} \end{array}$$

$$\begin{array}{r} 36 \\ 24 \end{array}$$

$$276 = 4 \text{ multiplied by } 69.$$

$$2 = \frac{1}{2} \text{ of } 4, \text{ or } 4 \text{ multiplied by } \frac{1}{2}.$$

$$\text{Miles } 278 = 4 \text{ multiplied by } 69\frac{1}{2}.$$

$$\text{Add } 10 = \text{the odd miles.}$$

$$288 = \text{all the miles.}$$

$$8 = \text{the furlongs in a mile.}$$

$$2307 = \text{all the furlongs, adding the 3 odd ones.}$$

$$40 = \text{the rods in a furlong.}$$

$$92280 = \text{the number of rods.}$$

$$16\frac{1}{2} = \text{the feet in a rod.}$$

$$553680$$

$$92280$$

$$46140 = \frac{1}{2} \text{ of } 92280.$$

$$1522620 = \text{the feet in } 4 \text{ deg. } 10 \text{ mil. } 3 \text{ fur.}$$

If the pupil should be at a loss about working the 2d and 3d. sums in long measure, or any others where halves or quarters are concerned, the example wrought under this remark will explain the difficulty. But it must be further remarked, however, that to divide by a whole number and a fraction, as $16\frac{1}{2}$, $69\frac{1}{2}$, &c. in reducing feet to rods, or miles to degrees, the divisor and dividend must be reduced to halves, before you divide, and then what remains will be halves.

Land Measure.

1. In 40 acres, 3 quarters (or rods) and 27 square rods, how many rods? Ans. 6547.

2. In 7647 square rods, how many acres?

Ans. 47 A. 3 roo. 7 rods.

Wine Measure.

1. In 50 hogsheads, how many gills?

Ans. 100800.

2. In 50400 pints, how many hogsheads?

Ans. 50.

REDUCTION.

Dry Measure.

1. In 340 bushels, 3 pecks, and 7 quarts, how many pints? Ans. 21822.

2. In 57435 pints how many bushels?
Ans. 897B. 1pk. 5qts. 1pt.

Time.

1. In 471 yrs. 3mo. 2w. how many seconds?
Ans. 13681785600.

2. If a person's age is 24 years, how many seconds old is he, allowing each year to be 365 days and 6 hours?
Ans. 662709600.

3. A good common clock clicks once every second; how many times does it click in a common year of 365 days?
Ans. 31536000 times.

Additional Examples.

1. In 875 pounds lawful money, how many sixpences? Ans. 35000.

2. In 17500 shillings how many moi-
clores at 36s. each?
Ans. 486 and 4s. over.

3. In 69888 farthings how many guineas, at 28s. ? Ans. 52 guin.

4. In 8976 pence, how many pistoles, at 22s. each? Ans. 34.

5. In 159840 half pence, how many crowns, at 6s. 8d. ? Ans. 999.

Problem.—To find how many times several sorts of coin of different values, but equal in number, are contained in any number of another kind.

Rule.—1. Reduce the several sorts of coin to the lowest denomination mentioned, and add them together for a divisor.

REDUCTIONS.

2. Reduce the given sum to the same denomination for a dividend, and the quotient found by division will be the number required.

Examples.

1. In 150 half joes at 48s. each, how many moidores at 36s., guineas at 28s., pistoles at 22s., dollars at 6s., shillings, and sixpences, of each the like number? Ans. 77, and 6d. besides.

⚡ The scholar will quickly see that the divisor and dividend must both be brought into sixpences.

2. A gentleman gave £500 towards educating 4 young men, to be distributed in the following proportion, viz.; that as often as A had £10, B should have £20, C £30, and D £40: what sum did each receive?

Ans. A £50; B £100; C £150; and D £200.

Another Example of much the same Nature.

Three men own 26 lb. of silver in partnership; A's part of it is 10 lb., and of the remaining part B's share is 3 lb. 5 oz. as often as C's is 4 lb. 7 oz. What is the whole share of B & C? Ans. B owns 6 lb. 10 oz., and C 9 lb. 2 oz.

⚡ To answer this last question, first reduce the 16 lb. (B's and C's part) to ounces, for the dividend; next, B's ratio or proportion to C's is 3 lb. 5 oz. = 41 oz., and C's ratio 4 lb. 7 oz. = 55 oz., and the sum of both, 96 oz., is the divisor by which divide the dividend, and the quotient 2, multiplied into 41 gives B's ounces, viz., 82 oz. = 6 lb. 10 oz.; and 55 multiplied by 2 gives 110 oz. = 9 lb. 2 oz., C's share.

I Questions for Examination.

1. What is Reduction?
2. How many Kinds of Reduction are ^{there,} and what are they called?
3. What is Reduction Descending?
4. What is Reduction Ascending?
5. How do you work Reduction Descending?
6. How do you perform Reduction Ascending?

GENERAL VIEW OF FRACTIONS.

A Fraction is a part, or the parts, of a unit. From the nature of the art or science of computing by numbers, fractions, in some form or other, are almost always intruding themselves into arithmetical calculations. Fractions are of two kinds, vulgar and decimal.

A Vulgar Fraction is represented by two numbers placed one above another, with a line drawn between them, thus; $\frac{1}{2}$ one half, $\frac{3}{4}$ three fourths. The number above is called the numerator, and the lower one the denominator.

The denominator shows how many parts the unit or 1 is divided into; and the numerator shows how many such parts are expressed by the fraction.

Vulgar Fractions are divided into Proper, Improper, Compound and Mixed.

1. A PROPER FRACTION, has the numerator less than the denominator; and is always less in value than a unit, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{13}{17}$, &c.

2. An IMPROPER FRACTION, has the numerator equal to, or greater than the denominator; as $\frac{4}{3}$, $\frac{13}{7}$, &c.

Proper, and Improper Fractions are both called Single Fractions.

If the numerator and denominator are the same, the fraction is equal to unit; as $\frac{3}{3}$, $\frac{5}{5}$, $\frac{9}{9}$, all of which are equal to one. If the numerator is greater than the denominator, the fraction expresses a quantity greater

OF FRACTIONS.

than unit, thus; $\frac{8}{4}$ expresses a quantity equal to 2 units; because if the unit is divided into four parts, and the numerator requires 8 such parts, it requires twice as much as the original unit, and consequently expresses the value of 2.

3. A MIXED NUMBER OR FRACTION is composed of a whole number and a Fraction at $3\frac{2}{3}$, $4\frac{7}{8}$, &c.

4. A COMPOUND FRACTION, is the fraction of a fraction, and known by the word *of*, as $\frac{1}{2}$ of $\frac{2}{3}$; $\frac{1}{7}$ of $\frac{3}{4}$ of $\frac{2}{5}$ of $\frac{1}{12}$, &c.

5. A WHOLE NUMBER, may be made a fraction by writing 1 under it for a denominator; thus, 4 may be expressed $\frac{4}{1}$, &c. ~~See~~

6. The COMMON MEASURE, of two or more numbers is any number that will divide each of them without a remainder. The greatest number that will do this, is called the GREATEST COMMON MEASURE; thus, 3 will divide 12, 18, and 24, without a remainder, and is called their COMMON MEASURE; 6 will likewise divide them all, and being the greatest number that will do so, is called their GREATEST COMMON MEASURE.

7. The COMMON MULTIPLE, is any number which can be measured by two or more numbers; thus, 75, 60, 45, and 30, can all be measured by 5 and 3; and are therefore called *common multiples* of 5 and 3: but the *least* number that can be measured by 5 and 3 is 15, which is therefore called their LEAST COMMON MULTIPLE.

8. A COMPOSITE NUMBER, may be divided by some other number without a remainder; all even numbers, therefore, except 2, are *composite numbers*.

9. A PRIME NUMBER, cannot be divided by any other number without a remainder. No even number, therefore, except 2, can be a *prime number*.

All *prime numbers*, except 2 and 5, have 1, 3, 7, or 9, in the units place; all odd numbers, therefore, that have not 1, 3, 7, or 9, in the units place are *composite numbers*.

OF FRACTIONS.

Every number, which is *not prime*, is a *multiple* of some *prime* (Euclid 7. 33) the regular series, therefore, of *odd numbers*, such as 3, 5, 7, 9, 11, &c. ~~&c.~~ must consist of *prime numbers*, and *multiples* of primes.

If a number cannot be divided by some number less than its square root, that number is a *prime*.

GENERAL RULE for finding all the PRIME numbers.

1. Place all the *odd numbers* in a row, continuing them as far as necessary.
2. Cancel every *third* figure, counting from the first *odd prime* (3.) the cancelled figures will be the *multiples* of that *prime* (3.)
3. Cancel every *fifth* figure, counting from the next *prime* (5) for the *multiples* of (5.)
4. Cancel every *seventh* figure, counting from (7,) every *ninth* figure, counting from (9,) every *eleventh* figure from (11,) and so on, till all the *multiples* are cancelled: the remaining figures are *PRIMES*.

EXAMPLE.

1. Find the *primes* in 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, &c. &c.
In the above example, counting every *third* figure from 3, cancels 9, 15, 21, 27, 33, &c. Every *fifth* figure from 5, cancels 15, 25, 35, &c.—Every *seventh* figure from 7, cancels 21, 35, &c.—Every *ninth* figure from 9, cancels 27, &c.—Every *eleventh* figure from 11, cancels 33, &c.—the cancelled figures viz. 9, 15, 21, 25, 27, 33, 35, are *MULTIPLES*; and the uncanceled figures, viz. 3, 5, 7, 11, 13, 17, 19, 23, 29, 31; are *PRIMES*.

EXAMPLE.

2. Find all the *primes* under 100—Ans. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
10. A PERFECT NUMBER is equal to the sum of all its aliquot parts.*

An aliquot part of a number is contained a certain number of times exactly in the number; but the number itself is not properly considered an aliquot part in arithmetical computations.

* The following are all the perfect numbers at present known.

$$\begin{aligned}
 6 &= 1 + 2 + 3. \\
 28 &= 1 + 2 + 4 + 7 + 14, \&c. \\
 496 & \\
 8128 & \\
 33550336 & \\
 8589869056 & \\
 137438691328 & \\
 2305843008139952128 & \\
 2317851639228158837784576 & \\
 9903520314282971830448816128 &
 \end{aligned}$$

DECIMAL FRACTIONS.

A Decimal Fraction is a sort of artificial number produced from a vulgar fraction. Decimal fractions were invented to render computations by fractions more simple and easy than they are by working by vulgar fractions, and the results by decimals are much more exact than you can make change, weight, or measure, of any kind. So that whoever thoroughly understands common decimals, will have no need of the most difficult part of vulgar fractions, except in matters of curiosity.

DECIMAL FRACTIONS.

A Decimal Fraction always has for its denominator a unit, with at least so many ciphers annexed as the numerator has places of figures; therefore, as the denominator is always 10, or 100, or 1000, &c., it need not be set down, for the numerator is made to express the value, placing a point or comma[†] before it, thus; $\frac{5}{10}$ is written or expressed .5; $\frac{5}{100}$ is .05; $\frac{5}{1000}$ is .005; &c.: By which it is shown that, in decimals, a single figure with a point before it, is so many tenths; and two places of figures, whether one is a cipher or not, are hundredths; and three places of figures are thousandths, &c. But it must be remembered that ciphers placed at the right hand of the last decimal figure, do not alter the value; thus, .5 is the same as .50; for $\frac{5}{10}$, and $\frac{50}{100}$, is each equal to $\frac{1}{2}$: yet every cipher placed at the left hand of a decimal figure, decreases its value ten times, as will plainly appear by the following table, in which the value of decimals decreases from units towards the right hand, in the same proportion that whole numbers increase in value towards the left hand.

T A B L E.

Whole Numbers.										Decimals.									
C Millions	X Millions	C Millions	X Thousands	C Thousands	X Thousands	C Hundreds	X Hundreds	C Tens	X Tens	C Tenths	X Hundredths	C Thousandths	X Thousandths	C Ten thousandths	X Ten thousandths	C Hundred thousandths	X Hundred thousandths	C Millionths	X Millionths
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9		

[†] The point or comma placed before a decimal, is called a separatrix.

DECIMAL FRACCTIONS.

ADDITION OF DECIMALS.

Rule.—Place the numbers so that the commas or separating points may all stand under one another, and then proceed exactly as in addition of whole numbers, and point off as many places for decimals in the sum, as are equal to the greatest number of decimals in either of the numbers added. — When there are whole numbers with decimals, place them to the left hand of the comma or separatrix.

Proof.—The same as in Simple Addition.

Reason.—The reason why a single figure in decimals, is set furthest to the left hand, instead of to the right, as in whole numbers, is because tens, hundreds, &c. in whole numbers, proceed from right to left; but in decimals, tenths, hundredths, &c., proceed from left to right; yet the real value increases the same way in decimals as in whole numbers, for although 10 is ten times less than 100, a tenth is ten times more than a hundredth.

DECIMAL FRACTIONS.

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EXAMPLES.

1. Add $5,207 + 331,6 + ,004 + 72,35 + 591, + 6,7032$.
Ans. $996,8642$.

OPERATION.

$$\begin{array}{r} 5,207 \\ 331,6 \\ ,004 \\ 72,35 \\ 591, \\ 6,7032 \\ \hline 996,8642 \end{array}$$

2. Add $72,5 + 32,071 + 2,1574 + 371,4 + 2,75$.
Ans. $480,8784$.

3. Add $30,07 + 200,71 + 59,4 + 3207,1$.
Ans. $3497,28$.

4. Add $3,5 + 47,25 + 927,01 + 2,0073 + 1,5$.
Ans. $981,2673$.

5. Add $52,75 + 47,21 + 724 + 31,452 + 3075$.
Ans. $855,7195$.

6. Add $3275 + 27,514 + 1,005 + 725 + 7,32$.
Ans. $4035,839$.

7. Add $27,5 + 52 + 3,2075 + 5741 + 2720$.
Ans. $2803,2816$.

SUBTRACTION OF DECIMALS.

Rule. — Place the numbers as taught in addition, and proceed as in subtraction of whole numbers. *

EXAMPLES.

1. From $873,421$ take $52,07325$.

OPERATION.

$$\begin{array}{r} 873,421 \\ 52,07325 \\ \hline 821,34775 \text{ Ans.} \end{array}$$

* The method of proof is the same as in Simple Subtraction; and the reason for placing the numbers is the same as in addition.

DECIMAL FRACTIONS.

2. From ,2754 take ,2371 Ans. ,0383.
3. From 271 take 215,7 Ans. 55,3.
4. From 270,2 take 75,4075 Ans. 194,7925.
5. From 107 take ,0007 Ans. 106,9993.

MULTIPLICATION OF TERMINATE DECIMALS.

R U L E.

Place the factors, and multiply them as in whole numbers, and from the product towards the right hand cut off as many places for decimals as there are in both factors together; but if there should not be so many places in the product, supply the defect with cyphers to the left-hand.†

E X A M P L E S.

1. Multiply 15,73 by ,0056 Ans. ,088088

O P E R A T I O N.

$$\begin{array}{r}
 15.73 \\
 ,0056 \\
 \hline
 9438 \\
 7865 \\
 \hline
 ,088088
 \end{array}$$

NOTE, There is no need to place the comma's or separating points under each other except in Addition and Subtraction.

2. Multiply 2,071 by 2,27 Ans. 4,70117.
 3. Multiply 5745 by ,0675 Ans. 387,7875.
 4. Multiply ,047 by ,046 Ans. ,002162.
 5. Multiply 8,04704 by ,2575 Ans. 2,072128.
- When any number of decimals is to be multiplied by 10, 100, 1000, &c. it is only removing the separating point in the multiplicand so many places towards the right-hand as there are cyphers in the multiplier: thus, $,578 \times 10 = 5,78$, $,578 \times 100 = 57,8$, $,578 \times 1000 = 578$, $,578 \times 10000 = 5780$.

Contracted MULTIPLICATION of DECIMALS.

R U L E.

Put the unit's place of the multiplier under that place of the multiplicand that is intended to be kept in the product; then invert the order of all the

† The method of Proof is the same as in Simple Multiplication.

DECIMAL FRACTIONS.

that is,

other figures, ~~and~~ write them all the contrary way; then in multiplying, begin at the figure in the multiplicand, which stands over the figure you are then multiplying with, and set down the first figure of each particular product directly one under the other, and have a due regard to the increase arising from the figures on the right-hand of that figure you begin to multiply at in the multiplicand.

In ~~That in~~ multiplying the figure left out every time next the right-hand in the multiplicand, if the product be 5, or upwards to 15, carry 1; if 15, or upwards to 25, carry 2; and if 25, or upwards to 35 carry 3, &c.

EXAMPLES.

6. Multiply 384,672158 by 36,8345, and let there be only four places of decimals in the product.

Contracted way.

384,672158
5438,63

115401647
23080329
3077377
115402
15387
1923

14169,2065

Ans. 14169,2065.

Common way.

384,672158
36,8345

1923360790
1538688632
1154016474
3077377264
2308032948
1154016474

14169,2065038510

7. Multiply 3,141592 by 52,7438, and leave only 4 places of decimals.

Ans. 165,6994.

8. Multiply 2,38645 by 8,2175, and leave only 4 places of decimals.

Ans. 19,6107.

DIVISION OF TERMINATE DECIMALS.

RULE. Proceed as in whole numbers, separating as many figures for decimals, in the quotient, as the dividend has more than the divisor.

DECIMAL FRACTIONS.

NOTE 1. If the divisor and dividend have both the same number of decimal parts, the quotient will be a whole number.

2. If the dividend has not so many places of decimals as are in the divisor, then so many cyphers must be annexed to the dividend as will make them equal, and the quotient will then be a whole number.

3. But if, when the division is done, the quotient has not so many figures as it should have places of decimals, then so many cyphers must be prefixed as there are places wanting.

EXAMPLES.

1. Divide 721,17562 by 2,257432. *Ans.* 319,467+

See the operation of this Example in page 227.

2. Divide 8564,825 by 6,321. *Ans.* 1354,9.

3. Divide 6,7258 by 647. *Ans.* ,010395+

4. Divide 246,1476 by 604,25. *Ans.* ,40736+

5. Divide ,0008136 by 678. *Ans.* ,0000012.

When numbers are to be divided by 10, 100, 1000, 10,000, &c. it is performed by placing the separating point in the dividend, so many places towards the left hand, as there are cyphers in the divisor.

Thus, $5784 \div 10 = 578,4$ | $5784 \div 1000 = 5,784$.

$5784 \div 100 = 57,84$ | $5784 \div 10000 = 5784$.

CONTRACTED DIVISION.

RULE.

Find what is the value of the first figure in the quotient; then by knowing the first figure's denomination, the decimal places may be reduced to any number, by taking as many of the left hand figures of the dividend as will answer them; and, in dividing, omit one figure of the divisor at each following operation.

NOTE, That in multiplying every figure left out in the divisor, you may carry one, if it be 5, or upwards to 15; if 15, or upwards to 25, carry 2; if 25, or upwards to 35, carry 3, &c.

DECIMAL FRACTIONS. 227

EXAMPLES.

1. ~~6~~ Divide 721,17562 by 2,257432, and let there be only three places of decimals in the quotient.

<i>Contracted.</i>	<i>Common way.</i>
2,257432)721,17562(319,467	2,257432)721,17562(319,467
..... 6772296	6772296
439460.	439460.
225743.	225743.
213717..	213717..
203169..	203168..
10548...	15048 120
9030...	9029 728
1518....	1518 3920
1354....	1354 4592
104.....	163 93280
158.....	158 02024
6.....	5 91256
=	=

2. ~~4~~ Divide 6109,2674 by 240,649 and let there be 4 places of decimals in the quotient. *Ans.* 25,3866.
3. ~~8~~ Divide 913,08 by 2187,2 and leave 3 places of decimals. *Ans.* ,427.

REDUCTION OF TERMINATE DECIMALS.

To reduce a Vulgar Fraction to a Decimal.

R U L E.

Add cyphers to the numerator, and divide by the denominator, the quotient is the decimal fraction required.

EXAMPLES.

1. Reduce $\frac{1}{4}$ --- to a decimal. 4)1,00(.25 *Ans.*
2. Reduce $\frac{1}{2}$ --- to a decimal. *Ans.* ,5
3. Reduce $\frac{3}{4}$ --- to a decimal. *Ans.* ,75
4. Reduce $\frac{3}{8}$ --- to a decimal. *Ans.* ,375
5. Reduce $\frac{5}{16}$ --- to a decimal. *Ans.* ,3125
6. Reduce $\frac{11}{14}$ of $\frac{10}{13}$ to a decimal, *Ans.* ,6043956+

Case I.

Case II.
Method First.

DECIMAL FRACTIONS.

To reduce quantities of several denominations to a Decimal.

RULE, Place the several denominations above each other letting the highest denomination stand at the bottom; then, divide each denomination, (beginning at the top) by its value in the next denomination the last quotient will give the decimal required.

E X A M P L E S.

1. → Reduce 3l. 18s. 8 $\frac{2}{5}$ d. to the decimal of a £.

OPERATION.

$$\begin{array}{r} 5 \overline{) 2,} \\ 12 \overline{) 8,4} \\ 20 \overline{) 18,7} \\ \hline \text{£. } 3,935 \text{ Ans.} \end{array}$$

2. → Reduce 9s. to the decimal of a £. Ans. .45.
3. → Reduce 16s. to the decimal of a £. Ans. .8.
4. → Reduce 8s. 4d. to the decimal of a £. Ans. .416.
5. → Reduce 16s. 7 $\frac{1}{2}$ d. to the decimal of a £. Ans. .8322916.
6. → Reduce 19s. 5 $\frac{1}{2}$ d. to the decimal of a £. Ans. .972916.
7. → Reduce 12 grains to the decimal of a lb. Troy. Ans. .002083.
8. → Reduce 12 drams to the decimal of a lb. Avoirdupoise. Ans. .046875.
9. → Reduce 2 qrs. 14 lb. to the decimal of an cwt. Ans. .625.
10. → Reduce 2 quarts, 1 pint to the decimal of a gallon. Ans. .625.
11. → Reduce 52 days to the decimal of a year. Ans. .142465+

Method Second.

Rule.—First, reduce all the denominations except the highest, to that of the lowest given, by Reduction Descending, for the numerator of a vulgar fraction; and 1 of the highest denomination to the same name of the lowest, for a denominator: then reduce the vulgar fraction to a decimal, and you will have the whole decimal of all the less denominations at one operation, which is generally much the easiest method.

Example.

Reduce the 3l. 18s. 8 $\frac{2}{5}$ d. in this case, to the decimal of a £. by this Second Method.

First, 18s. 8 $\frac{2}{5}$ d. reduced to 5ths of a penny, is 1122, the numerator; and £1. reduced to 5ths of a penny is 1200, the denominator. Then, $\frac{1122}{1200}$ reduced to a decimal, is .935 = the decimal part found by the first method.

DECIMAL FRACTIONS.

Case III.

To find the value of any Decimal Fraction, in the known parts of an Integer.

RULE. Multiply the decimal given, by the number of parts of the next inferior denomination, cutting off the decimals from the product; then multiply the remainder by the next inferior denomination; thus proceeding, till you have brought in the least known parts of the integer.

EXAMPLES.

1. ~~18~~ What is the value of .8322916 of a £. ?

$$\begin{array}{r} .8322916 \\ \times 20 \\ \hline 16,6458320 \\ \times 12 \\ \hline 7,7499840 \\ \times 4 \\ \hline 49999360 \end{array}$$

Ans. 16s. 7½d.
2. ~~19~~ What is the value of .002084 of a lb. Troy ?
Ans. 12,00384 grs.
3. ~~20~~ What is the value of .046875 of a lb. Avoirdupoise ?
Ans. 12 drams.
4. ~~21~~ What is the value of .625 of a cwt.
Ans. 2 qrs. 14 lb.
5. ~~22~~ What is the value of .625 of a gallon ?
Ans. 2 quarts, 1 pint.
6. ~~23~~ What is the value of .071428 of an hogshead of wine ?
Ans. 4 gal. 1 quart, 999856.
7. ~~24~~ What is the value of .0625 of a barrel of beer ?
Ans. 2 gallons, 1 quart.
8. ~~25~~ What is the value of .142465 of a year ?
Ans. 51,999725 days.

DECIMAL TABLES OF COIN, WEIGHT, AND MEASURE.

TABLE I. ENGLISH COIN. 1 <i>l.</i> the Integer.		<i>Farths.</i>	<i>Decimals.</i>	<i>Grains.</i>	<i>Decimals.</i>
		3	,0625	12	,025
		2	,041666	11	,022916
		1	,028333	10	,020833
<i>Sh.</i>	<i>dec.</i>	<i>Sh.</i>	<i>dec.</i>		
19	,95	9	,45	9	,01875
18	,9	8	,4	8	,016666
17	,85	7	,35	7	,014583
16	,8	6	,3	6	,0125
15	,75	5	,25	5	,010416
14	,7	4	,2	4	,008333
13	,65	3	,15	3	,00625
12	,6	2	,1	2	,004166
11	,55	1	,05	1	,002083
10	,5				

TABLE III. TROY WEIGHT. 1lb. the Integer. <i>Ounces the same as Pence in the last Table.</i>		<i>Penny weight.</i>	<i>Decimals.</i>
		10	,041666
		9	,0375
		8	,033333
		7	,029166
		6	,025
		5	,020833
		4	,016666
		3	,0125
		2	,008333
		1	,004166

TABLE IV. AVOIRUP. WT. 112lb. the Integer.		<i>Qrs.</i>	<i>Decimals.</i>
		3	,75
		2	,5
		1	,25

<i>Farths.</i>	<i>Decimals.</i>	<i>Grains.</i>	<i>Decimals.</i>	<i>Pounds.</i>	<i>Decimals.</i>
3	,003125	12	,002083	14	,125
2	,0020833	11	,001910	13	,116071
1	,0010416	10	,001736	12	,107143
		9	,001562	11	,098214
		8	,001389	10	,089286
		7	,001215	9	,080357
		6	,001042	8	,071428
		5	,000868	7	,0625
		4	,000694	6	,053571
		3	,000521	5	,044643
		2	,000347	4	,035714
		1	,000173	3	,026786
				2	,017857
				1	,008928

TABLE II. ENG. COIN 1 <i>Sh</i> Long Meaf. 1 <i>Foot</i> the Integer.		1 <i>Oz.</i> the Integer. <i>Penny weights the same as shillings in the first Table.</i>	
<i>Pence and Inches.</i>	<i>Decimals.</i>		
6	,5		
5	,416666		
4	,333333		
3	,25		
2	,166666		
1	,083333		

<i>Ounces.</i>	<i>Decimals.</i>
8	,004464
7	,002906

DECIMAL FRACTIONS.

Decimal TABLES of COIN WEIGHT, and MEASURE.

500	,284091	80	,219178	TABLE X. CLOTH MEASURE. 1 Yard the Integer. Qrs. the same as Table IV.	
400	,227272	70	,191751		
300	,170454	60	,164333		
200	,113636	50	,136916		
100	,056818	40	,109589	Nails. Decimals.	
90	,051136	30	,082192		
80	,045454	20	,054794	2	,125
70	,039773	10	,027397	1	,0625
60	,034091	9	,024657	TABLE XI. LEAD WEIGHT. A Fother the Integer.	
50	,028409	8	,021918		
40	,022727	7	,019178		
30	,017045	6	,016438		
20	,011364	5	,013698	Hund. Decimals.	
10	,005682	4	,010959		
9	,005114	3	,008219	10	,512820
8	,004545	2	,005479	9	,461538
7	,003977	1	,002739	8	,410256
6	,003409	1 Day the Integer.		7	,358974
5	,002841			6	,307692
4	,002273	Hours.	Decimals.	5	,256410
3	,001704	12	,5	4	,205128
2	,001136	11	,458333	3	,153846
1	,000568	10	,416666	2	,102564
		9	,375	1	,051282
Feet.	Decimals.	8	,333333	Qrs. Decimals.	
2	,0003787	7	,291666		
1	,0001893	6	,25	2	,025641
		5	,208333	1	,012820
Inches.	Decimals.	4	,166666	Pound. Decimals.	
6	,0000947	3	,125		
3	,0000473	2	,083333	14	,0064102
1	,0000156	1	,041666	13	,0059523
TABLE IX. TIME.		Minutes.	Decimals.	12	,0054945
		30	,020833	11	,0050360
1 Year the Integer		20	,013888	10	,0045787
Months the same as		10	,006944	9	,0041208
Pence in the second		9	,00625	8	,0036630
Table.		8	,005555	7	,0032051
Days.	Decimals.	7	,004861	6	,0027472
365	1,000000	6	,004166	5	,0022893
300	,821918	5	,003472	4	,0018315
200	,547945	4	,002777	3	,0013730
100	,273972	3	,002083	2	,0009157
90	,246575	2	,001388	1	,0004576
		1	,000694		

DECIMAL FRACTIONS.

Decimal TABLES of COIN, WEIGHT, and MEASURE.

6	.003348	80	.317460	Pints.	Decimals.	
5	.002790	70	.27777	3	.005952	
4	.002232	60	.238095	2	.003968	
3	.001674	50	.198412	1	.001984	
2	.001116	40	.158730	TABLE VII. MEASURE. Liquid. Dry. 1 Gallon. 1 Quarter Integer.		
1	.000558	30	.119047			
TABLE V. Avoirdup. Wt. 1 lb. the Integer.		20	.079365	Pint.	Dec.	Engh.
		10	.039682	4	.5	4
Oz.	Decimals.	9	.035714	3	.375	3
3	.000418	8	.031746	2	.25	2
2	.000279	7	.027777	1	.125	1
1	.000139	6	.023809	Q. pt. Dec. Pk.		
Ounces.		5	.019841			
		4	.015873	3	.09375	3
Pints.		3	.011904	2	.0625	2
		2	.007936	1	.03125	1
8	.5	1	.003968	Decimals. Q. pks.		
7	.4375	TABLE VIII. LONG MEASURE. 1 mile the Integer.				
6	.375	Gallons.	Decimals.	Decimals.	Pints	
5	.3125	30	.476190	.005859	3	
4	.25	20	.317460	.003906	2	
3	.1875	10	.158730	.001953	1	
2	.125	9	.142857	TABLE VIII. LONG MEASURE. 1 mile the Integer.		
1	.0625	8	.126984			
Drams.		7	.111111	Yeras.	Decimals.	
		6	.095238	1000	.568182	
TABLE VI. LIQUID MEAS. 1 tun the Integer		5	.079365	900	.511364	
		4	.063492	800	.454545	
Gallon.	Decimals.	3	.047619	700	.397727	
100	.396825	2	.031746	600	.340909	
90	.357141	1	.015873			

COMPOUND MULTIPLICATION

COMPOUND MULTIPLICATION

Is a very useful rule in business, and teaches to find the value of goods, &c., by knowing the price of one thing of the kind.

Rule.—Multiply the price, weight, or measure, of one thing or quantity by the number of things or quantities whose value, weight, or measure is wanted, and carry as in compound addition.

Federal Money.

To find the value of any number of things in Federal Money;—as cents are hundredths of a dollar, and mills are thousandths of a dollar, multiply the price and quantity together, after first reducing the parts if there are any, of the highest denomination to the decimal of the highest, and point off as many places for decimals in the product as the rule of the Multiplication of Decimals requires; then the figures to the left hand of the decimal point will be dollars, and those on the right hand will be decimals of a dollar.

Proof and Reason of the Rule.—The same as in multiplication of decimals.

Examples.

1. What cost 574 lb. of sugar, at $12\frac{1}{2}$ cents per pound?—Here $12\frac{1}{2}$ cents = 125 mills, which = .125 of a dollar; therefore, 574 lb. multiplied by .125 = \$71,750; that is, \$71,75, because in reckoning dollars and decimals of a dollar, the two left hand decimal figures are cents, and the next decimal figure is mills.

2. What cost $6\frac{3}{4}$ gallons of wine, at \$1,67 a gallon? Ans. \$11,2725.

Here $6\frac{3}{4}$ expressed decimally, is 6,75, and $6,75 \times 1,67 = 11,2725$, which is \$11,27cts., and $2\frac{1}{2}$ mills, because 5 tenths of a mill is 1 half.

3. What cost 4 cwt. 3 qrs. 9 lb. of rice, at \$4,75 per cwt.? Ans. \$22,94,4 nearly.

The Operation.

COMPOUND MULTIPLICATION

First reduce the 4 cwt. 3 qrs. 9 lb. to the decimal of a hundred gross, by Case 2d of Reduction of Decimals, thus.

First Method.

$$\begin{array}{r} 28 \overline{) 9} \\ 4 \overline{) 3.3214+} \\ \hline \text{Cwt. } 4.8303+ \end{array}$$

Second Method.

3 qrs. 9 lb. = 93 lb. the numerator; and 1 cwt. = 112 lb. the denominator; and $\frac{93}{112}$ reduced to a decimal, is .8303+, to which prefix 4 cwt. and it will be $4.8303+$, as before. Then,

$$\begin{array}{r} 4.8303 \\ 4.75 \\ \hline 241515 \\ 338121 \\ \hline 193212 \\ \$22,943925 \text{ Ans.} \end{array}$$

Here as the figure next to mills (9) is so near the value of 10, the mills may be reckoned 4.

4. What is the value of 3574 feet of boards, at \$5.25 a thousand?

Ans. \$18,7635. That is }
\$18. 76 cts. 3½ m.

To solve this question, the 574 feet are reduced to the decimal of a thousand, by prefixing a comma; so the whole is 3.574, which multiply by \$5.25. — Proceed in the same manner in all cases of a similar nature.

English or Lawful Money.

CASE I.

When the the quantity does not exceed 12.

Rule.—Multiply the price by the quantity, and carry as in addition of Lawful Money.

Proof.—After learning Compound Division, divide the product by the multiplier, and the quotient will be the multiplicand; but,

COMPOUND MULTIPLICATION.

before Compound Division is learned, after finding the answer as the rule directs, find the answer again by multiplying by two numbers whose sum is the whole multiplier, and if the sum of the products is the same as the first answer, you may conclude the work is right. The rule and method of proof is the same for all compound numbers.

Examples.

1. What cost 4 lb. of coffee, at 1s. 6d. a pound?

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 1 \quad 6 \\ \times 4 \\ \hline 6 \quad 0 \end{array} \text{ Ans.}$$

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 13 \quad 6 \\ \times 3 \\ \hline 4 \quad 6 \end{array} \text{ Price of 3 lb.}$$

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 1 \quad 6 \\ \times 1 \\ \hline 1 \quad 6 \end{array} \text{ Price of 1 lb.}$$

$$\text{Proof. } \begin{array}{r} \text{s.} \quad \text{d.} \\ 6 \quad 0 \\ \times 4 \\ \hline 24 \quad 0 \end{array} \text{ Price of 4 lb.}$$

Explanation.

The first answer is found by multiplying 6d. by 4, which produces 24d. or just 2s. Then 4 times 1s. is 4s., and 2 to carry is 6s., the answer.

In proving the work, as 3 and 1 are 4, the price 1s. 6d. is first multiplied by 3, and then by 1, and the sum of the products is 6s. 0d., the same as the first answer. The result in proving the work would be the same if the price were multiplied by 2 and 2, instead of 3 and 1.

2. What will 9 acres of land cost, at £1. 17s. 8d. an acre?

$$\text{Ans. } £16 \quad 19s.$$

In example 2d, I say (after setting 9 under the 8d.) 9 times 8 are 72 pence, = 6s.; then 9 times 17s. multiplying one figure at a time, and carrying 6, is 159s., which is £7 19s.; and lastly, 9 times £1 is 9, and 7 to carry makes 16; so the whole is £16 19s.

3. What will 12 yards of cloth cost, at £2 5s. 4½d. a yard?

$$\text{Ans. } £27 \quad 4s. \quad 6d.$$

CASE II.

When the multiplier or quantity is more than 12.

COMPOUND MULTIPLICATION.
Rule. Multiply by two or more such numbers, as, being multiplied together, will produce the whole given quantity.

Examples.

1. What will 120 bushels of corn come to at 4s. 6d. a bushel? [$10 \times 12 = 120$.]

$\frac{3}{4}$ $\frac{d.}{6}$ Price of 1 bushel.

$\underline{\text{£}} 2 \frac{5}{12} \frac{10}{12}$ Price of 10 bushels.

$\underline{\text{£}} 27 \ 0 \ 0$ Price of 120 bushels. Ans.

Explanation.—If 10 be multiplied by 12, the product is 120, the given quantity; therefore 10 times the price of 1 bushel is the price of 10 bushels; and 12 times the price of 10 bushels is the price of 120 bushels.

Note.—The numbers taken for multipliers, are called aliquot parts of the given quantity; and no aliquot part greater than 12 is used as a multiplier, in Compound Multiplication.

2. What will 144 gallons of molasses cost at 2s. 6 $\frac{1}{2}$ d. a gallon? [$12 \times 12 = 144$.]

Ans. $\underline{\text{£}} 18 \ 6s.$

3. What cost 27 gallons, at 9s. 4d. a gallon? Ans. $\underline{\text{£}} 12 \ 12s.$

4. What cost 24 bushels at 3s. 5 $\frac{1}{2}$ d. a bushel?

Ans. $\underline{\text{£}} 3 \ 19s.$

5. What cost 28 $\frac{3}{4}$ pounds of sugar at 1s. 8d. a pound?

$\frac{3}{4}$ $\frac{d.}{8}$ Price of 1 lb.

$\frac{6}{7}$ $\frac{s.}{8}$ Price of 4 lb.

$\underline{\text{£}} 2 \ 6 \ 8$ Price of 28 lb.

$\frac{0}{10}$ $\frac{10}{10}$ Price of $\frac{1}{4}$ lb.

$\underline{\text{£}} 2 \ 7 \ 11$ Ans.

To multiply any number by a fraction, multiply by the numerator, and divide by the denominator: Or, work as in this example!

COMPOUND MULTIPLICATION.

CASE III.

When the quantity is such a number, as that no two or more numbers multiplied together will produce it exactly.

Rule. Multiply by such numbers as come the nearest to it, and for the number which is wanting, multiply the price of one by the said number wanting, and add the products together; the sum will be the answer.

Examples.

1. Multiply £ 285 14s. 6 $\frac{3}{4}$ d. by 17. Ans. £ 4857 7s. 6 $\frac{3}{4}$ d.

$$\begin{array}{r}
 \text{£.} \quad \text{s.} \quad \text{d.} \\
 285 \quad 14 \quad 6\frac{3}{4} \\
 \hline
 1152 \quad 18 \quad 3 \text{ value of 4.} \\
 \hline
 4571 \quad 13 \quad 0 \text{ value of 16.} \\
 285 \quad 14 \quad 6\frac{3}{4} \text{ value of 1.} \\
 \hline
 4857 \quad 7 \quad 6\frac{3}{4} \text{ value of 17.}
 \end{array}$$

As $4 \times 4 = 16$, I first multiply the given multiplicand by 4, and that product by 4, which gives the product or value for 16, to which

the upper line or value of 1 is added, and the sum is the value of 17. The same answer would have been found by multiplying the upper line by 5, and that product by 3, and adding twice the upper line to the last product.

2. What is the value of 23 clls of Holland, at 15s. 6 $\frac{1}{2}$ d. per ell? Ans. 1 l. 15s. 5 $\frac{1}{2}$ d.

3. What is the value of 46 bushels of wheat, at 4s. 7 $\frac{1}{2}$ d. per bushel? Ans. 10 l. 11s. 9 $\frac{1}{2}$ d.

4. What cost 59 yards of cloth, at 7s. 10d. per yard? Ans. 23 l. 2s. 2d.

5. What will 117 cwt. of raisins cost, at 1 l. 2s. 3d. per cwt.? Ans. 130 l. 3s. 3d.

6. What will 112 lb., or 1 cwt. gross weight come to at 1s. 6d. per pound? Ans. 8 l. 8s.

This last question, or any other, where the quantity is 112, is solved by multiplying by 4, and by 4, and by 7; or by 8, by 7, and by 2, because $4 \times 4 \times 7 = 112$; also, $8 \times 7 \times 2 = 112$.

COMPOUND DIVISION.

Weights and Measures.

1. How much is 15 times 3 tons, 4 cwt. 3 qrs. 7 lb., avoirdupois? Ans. 48 t. 12 cwt. 0 qrs. 21 lb.

$$\begin{array}{r}
 \text{t.} \quad \text{cwt.} \quad \text{qrs.} \quad \text{lb.} \\
 3 \quad 4 \quad 3 \quad 7 \\
 \hline
 16 \quad 4 \quad 0 \quad 7 \text{ weight of 5.} \\
 3
 \end{array}$$

$$\begin{array}{r}
 48 \quad 12 \quad 0 \quad 21 \text{ weight of 15.}
 \end{array}$$

2. Multiply 5 t. 15 cwt. 2 qrs. 10 lb. by 24.
Ans. 138 t. 5 cwt. 0 qrs. 16 lb.

3. How much is 19 times 3 bushels, 3 pecks, and 5 quarts? Ans. 74 b. 0 pk. 7 qts.

II Questions for Examination.

1. What is Compound Multiplication, or what does it teach?
2. How do you multiply a compound number by a single one?
3. How do you proceed when the multiplier exceeds 12, and can be divided into aliquot parts?
4. When no numbers multiplied together will exactly produce the whole multiplier?
5. How is a number multiplied by a fraction?
6. How is Compound Multiplication proved?

COMPOUND DIVISION.

Compound Division teaches to divide a compound number, as pounds, shillings, pence, &c., by a simple divisor, without first reducing the dividend to one denomination.

Case I.

When the Divisor does not exceed 12.

Rule.—First, find how often the divisor is contained in the highest denomination of the dividend, and set the quotient under it; next, reduce what remains to the next denomination, and add it to that denomination, and find how many times the divisor is contained in that, and so on till the whole is divided.

COMPOUND DIVISION.

Proof.—Multiply the quotient by the divisor, adding in the remainder*, when there is any, and if the work is right, the product will be equal to the dividend.

Reason.—The same in nature as in Simple Division.

Examples.

1. Divide 24 l. 7 s. 5 d. by 2. Ans. 12 l. 3 s. 8 $\frac{1}{2}$ d.

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \\ 2 \overline{) 24 \quad 7 \quad 5} \\ \underline{12} \quad \quad \quad \\ \quad 3 \quad \quad \quad \\ \quad \quad 8 \frac{1}{2} \quad \quad \\ \quad \quad \quad 2 \end{array}$$

Proof 24 7 5

Explanation.—First, in dividing, the twos in 2, will go 1 time, and in 4 two times, which are set down. Then the twos in 7 s. are 3 times, and 1 remains, which reduced to pence and added to the 5 d. makes 17, and the twos in 17 are 8 times and 1 remains, which placed over the divisor is $\frac{1}{2}$. Then to prove the work, the product is multiplied by the divisor, and as the product is equal to the dividend, the work is right.

2. Divide 75 l. 14 s. 7 $\frac{1}{4}$ d. by 3. Ans. 25 l. 11 s. 6 $\frac{1}{2}$ d.

3. Divide 28 l. 2 s. 1 $\frac{1}{2}$ d. by 6. Ans. 4 l. 13 s. 8 $\frac{1}{3}$ d.

4. Divide 135 l. 10 s. 7 d. by 9. Ans. 15 l. 11 s. 2 d.

5. Divide 6 degrees, 34 English miles, and 3 furlongs, by 5. Ans. 1 deg. 20 mi. 6 $\frac{1}{5}$ fur.

As a difficulty occurs in proving the work in solving the last question, the work, proof, and explanation are given, as follows.

$$\begin{array}{r} \text{Deg.} \quad \text{mi.} \quad \text{fur.} \\ 5 \overline{) 6 \quad 34 \quad 3} \\ \underline{1} \quad \quad \quad \\ \quad 20 \quad \quad \quad \\ \quad \quad 6 \frac{1}{5} = \text{the quotient.} \\ \quad \quad \quad 5 \\ \underline{\quad} \quad \quad \quad \\ \quad \quad 33 \frac{1}{2} \quad 7 = \text{the value of the dividend.} \end{array}$$

Here the quotient multiplied by 5 is exactly the same in value as the dividend, though expressed in a different form. The cause of the difference is, that in dividing the miles, $3 \frac{1}{2}$ remained, = 28 furlongs; but in multiplying $6 \frac{1}{5}$ furlongs by 5, the product 31 is 3 miles and 7 furlongs. In order to have the work prove when there are halves or quarters belonging to any of the

* Adding in the remainder is the same as multiplying the fraction and carrying the product to the whole number. Thus, if the quotient be 4, the remainder 3, and the divisor 5, then $5 \times 4 + 3 = 23$; and $4 \frac{3}{5} \times 5 = 23$; and so of any other remainder or fraction.

COMPOUND DIVISION.

denominations, you must always carry as much to the left hand when you multiply, as you do to the right hand in dividing.

CASE II.

When the divisor exceeds 12, and can be divided into aliquot or component parts, divide continually by those parts.

Examples.

1. Divide 48 l . 13 s . 7 d . by 15. Ans. 3 l . 4 s . 10 $\frac{13}{15}$ d .

Operation. $5 \times 3 = 15$.

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 5 \overline{) 48 \quad 13 \quad 7} \\ \underline{3} \quad 9 \quad 14 \quad 8 \frac{3}{5} \\ \underline{3} \quad 4 \quad 10 \frac{13}{15} \text{ quotient.} \\ \underline{9} \quad 14 \quad 8 \frac{3}{5} \\ \underline{48} \quad 13 \quad 7 \text{ proof.} \end{array}$$

Explanation. In dividing the 9 l . 14 s . 8 $\frac{3}{5}$ d . by 3, after dividing the pence, 2 remained, which reduced to 5ths were 10, which added to 3, made $\frac{13}{5}$, which was divided by 3 by multiplying $\frac{1}{3}$ and $\frac{13}{5}$ together; thus, $\frac{1}{3} \times \frac{13}{5} = \frac{13}{15}$. In the same manner all such fractions are divided. But the easiest and plainest method of performing compound division when the divisor exceeds 12, is by Long Division, for then there will be but one remainder, and that will be the true one.

2. Divide 48 l . 13 s . 7 d . by 15, working by Long Division.

Operation.

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 15 \overline{) 48 \quad 13 \quad 7} (3 \text{ l.} \\ \underline{45} \quad \quad \quad \\ 30 \quad \quad \quad \\ 15 \overline{) 73} (4 \text{ s.} \\ \underline{60} \quad \quad \quad \\ 13 \quad \quad \quad \\ 12 \quad \quad \quad \\ 15 \overline{) 63} (10 \frac{13}{15} \text{ d.} \\ \underline{15} \quad \quad \quad \\ 13 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ \underline{3} \quad \underline{4} \quad \underline{10 \frac{13}{15}} \\ 15 \overline{) 48 \quad 13 \quad 7} \\ \underline{45} \quad \underline{12} \quad \underline{6 \frac{5}{15}} \text{ remainder.} \\ \underline{48} \quad 13 \quad 7 \text{ Proof.} \end{array}$$

COMPOUND DIVISION.

Another Method of Proof.

$$\begin{array}{r}
 \text{£. s. d.} \\
 3 \quad 4 \quad 10 \\
 \hline
 16 \quad 4 \quad 2 \\
 3 \\
 \hline
 48 \quad 12 \quad 6 \\
 1 \quad 1 = 13 \text{d. the remainder} \\
 \hline
 48 \quad 13 \quad 7 \text{ proof as before.}
 \end{array}$$

Explanation. The division is performed thus. The divisor 15 is contained in the 48 £. , 3 times, and 3 remains, which multiplied by 20, and the 13 s. being added, makes 73 s. , which divided by 15 gives 4 s. , and 13 remain, which multiplied by 12, and the 7 d. being taken in, makes 163 d. , which divided by 15 gives 10 d. for the quotient, and 13 remain, which are $\frac{13}{15}$ of another penny.

By the first method of proof the $\frac{13}{15}$ is multiplied by 5, which produces $\frac{65}{15}$, or 4 $\frac{5}{15}$; the $\frac{5}{15}$ is set down, and 4 is carried when the pence are multiplied. When the product is multiplied by 3, three times $\frac{5}{15}$ is $\frac{15}{15}$ or 1, which is carried when the 6 d. are multiplied. Nothing except the fractional parts need explaining.

The other method of proof is the same as the first, excepting that the fraction is left out, and the remainder is added in the last step. And, always, in all denominations, the remainder reduced to the highest denomination, as pence to shillings and pence, &c., when the remainder is large enough to be reduced, and then added as in the second method, will give a true result. This last method of division and proof is the easiest, when the divisor exceeds 12. (Troy weight.)

3. Divide 23 lb. 7 oz. 6 pwt. 12 grs. by 21.

Ans. 1 lb. 10 oz. 9 pwt. 20 grs.

4. Divide 375 mbs. 2 fur. 7 po. 2 yds. 1 ft. 2 in. , (Long Measure,) by 39.

Ans. 9 mbs. 4 fur. 39 po. 2 ft. 8 in.

REDUCTION OF COINS AND CURRENCIES.

5. What is the price of 1 lb. of raisins, at £ 6 6s. per cwt. gross, or 112 lb.

Ans. 1s. 1½ d.
The usual method of finding the price of 1 lb., having the price of 1 cwt. given, is to divide by 8, by 7, and by 2, continually, because $8 \times 7 \times 2 = 112$. But when there are remainders, it is easier to divide by 112 at once, according to the method in example 2.

Remark.—From the nature of compound division it will easily be perceived that if the price, weight, or measure of any number of things be divided by that number, the quotient will be the price, weight, or measure of one thing, which makes compound division, as well as compound multiplication, very useful in many branches of business.

I Questions for Examination.

1. What is compound division?
2. How do you perform compound division when the divisor does not exceed 12?
3. How do you proceed when the divisor is more than 12?
4. How is compound division proved?

REDUCTION OF COINS AND CURRENCIES.

N. E. & I.
For changing Shillings and Pence into Cents and Mills.
New-England and Virginia Currency. 6 shillings to a dollar.

	Shill. 0		Shill. 1		Shill. 2		Shill. 3		Shill. 4		Shill. 5	
Pence.	cts.	m.	cts.	m.	cts.	m.	cts.	m.	cts.	m.	cts.	m.
0½	0	3	17	0	33	6	50	3	67	0	83	6
0¾	0	7	17	4	36	0	50	7	67	4	84	0
0½	1	1	17	8	34	3	51	1	67	8	84	4
1	1	4	18	1	34	7	51	4	68	1	84	7
2	2	8	19	4	36	1	52	8	69	5	86	1
3	4	2	20	8	37	5	54	2	70	9	87	5
4	5	6	22	2	38	9	55	6	72	3	88	9
5	6	0	23	6	40	3	57	0	77	7	90	3
6	8	3	25	0	41	7	58	3	75	0	91	7
7	9	7	26	4	43	1	59	7	76	4	93	0
8	11	1	27	8	44	5	60	1	77	8	94	4
9	12	5	29	2	45	8	62	5	79	2	95	8
10	13	9	30	6	47	2	63	9	80	6	97	2
11	15	3	32	0	48	6	65	3	82	0	98	6
12	16	7	33	3	50	0	66	7	83	3	100	0

† The New-England States are New-Hampshire, Massachusetts, Rhode-Island, Connecticut, Vermont, and Maine. In all these States, and also in Virginia, Kentucky, and Tennessee, 6s. are a dollar. When New-England and Virginia currency, or only that of New-England, all the others are understood.
‡ The reason why the value of a dollar in shillings is different states, is because the paper currency depreciated in value more in some States than in others.

REDUCTION OF COINS AND CURRENCIES.

Use of the Table. Suppose you want to know how many cents and mills are contained in 1s. and 1d. Look in the 1s. column, and against 1d., or 1 in the pence column, and under 1s. and against 1d., are 18 cts. 10m.; and so of any other number given and required.

In New York and North Carolina, 8s. = \$1. In New Jersey, Pennsylvania, Delaware, and Maryland, 7s. 6d. = \$1. In South Carolina and Georgia, 4s. 8d. = \$1. In Canada and Nova Scotia, 5s. = \$1. In English or Sterling money, 4s. 6d. = \$1. Therefore,

To find how many cents and mills any number of shillings and pence in the last mentioned currencies are equal to, find the shillings and pence in Table I.; then to or from the corresponding number of cents and mills, add or subtract as follows, viz. For,

New York and North Carolina, - - - - - subtract $\frac{1}{4}$.
 New Jersey, Pennsylvania, }
 Delaware, and Maryland, } - - - - - subtract $\frac{1}{5}$.
 South Carolina and Georgia, - - - - - add $\frac{2}{7}$.
 Canada and Nova Scotia, - - - - - add $\frac{1}{5}$.
 English or Sterling Money, - - - - - add $\frac{1}{3}$.

Example.

In 2s. 8d. New York and North Carolina currency, how many cents and mills? Ans. 33 cts. 3m.

Observe, in finding the answer to this question, the value of 2s. 8d. in cents and mills is 44 cts. 4m. by the table, the 4th part of which subtracted, leaves 33 cts. 3m.

T A B L E II.

For changing Cents into Shillings, Pence and Farthings.
 New England and Virginia Currency.

Cents.	s.	d.	Cents.	s.	d.	This table is computed to the nearest farthing. For New-York and North Carolina Currency, add $\frac{1}{3}$. For New Jersey, Pennsylvania, Delaware, and Maryland, add $\frac{1}{4}$. For South Carolina and Georgia, subtract $\frac{2}{7}$. For Canada and Nova Scotia, subtract $\frac{1}{5}$. For English or Sterling money, subtract $\frac{1}{4}$, from the shillings, pence, &c. New-England currency.
0 $\frac{1}{4}$	0	0 $\frac{1}{4}$	20	1	2 $\frac{1}{2}$	
0 $\frac{1}{2}$	0	0 $\frac{1}{2}$	30	1	9 $\frac{1}{2}$	
1	0	0 $\frac{3}{4}$	40	2	3 $\frac{3}{4}$	
2	0	1 $\frac{1}{2}$	50	3	0	
3	0	2 $\frac{1}{4}$	60	3	7 $\frac{1}{4}$	
4	0	3	70	4	2 $\frac{1}{2}$	
5	0	3 $\frac{1}{2}$	80	4	9 $\frac{1}{2}$	
6	0	4 $\frac{1}{4}$	90	5	4 $\frac{3}{4}$	
7	0	5	100	6	0	
8	0	5 $\frac{3}{4}$				
9	0	6 $\frac{1}{2}$				
10	0	7 $\frac{1}{4}$				

REDUCTION OF COINS AND CURRENCY &c.

To find how many shillings, Pence, and Farthings in the different currencies are equal to a given number of cents, by Table II.

Rule. Add or subtract, to or from, the shillings, &c. in the table, as follows, viz. For

New York and North Carolina,	add $\frac{1}{3}$.
New Jersey, Pennsylvania, }	add $\frac{1}{4}$.
Delaware, and Maryland, }	
South Carolina and Georgia,	subtract $\frac{2}{9}$.
Canada and Nova Scotia,	subtract $\frac{1}{6}$.
English or Sterling money,	subtract $\frac{1}{4}$.

Example.

How many shillings, &c. New York currency is equal in value to 51 cents? Ans. 4s. 1d.

Observe, that the value of 51 cents in Table II., is 3s. $0\frac{7}{8}$ d., which increased by $\frac{1}{4}$ of itself, is 4s. 1d.

T A B L E III.

Showing the value of several pieces of coin, in Federal Money, and in the different currencies of the United States.

Coins.	Value in Federal Money.	N. Hamp. Mass. R. Island, Conn. and Virginia.	New York and N. Carolina.	New Jersey, Penn. Delaware, and Maryland.	S. Carolina and Georgia.
$A\frac{1}{2}$ Dime - - - -	$\$$ $\frac{5}{16}$	$\pounds$ $\frac{3}{16}$ s. $\frac{3}{4}$ d.	$\pounds$ $\frac{3}{16}$ s. $\frac{3}{4}$ d.	$\pounds$ $\frac{3}{16}$ s. $\frac{3}{4}$ d.	$\pounds$ $\frac{3}{16}$ s. $\frac{3}{4}$ d.
$\frac{1}{16}$ of a dollar, ($\frac{1}{2}$ d.) - -	0 5	0 0 $3\frac{3}{4}$	0 0 $4\frac{1}{2}$	0 0 $4\frac{1}{2}$	0 0 $2\frac{1}{2}$
$\frac{1}{8}$ a Pistareen - - -	0 8	0 0 $6\frac{5}{8}$	0 0 $8\frac{5}{8}$	0 0 $7\frac{13}{20}$	0 0 $4\frac{19}{25}$
$\frac{1}{4}$ of a dollar, (9d.) -	0 12	0 0 9	0 0 8	0 0 7	0 0 7
A Pistareen, - - -	0 17	0 1 $0\frac{6}{16}$	0 1 $4\frac{8}{16}$	0 1 $3\frac{3}{10}$	0 0 $9\frac{13}{25}$
An English Shilling - -	0 22	0 1 4	0 1 $9\frac{1}{3}$	0 1 8	0 1 $0\frac{5}{9}$
$\frac{1}{2}$ of a dollar, - - -	0 25	0 1 6	0 2 0	0 1 $10\frac{1}{2}$	0 1 2
$\frac{1}{2}$ a dollar, - - -	0 50	0 3 0	0 4 0	0 3 9	0 2 4
A Dollar, - - - -	1 00	0 6 0	0 8 0	0 7 6	0 4 8
French Crown - - - -	1 10	0 6 7	0 8 $9\frac{1}{2}$	0 8 3	0 5 $1\frac{1}{2}$
Fr. Guinea, ^{per 5. 2. 11.} 5 5	5 5	1 7 $3\frac{1}{4}$	1 16 $4\frac{1}{2}$	1 14 0	1 1 $2\frac{1}{2}$
Eng. Guinea, 5 6	5 6	1 8 0	1 17 4	1 15 0	1 1 $3\frac{1}{3}$
$\frac{1}{2}$ Johanna, 9 0	8 00	2 8 0	3 4 0	3 0 0	1 17 4
Pistole, - - 4 5	3 53	1 1 $2\frac{1}{2}$	1 8 3	1 6 6	0 16 5
Moidore, - - 6 18	6 0	1 16 0	2 8 0	2 5 0	1 8 0
Dubloon, - - 17 0	14 66	4 8 0	5 16 0	5 12 0	3 10 0
Mark, - - - - -	2 96	0 17 $9\frac{1}{3}$	1 3 $8\frac{5}{9}$	1 2 $2\frac{2}{3}$	0 13 10
Noble, - - - - -	1 68	0 8 $10\frac{2}{3}$	0 11 $10\frac{2}{3}$	0 11 $1\frac{1}{3}$	0 6 11

REDUCTION OF COINS AND CURRENCIES.

The following are the rates at which foreign coins and currencies are valued at the Custom-Houses of the United States.

	Dols.	cts.
A pound sterling of Great Britain, at -	4	44
A pound sterling of Ireland, - - - - -	4	10
A livre tournois of France, - - - - -	0	18½
A real of plate of Spain, - - - - -	0	10
A real of vellon of Spain, - - - - -	0	5
A millrea of Portugal, - - - - -	1	24
A florin or guilder of the United Netherlands, -	0	40
A mark banco of Hamburgh, - - - - -	0	33½
A rix dollar of Denmark, - - - - -	1	00
A tale of China, - - - - -	1	48
A piagoda of India, - - - - -	1	84
A rupee of Bengal, - - - - -	0	50

CASE I.

Section I. To reduce the currency of New-England, Virginia, Kentucky, and Tennessee, to Federal Money.

Rule.—If the given sum is pounds only, annex a cipher to it, and divide by 3; the quotient will be dollars:—if any thing remain, annex more ciphers, and continue the division three figures further for cents and mills. But, if there are shillings, pence, &c. in the given sum, reduce the whole to the decimal of a pound, and proceed as before remembering to cut off one figure less than the whole number of decimals, for cents and mills.

Reason.—The reason for adding a cipher to the pounds, and dividing by 3, is because that when pounds are multiplied by 10, the product is so many times 2 shillings, and 2 shillings is the third of a dollar. And, one decimal less than the whole must be cut off because the cipher added or annexed to the pounds, before dividing, is not a decimal.

Proof.—Section 2d proves section 1st, in all the cases.

Examples.

1. Reduce £ 6 to Federal Money.

Ans. \$ 20.

REDUCTION OF COINS AND CURRENCIES.

$$\begin{array}{r} \text{£} \\ 3 \overline{) 60} \\ 20 \end{array}$$

Explanation. Here a cipher is annexed to £6, and 60 divided by 3, gives 20 for the quotient, which is dollars.

2. In £7 13s. 6d., how many dollars, cents, and mills? Ans. \$25. 58cts. 3 $\frac{1}{2}$ m.

$$\begin{array}{r} 12 \overline{) 6.0} \\ 20 \overline{) 13.5} \\ 3 \overline{) 71.675} \\ \$25.583\frac{1}{2} \end{array}$$

Explanation. The 6d., 13s., and £7, are set orderly under each other, for the integral or whole number parts of the dividends, and 12, 20, and 3, for divisors, because 12 pence make a shilling, 20 shillings a pound, and pounds must be divided by 3, according to rule.† In dividing by 3, a cipher was supposed to be annexed to the decimal of a pound (.675) which makes four decimals, therefore three (one less) are cut off for cents and mills, as the rule directs. In the same manner find the answers to the following questions,

3. Change £58 into Federal money, --- Ans. \$193, 33 $\frac{1}{3}$.
4. Change £16 10s. 3d., --- Ans. --- 55, 04 $\frac{1}{2}$.
5. Change £12 14s. 7 $\frac{1}{2}$ d., --- Ans. --- 42, 53 $\frac{2}{3}$ +
6. Change £27 11s. 4 $\frac{1}{2}$ d., --- Ans. --- 91, 89 $\frac{2}{3}$ +

Section II. To reduce Federal Money to New-England, Virginia, Kentucky, and Tennessee currency.

Rule. - Multiply the dollars and decimal of a dollar by 3, and from the product cut off one figure more than the whole number of decimals: then find the value in shillings, pence, &c., of the decimal so cut off.

As this rule is exactly the reverse of that in Section 1st, the reason and method of proof need no explanation.

Examples.

1. In \$50, 75, how many pounds, shillings, and pence? Ans. £15 4s. 6d.

$$\begin{array}{r} \$50, 75 \\ \text{£} 15, 225 \\ \quad 20 \\ \hline 3, 4, 500 \\ \quad 12 \\ \hline 6, 000 \end{array} \left. \begin{array}{l} \text{The} \\ \text{operation.} \end{array} \right\}$$

2. In \$25, 583 $\frac{1}{2}$, how many pounds, shillings, and pence? Ans. £7 13s. 6d.

† The shortest, and after a little practice, the easiest method of reducing shillings, pence, and farthings to the decimal of a pound, is by what is called inspection, thus; if the shillings are even, half their number, with a comma prefixed, is their decimal. But if the shillings are odd, annex 5 to half their number; then, if the pence and farthings, reduce them all to farthings, and increase the number by 1 when they are above 12, and by 2 when they are above 36, and add them to the decimal of the shillings which, if the shillings are even, must have two ciphers annexed to half their number; but if odd, 5 and one cipher must be annexed, and the farthings, if any, added.

Example. Reduce £7 13s. 6d. to the decimal of a pound.

$$\begin{array}{l} \text{£} 7, 6 = \text{half the even number of shillings.} \\ \quad 5 = \text{annexed for the odd shilling.} \\ \quad 25 = \text{the farthings in 6d. increased by 1.} \\ \hline 7, 675 = \text{the answer.} \end{array} \quad \text{Ans. } \text{£} 7, 675.$$

Here two ciphers are supposed to be annexed to 6, and one cipher annexed to 5.

Reason. As any number of shillings are 20ths of a pound, half the number must be tenths of a pound: 5 is annexed for an odd shilling, because $\frac{5}{10}$ is equal to one half.

The reason why the farthings are increased by 1 when they are above 12, and by 2 when they exceed 36, is because that if 960, the farthings in a pound, be increased by its 24th part, the sum will be just 1000, and the third decimal figure is thousandths. Therefore, when the number of farthings is more than half of 24, then 1 is added; and when the farthings are nearer 48 than 24, then 2 is added. - The method by inspection gives the decimal of a pound to three places only.

§ The value of the decimal of a pound may be found by inspection, as follows.

Double the first figure, or place of 10ths, for shillings, and if the second figure be 5 or more, reckon another shilling; then call the figures in the second and third places, after 5, if contained, is deducted, so many farthings; deducting 1, when they are above 12, and 2, when they exceed 36.

Example. Find the value of .785 of a pound, by inspection.

REDUCTION OF COINS AND CURRENCIES.

CASE II.

Section I. To reduce or change New-York and North-Carolina currency to Federal Money.

Rule. Proceed in all respects as in changing New-England currency, excepting that you must divide by 4, instead of 3.

Reason. The reason for dividing by 4, is because 8s. New-York currency make a dollar, and 8s. is 4 tenths of a pound.

Examples.

1. Change £ 5 4s. 6d. to Federal Money.

$$\begin{array}{r} 12 \overline{) 6,0} \\ 20 \overline{) 4,5} \\ 4 \overline{) 5,225} \\ \hline \$ 13,06\frac{1}{4} \text{ Ans.} \end{array}$$

2. Change £ 7 5s. 6½d. to dollars and cents.
Ans. \$ 18, 19 +.

Section II. To reduce Federal Money to New-York and North-Carolina currency.

Rule. The same as for N. England currency, only use 4, instead of 3, for a multiplier.

Examples.

1. Change \$ 13, 06½ to New-York and N. Carolina currency.

$$\begin{array}{r} \$ 13,06\frac{1}{4} \\ 4 \\ \hline £ 5,225 \\ 20 \\ \hline 3,4,500 \\ 12 \\ \hline d. 6,000 \end{array} \quad \text{Ans. } £ 5 \text{ } 4s. \text{ } 6d.$$

2. Change \$ 18, 14 to New-York and N. Carolina currency. Ans. £ 7 5s. 6½d. nearly.

CASE III.

Section I. To change New-Jersey, Pennsylvania, Delaware, and Maryland currency to Federal Money.

Rule. The same as for New-England currency, only remember to subtract ⅕ of the answer from itself.

Reason. The reason for subtracting ⅕ is, because 7s. 6d. of Pennsylvania currency make a dollar, and ⅕ of 7s. 6d. is 1s. 6d.

REDUCTION OF COINS AND CURRENCIES.

Examples.

1. Change or reduce £ 471 to Federal Money.

$$\begin{array}{r} \text{£} \\ 3) 4710 \\ 5) \$1570 \text{ New-Eng. currency.} \\ \text{Subtract} \quad - \quad 314 = \frac{1}{5} \\ \hline \$1256 \text{ Ans.} \end{array}$$

2. Change £ 240 9s. 10½d. to Federal Money.

$$\text{Ans. } \$641, 31\frac{2}{3}.$$

Section II. To change Federal Money to New-Jersey, Pennsylvania, Delaware, and Maryland currency.

Rule. First find the value in New-England currency, and then add $\frac{1}{4}$ of the answer to itself.

Reason. The reason why $\frac{1}{4}$ the value of New-England currency must be added to itself, is because $\frac{1}{4}$ of 6s. is 1s. 6d., which added to 6s. (a dollar in New-Eng.) is 7s. 6d.

Examples.

1. Change \$1256 to Pennsylvania, &c. currency.

$$\begin{array}{r} \$1256 \\ 3 \\ 4) 3768 \text{ l. in New-England.} \\ \text{Add} \quad - \quad 942 = \frac{1}{4} \\ \hline £4710 \text{ Ans.} \end{array}$$

2. Change \$641, 31⅔ to Pennsylvania currency.

$$\text{Ans. } £240 \text{ 9s. } 10\frac{1}{2} \text{ d.}$$

CASE IV.

Section I. To change South-Carolina and Georgia currency to Federal Money.

Rule. Find the value as if it were New-England currency, and multiply it by 9 and divide the product by 7; the quotient will be the answer.

Reason. Because 4s. 8d. ^{or 56d.} = \$1 in South Carolina, is worth as much as 6s. or 72d. in New-England, and therefore 7d. in S. Carolina, is worth 9d. in New-England.

Examples.

1. Change £ 5 12s. 4d. S. Carolina and Georgia currency, to Federal Money.

$$\begin{array}{r} 12) 5.0 \\ 20) 12.333 \\ 3) 5.6166 \\ \$18,722 = \text{the value in New-England, (Case 1.)} \\ 9 \\ 7) 168,498 \\ \$24,071 \text{ Ans.} \end{array}$$

2. Reduce £ 14 S. Carolina, &c. currency, to Federal Money. Ans. \$60.

REDUCTION OF COINS AND CURRENCIES.

Section II. To Reduce Federal Money to South-Carolina and Georgia currency.

Rule. - Find the value in New-England currency, by Case I., which divide by 9, and multiply the quotient by 7.

Examples.

1. Change \$24,071 to S. Carolina and Georgia currency.

$$\begin{array}{r} \$24,071 \\ 9 \overline{) \$7,221 \frac{3}{4}} \text{ in New-England.} \\ \underline{1,802 \frac{3}{4}} \end{array}$$

\$5,616 $\frac{5}{20}$ nearly.

s. 12,332 $\frac{0}{12}$

Ans. £5 12s. 4d. nearly.

d. 3,960 $\frac{4}{4}$

qrs. 3,450

2. Change \$60 to S. C. and Georgia currency.

Ans. £14.

CASE V.

Section I. To reduce or change Canada and Nova-Scotia currency to Federal Money.

Rule. - First find the value in New-England currency, by Case I., and add the 5th part of it to itself, and the sum will be the answer.

Reason. - $\frac{1}{5}$ is added because 5s. in Canada and Nova-Scotia are equal to 6s. in New-England.

Examples.

1. Change £264 6s. 3d. Canada and Nova-Scotia currency to Federal Money.

$$12 \overline{) 310}$$

$$20 \overline{) 625}$$

$$3 \overline{) 264,312 \frac{5}{12}}$$

$$5 \overline{) \$881,041 \frac{2}{3}} \text{ in New-Eng. currency.}$$

$$176,208 = \frac{1}{5}$$

$$\$1057,249 \frac{2}{3} \text{ Ans.}$$

2. Change £18 Canada currency to Federal Money. Ans. \$72.

Section II. To reduce Federal Money to Canada and Nova-Scotia currency.

Rule. Change the Federal Money to New-England currency, by Case I., and subtract $\frac{1}{5}$ of itself.

SINGLE RULE OF THREE.

Examples.

1. Change \$72 to Canada and Nova-Scotia currency.

$$\frac{\$72}{3}$$

b) $\frac{\$21,6}{3,6} = \frac{1}{6}$ New-England currency.

\$18 Ans.

2. Change \$34,66 to Canada and Nova-Scotia currency. Ans. £8 13s. 3½d. +

SINGLE RULE OF THREE.

The Single Rule of Three is a rule of three numbers given to find a fourth.

The first number in order is that whose value or effect* is given or known.

The second given number is the value or effect of the first.

The third given number is that whose value or effect is required. And,

The fourth number, when found, is the value or effect of the third.

Nature of the Rule. In the Single Rule of Three the first number is always of the same kind, or nature, as the third; and the second number is of the same kind or nature, as the fourth or answer.

The numbers given in the rule of three, are called terms.

Stating a Question. - Stating a question in the rule of three, is putting the three given numbers in a state of preparation, or setting them down in such order as that the fourth term or answer may be most conveniently obtained.

[Remark. - In many different arts and sciences there is so great a connexion between some of their parts, that after learning one, another may be learned much easier. Thus

* The value or effect in the Rule of Three is what the first or third term or number gives, costs, &c., whether it be price, weight, measure, time, distance, or whatever.

SINGLE RULE OF THREE DIRECT.

The stating of a question in the rule of three is nearly related to parsing a sentence in grammar. For the 1st term is always the nominative case to the verb gives, costs, &c., expressed or understood, and the 2d term is the objective case governed by the verb, as will appear by what follows.]

The nature and use of the Rule of Three will be more fully explained by the following preparatory questions belonging to that rule, which may be easily solved mentally, by reason only, without any knowledge of the common method of working by the rule of three.

1. If 2 pounds of sugar cost 25 cents, what will 4 pounds cost at that rate? Ans. 50 cents.

2. If 300 acres of land are worth \$1000, what are 900 acres worth? Ans. \$3000.

In solving the two preceding questions, it is plain that double the quantity is worth double the value, as in the first question; and that triple the quantity is worth triple the value, as in the second.

3. If 3 tons of hay are worth \$20, what are 5 tons worth at that rate? Ans. \$33 $\frac{1}{3}$.

In order to solve the third question, by reason, ^{only} the price of 1 ton must first be found, and that price multiplied by 5, will be the value of 5 tons. Now the price of 1 ton is found by dividing \$20, the price of 3 tons, by 3, which gives \$6,66 $\frac{2}{3}$, which multiplied by 5, gives \$33 $\frac{1}{3}$, the value of 5 tons, which was required. In the same manner most other questions in the Rule of Three ^{Direct} may be solved on general principles, either mentally or by using the slate; but for the sake of greater ease and expedition, methods founded on these general principles have been ^{easy} invented which give the same result in a more ^{neat}, and convenient manner.)

SINGLE RULE OF THREE DIRECT.

The Single Rule of Three Direct, is a rule of three numbers given to find the fourth or answer, ^{where} more requires more, or less requires less.

More requires more when the third term is greater than the first, and requires the fourth, or answer, to be greater than the second. Thus, in this question. If 2 lb. cost 8 cents, what will 3 lb. cost? Here more requires more; for the 3rd term 3 lb. is greater than the first

SINGLE RULE OF THREE DIRECT.

Term 2nd, and requires the answer, or 4th term to be more or greater than the 2^d term.

Less requires less when the 3rd term is less than the 1st, and requires the 4th term or answer to be less than the 2^d. Thus, if 3 lb. cost 8 cents, what will 2 lb. cost? Here 2 lb. is less than 3 lb., and requires that the answer or 4th term should be less than 8 cents, the 2^d term.

In the single Rule of Three Direct, the product of the second and third terms, when multiplied together, is always equal to the product of the first and fourth term or answer, when multiplied together. Also, the fourth term or answer, bears the same proportion to the third term, as the second does to the first, for which reason the rule of three is likewise very properly called the Rule of Proportion.

RULE.

1. State the question by setting down the three given numbers in a line, with colons between them, so that the number whose value or effect is known, shall be the first or left hand term. The value, cost or effect of the first term, must be the second term; ^{then} the remaining number whose value, cost, effect, &c., is required, must be the third term.

2. After the question is stated, and the first and third terms are reduced to the lowest denomination contained in either of them, and the second term is reduced to the lowest denomination given in that, multiply the second and third terms together, and divide their product by the first term, and the quotient will be the answer.†

If any thing remains after dividing by the first term, reduce it to the next lower denomination, and divide by the first term as before, and so proceed with every denomination to the lowest.

When any of the terms of a statement consist partly of fractions, reduce them to decimals if necessary, and multiply and divide according to the rules of multiplication and division of decimals. When decimals are used, the operation is called the Rule of Three in Decimals.

† The rule here given is the one generally used, and is almost always the easiest, but where fractions do not accumulate too fast, the answer may often be found more expeditiously by one of the following methods.

1. Divide the 2^d term by the 1st, and multiply the 3^d term by the quotient. Or
2. Divide the 3^d term by the 1st, and multiply the 2^d term by the quotient. Or
3. Divide the 1st term by the 2^d, then divide the 3^d term by the quotient. The result of either will be the answer.

SINGLE RULE OF THREE DIRECT.

Proof.—Invert the order of the question and statement, by saying as the 3d term is to the 4th or answer, so is the 1st to the 2d; remembering to add in the remainder when you multiply the 2d and 3d terms, in proving the work, when any thing remained in the first work.

Reason of the Rule.—The reason why the product of the 2d and 3d terms must be divided by the 1st, is because the 2d term is as much more than the cost or value of 1, as the 1st term is greater than 1.

Example I.

If 5 lb. of sugar cost 63 cents, what will 7 lb. cost at that rate?

$$\begin{array}{r} \text{lb.} \quad \text{cts.} \quad \text{lb.} \\ 5 : 63 :: 7 \\ \hline 5 \overline{) 441} \\ 88 \frac{1}{5} \text{ cts. Ans.} \end{array}$$

Proof.

$$\begin{array}{r} \text{lb.} \quad \text{cts.} \quad \text{lb.} \\ 7 : 88 \frac{1}{5} :: 5 \\ \hline 7 \overline{) 441} \\ 63 \text{ cts.} = \text{the price of 5 lb.} \end{array}$$

Explanation.

In the first statement the thing whose value or cost is known, is 5 lb. of sugar, which must therefore be the 1st term. The value of the 1st term is 63

cents, which must be the 2d term, according to the rule. Then, the thing whose value is required, viz. 7 lb. of sugar, must be the 3d term. And, the 2d and 3d terms being multiplied together, and the product divided by the 1st term, gives $88 \frac{1}{5}$ cents for the value of 7 lb. The $\frac{1}{5}$ of a cent is found by placing the remainder 1, over the divisor 5. If the remainder 1, had been reduced to mills, and divided by 5, the answer would be 88 cts. 2 m.

The second statement and work prove the first, for by saying if 7 lb. cost $88 \frac{1}{5}$ cts. what will 5 lb. cost, and working as before, the value of 5 lb. comes out 63 cents, as it ought to, therefore all the work is right.

In multiplying $88 \frac{1}{5}$ by 5, the 88 is multiplied and the remainder 1, is added in.

SINGLE RULE OF THREE DIRECT.

Remark.—A person of a good genius, but who had never learned the "Golden Rule of Three," would solve the question under example first, by first finding the value of 1 lb. of sugar, by dividing 63 cents by 5, and then multiplying the 7 lb. by the price of 1 lb., either mentally, or on the slate or a piece of paper. By the last method the work would be as follows.

$$\begin{array}{r} 5 \overline{) 63 \text{ cts.}} \\ 12 \frac{3}{5} \text{ cts.} = \text{the price of 1 lb.} \\ 7 \\ \hline 88 \frac{1}{5} \text{ cts.} = \text{the price of 7 lb.} \text{ Ans.} \end{array}$$

This last method of solving questions in the rule of three, is called the analytical method, or solving them on general principles, the reason appearing plain at every step, in most cases, but in some questions it is much more difficult to solve questions in this way than by the general or common rule, which is founded on the analytical method. Besides, in the last method, you generally have to multiply a fraction to find the value of the quantity whose price is required, as $\frac{3}{5}$ by 7 in this example, by general principles, which was avoided by the common rule.

Example II.

If 25 gallons of molasses cost \$7.50, what is 17 hogsheads of the same kind worth?

$$\begin{array}{rclcl} \text{gal.} & \text{cts.} & \text{hhd.} & & \text{Ans. } \$321.30 \\ 25 & : 750 & :: 17 & & \end{array}$$

$$\begin{array}{r} 63 \\ 51 \\ \hline 102 \\ 1071 \text{ gallons.} \\ \hline 750 \\ 53550 \\ 7497 \\ \hline 25 \overline{) 803250} \text{ } \$321.30. \\ 75 \\ \hline 53 \\ 50 \\ \hline 32 \\ 25 \\ \hline 75 \\ 75 \\ \hline 0 \end{array}$$

SIXTH RULE OF THREE DIRECT.

In example 2d, as the first term is gallons, and the third is hogsheads, the last are reduced to gallons before multiplying, so that the 1st and 3d terms may be of one name.

Example III.

If $9\frac{1}{2}$ lb. of coffee cost $\$2, 12\frac{1}{2}$, what will 19 lb. cost at that rate?

Ans. $\$4, 17$ ds. $1\frac{1}{9}$ m.

$$\begin{array}{lcl} \text{lb.} & \text{cts.} & \text{lb.} \\ 9\frac{1}{2} & : 2\ 12\frac{1}{2} & :: 19 \\ \underline{2} & & \underline{19} \\ 19 & : 4\ 25 & :: 38 \\ & 38 & \end{array}$$

$$\begin{array}{r} 3400 \\ 1275 \\ \hline 19 \overline{) 16150} \quad \begin{array}{l} 834 \text{ half cents.} \\ \text{or} \\ 417 \text{ cents.} \\ \text{or} \\ \$4, 17. \end{array} \\ \underline{152} \\ 65 \\ \underline{57} \\ 80 \\ \underline{76} \\ 4 \\ \underline{5} \\ 19 \overline{) 20} \quad 1\frac{1}{9} \text{ m.} \\ \underline{19} \\ 1 \end{array}$$

Explanation.—Before working this statement, as the first term contains a half, the 1st and 3d must both be brought into halves; and as the 2d term also contains an odd half, that must be brought into halves likewise. Then, as the 2d term is reduced to halves, the answer will come out in halves, that is, in half cents; and the remainder 4, is half cents, which is multiplied by 5, in reducing, because half a cent is 5 mills.

Example III.

If $7\frac{1}{2}$ yards of cloth cost $\$15$, what will $9\frac{1}{8}$ yards come to at that rate?

Ans. $\$18, 25$.

SINGLE RULE OF THREE DIRECT. Statement.

$$\begin{array}{ccc} \text{2ds.} & \text{\$/} & \text{2ds.} \\ 7\frac{1}{2} & : 15 & :: 9\frac{1}{8} \end{array}$$

Here as the first and third terms have fractions, they must both be reduced to 8ths ($\frac{1}{2} = \frac{4}{8}$) or both the fractions must be reduced to decimals. For explaining the method by decimals, we will work by them.

Now .5 is the decimal of $\frac{1}{2}$, and .125 is the decimal of $\frac{1}{8}$, found by adding ciphers to the numerator, and dividing by the denominator. The statement and operation by decimals, will therefore be as follows.

$$\begin{array}{ccc} \text{2ds.} & \text{\$/} & \text{2ds.} \\ 7.5 & : 15 & :: 9.125 \end{array}$$

$$\begin{array}{r} 45 \ 625 \\ 91 \ 25 \\ \hline 7.5 \overline{) 136.875} \quad \begin{array}{l} \text{Dls.} \quad \text{cts.} \\ 18.25 \end{array} \\ \underline{75} \\ 618 \\ \underline{600} \\ 187 \\ \underline{150} \\ 375 \\ \underline{375} \\ 0 \end{array}$$

In working this question three decimals are cut off in the product, because there are three in both factors. And, in dividing, two decimals are cut off in the quotient, because there ^{are} three in the dividend, and there is one in the divisor; and there must always be as many decimals in the divisor and quotient counted together, as there are in the dividend.

Statements and Answers.

If the student multiplies the 2d and 3d terms together, and divides by the first, the quotient will be the answer, in each of the following statements; this will give the scholar a clearer idea of the nature of solving questions which are not stated. As the

SINGLE RULE OF THREE DIRECT.

numbers have no denominations annexed to them, an unlimited variety of questions may be formed from them. For as the 1st and 3d terms are supposed to be of a name, they may be minutes, hours, days, ounces, pounds, hundreds, or any other denomination; and the 2d term may be pence, shillings, cents, dollars, &c. Thus, in the 1st example, if the 1st and 3d terms are gallons, and the 2d is shillings, it will be, if 7 gallons cost 15 shillings, what will 9 gallons cost? Here the answer is found to be $19\frac{2}{7}$ shillings: for the answer is always of the same name as the 2d term. Again, if the 1st and 3d terms are called hundreds, and the 2d dollars, it will be, if 7 cwt. costs or gives \$15, ^{what} will 9 cwt. cost or give, &c.? Ans. \$ $19\frac{2}{7}$ &c.

Examples.

First term.	Second term.	Third term.	Ans.
7	15	9?	$19\frac{2}{7}$.
5	9	8?	$14\frac{2}{5}$.
37	120	28?	$90\frac{30}{37}$.
8,5	73	5,75?	49,382+.
15	28	45?	84.
$7\frac{1}{2}$	19	30?	120.
14	$6\frac{1}{2}$	70?	$32\frac{1}{2}$.

Questions for Exercise.*

1. If 3 bushels of corn cost \$2,25, what will 257 bushels cost? Ans. \$192,75.
2. If 28 bushels of corn cost \$21,28, what is it per bushel? Ans. 76 cents.
3. What will 64 thousand of boards come to, at \$5,50 per thousand? Ans. \$407.
4. If 7 lb. of rice cost 25 cts., how much may be bought for \$243,50? Ans. 60 cwt. 3 qrs. 21 lb.
5. If 3 lb. of the best hyson tea cost £1 13s., what is 79 lb. worth? Ans. £32 11s. 9d.

* In our arithmetics, every question is called an example, but this is improper in the rule of three, unless the question is worked out.

SINGLE RULE OF THREE DIRECT.

6. A owes B £ 98 2s., New-England currency, 6s. being a dollar. How many dollars will pay the debt? Ans. \$ 327.

7. A person rode in a stage till his stage fare was \$ 3, 52½, at 5 cents per mile. How far did he go? Ans. 70½ miles.

8. A schoolmaster taught a school 3 months and 9 days, at \$ 15 per month: what came his wages to, 22 days being a school month? Ans. \$ 51, 13 ¹¹/₂₂.

9. A laborer worked for a farmer 6 months and 15 days, for \$ 10 a month: how much did he earn in the whole time? Ans. \$ 65, 76 ²⁴/₂₆.

~~26~~ 26 days are a working month.

10. A ^{man} owes in all, \$ 1700, but is worth only \$ 1100: how much can he pay on a dollar? Ans. 64 ¹²/₁₇ cents.

11. If I buy a hogshead of wine for \$ 62, 78, how much water may be added to it, so that the mixture may be worth 86 ⁵/₈ a gallon? Ans. 10 gallons.

12. If a town is taxed \$ 1500, and the sum of all the inventories is \$ 2100, and A's inventory is \$ 19, how much is A's tax? Ans. \$ 13, 57 ⁵/₇.

13. What is the interest of \$ 378 for one year, at 6 per cent. per annum?

Ans. \$ 22, 68.

Note. - 6 per cent. means £ 6 or \$ 6 for every 100; and per annum, means for a year.

14. What will 49 thousand, 3 hundred, and 25 casts of staves come to at \$ 10 per thousand? Ans. \$ 493, 62½.

N.B. - In counting staves, 3 are cast or thrown at a time: 40 casts make a hundred, and 10 hundred a thousand, so that 1200 staves go for a thousand.

SINGLE RULE OF THREE DIRECT.

15. What will 84 bundles of hoops come to at \$25 per thousand, reckoning 30 bundles for a thousand? Ans. \$70.

Hoops are generally bound in bundles of 40 each, 3 bundles making 1 hundred, and 10 such hundreds, or 30 bundles, 1 thousand. They are sometimes bound 30 in a bundle, and then 4 bundles make 1 hundred and always 10 hundred a thousand.

16. If a watch gain 5 minutes of time in 7 days, how much would it gain in a year of 365 days? Ans. 4h. 20 $\frac{5}{8}$ m.

SINGLE RULE OF THREE INVERSE.

Though questions in the Single Rule of Three generally fall under the direct rule, because, in most cases where proportion is concerned, if the 3d term exceeds the first, the 4th term or answer must exceed the 2d; or, if the 3d term is less than the first, ~~and~~ ^{it} requires the 4th to be less than the 2d. But, there are cases in which more requires less, and others in which less requires more. More is said to require less, when the 3d term is greater than the first, and requires the answer to be less than the 2d. Less is said to require more, when the 3d term is less than the first, and requires the answer to be greater than the 2d term. Questions of such a nature are said to be in Inverse Proportion, and belong to the Rule of Three Inverse.

R U L E S.

State the question as in the Single Rule of Three Direct, and proceed in every respect as in that rule, excepting that you must multiply the first and 2d terms together, and divide by the third.

SINGLE RULE OF THREE INVERSE.

Proof.—As the 3d term is to the answer, so is the first term to the second.

Nature of the Rule.—In the Single Rule of Three Inverse, the product of the 1st and 2d term is equal to the product of the 3d and 4th or answer.

Reason of the Rule.—The reason why the first and second terms are multiplied together, is because the product of those two numbers would be the answer supposing the third term to be 1, but when the third term is more than 1, the product of the first and second is as many times more than the answer as the third term is more than 1.

Example.

1. If 10 men can mow a field in 8 days, in how long a time can 20 men do the same? Ans. 4 days.

The numbers in this question are easy to compare, and it is plain that double the number of men can do the work in half the time. We will now find the answer by rule.

$$\frac{M.}{10} : \frac{D.}{8} :: \frac{M.}{20}$$

$$2 \overline{) 80} \\ 4 \text{ days.}$$

Explanation.

Here the 3d term 20, is greater than the 1st term 10, and requires the 4th term or answer to be less than the 2d, therefore more requires less.

Questions for Exercise.

1. If 48 mowers can mow a quantity of grass in 12 days, in what time will 192 mowers do it? Ans. 3 days.

2. If a man perform a journey in 6 days, when the day is 8 hours long, in what time will he do it when the day is 12 hours long? Ans. 4 days.

3. If I lend my friend £100 for 180 days, how long ought he to lend me £1000 to return my kindness? Ans. 18 days.

4. How many yards of matting half a yard wide, will cover a room 18 feet wide and 30 feet long? Ans. 120 yards.

5. Suppose 800 soldiers are provided with provision for 2 months; how many soldiers must depart to make the provisions last 5 months? Ans. 480.

SINGLE RULE OF THREE DIRECT AND INVERSE.

The following additional explanations and questions are given to assist the scholar and to prove the student's skill in solving such questions as may be proposed, in the rule of three, when it is not said to which rule they belong, whether ^{to} direct or inverse.

Directions.

1. State the question as before taught.
2. Consider whether the answer, according to the nature of the question, must be more or less than the 2d term: if more, multiply the 2d term and the greater of the other two terms together for a dividend, and divide by the other term. But if the answer ought to be less than the 2d term, then multiply the 2d term and the least of the other two, for a dividend, and divide by the greater.

Examples.

1. Suppose the question to be:— If 8 dollars will buy 12 books, how many books will 2 dollars buy?

This question when stated, will stand thus.

$$\begin{array}{ccccc} \text{dolls.} & & \text{bks.} & & \text{dolls.} \\ 8 & : & 12 & :: & 2 \end{array}$$

Here, though the 3d term is less than the 1st ^{and}, it is plain that the 4th term or answer must be less than the 2d; for 2 dollars will not buy so many books as 8 dollars: the question therefore belongs to the rule of three direct, and is solved by multiplying the 2d and 3d terms, and dividing by the 1st term, and the answer is 3 books.

2. But suppose the question to be,

If 8 men can frame a building in 12 days, in what time can 2 men do the same?

SINGLE RULE OF THREE.

This question, when stated, will stand like the former, thus.

$$\begin{array}{ccccc} M. & & D. & & M. \\ 8 & : & 12 & :: & 2 \end{array}$$

But this last statement is of a very different nature from the first; for here, though the 3d term is less than the 1st, the 4th term or answer must be greater than the 2d; because 2 men require more days to do the work than 8 men; therefore the question belongs to the rule of three inverse, and the answer (48 days) is found by multiplying the 2d term and the greater of the other two, viz., 8, together for a dividend, and dividing by 2, the 3d term; and so in all other questions and statements of a similar nature.

Questions for Exercise.

The four first may be solved mentally, and the reasons given, &c.

1. If 4 bushels of wheat are worth \$5, what are 16 bushels worth? Ans. \$20. Q. Does this question belong to the rule of three direct, or the rule of three inverse? A. To the rule of three direct. Q. Why? A. Because the quantity whose price is required is greater than that whose price is given, and requires the value to be greater.

2. If 3 men clear up a piece of land in a week, in how long a time would 1 man do the same work? Ans. in 3 weeks. Q. Does this question belong to the rule of three direct, or inverse? A. To the rule of three inverse. Q. Why? A. Because the number of men to do the work in a certain time unknown, is less than the number who do it in a given time, and therefore requires a longer time to do it in, so less requires more; that is, less men require more time.

3. If 15 lb. of coffee cost \$2.25, what is $7\frac{1}{2}$ lb. worth? Ans. \$1.12 $\frac{1}{2}$.

SINGLE RULE OF THREE.

4. If 3 yards of cloth a yard wide, will make a garment, how many yards half a yard wide will make the same? Ans. 6 yards.

(Exercises for the Slate.)

5. If a man's yearly income (365 days to a year) is \$474, 50, how much is it per day? Ans. \$1, 30.

6. If the interest of \$100 for 1 year, is \$6, what is the interest of \$912 for the same time? Ans. \$54, 72.

7. When boards are \$9 per thousand, what is the price of a square foot?

Ans. 9 mills.

8. If a staff 3 feet long cast a shadow 4 feet in length, how many feet high is a building whose shadow at the same time measures 80 feet? Ans. 60 feet.

9. A water tub holding 50 gallons, when full, has a pipe leading to it which will fill it in an hour, and a hole in the bottom will let out 5 gallons in an hour. In how long a time will the tub be full, if the pipe be set to running, and the tap be left out, supposing the tub empty at first? Ans. 1 h. $6\frac{2}{3}$ m.

10. If a board is 8 inches wide, how much in length will make a square foot, or 144 square inches? Ans. 18 inches.

As 12 inches wide are to 12 inches in length, so are any other number of inches wide (different from 12) to the number of inches in length to make a square foot, inversely.

11. If my purse and money together are worth \$70, and the money is worth 10 times as much as the purse, how much money is there in the purse?

Ans. \$63 $\frac{9}{11}$.

SINGLE RULE OF THREE.

12. John and James together can hoe an acre of corn in 2 days, but John can hoe 3 rows as quick as James can hoe 2. In what time can each hoe it alone, at that rate? Ans. John in $3\frac{1}{3}$ days, and James in 5.

Explanatory Remark.—In this question it will be found by a little consideration that John hoes $\frac{3}{5}$, and James $\frac{2}{5}$, of an acre in 2 days.

13. If a man perform a journey in 5 days, when the days are 15 hours long, in how many days will he perform the same when the days are 10 hours long?

Ans. $7\frac{1}{2}$ days.

14. If when wheat is \$1.50 per bushel, a penny-worth of bread weighs 4 oz. what will a penny-worth weigh when wheat is worth \$2 per bushel? Ans. 3 oz.

15. As I was beating on the forest ground,
Up started a hare before my fleet grey hounds;
The dog being swift on foot did nimbly run,
To her eighteen rods just twenty one.
The distance that she started up before,
Was fifteen rods, precisely, and no more;
And now I wish you would to me declare,
How far he ran before he caught the hare?

Ans. 105 rods.

I Questions for Examination.

1. What is the ^{single} Rule of Three?
2. What is the ^{single} rule of three direct?
3. What is stating a question in the rule of three?
4. How do you state a question?
5. How do you work a statement in the rule of three direct?
6. How is a statement worked in the rule of three inverse?

S I M P L E I N T E R E S T .

7. What is the rule of three inverse?
8. How are operations in the rule of three proved?

I N T E R E S T .

Interest is a premium or sum paid or allowed by the borrower of money to the lender, for the use of the sum lent.*

Interest is of two kinds, simple and Compound.

Simple Interest is that which is allowed for the use of the principal only.

Compound Interest is that which arises from the principal and interest being added together.

Principal is the money lent or let out.

The Rate per cent. is the sum allowed for the use of $\text{£} 100$ or $\text{£} 100$. — Per annum means for a year.

Amount is the sum of the principal and interest added together.

S I M P L E I N T E R E S T .

General Rule.

Multiply the principal by the rate per cent., and cut off the two right hand figures of the highest denomination; the figures on the left hand will be interest: reduce the right hand figures to the next lower denomination, and cut off as before, and so on, to the lowest denomination. The whole interest so found will be for one year. Then multiply the interest of one year by the number of years. If there are odd months, &c. with years, take proportional parts of a year.

Proof. When interest is found by multiplication, the proof is the same as for that rule.

Reason. The reason why the principal is multiplied by the rate, and two figures cut off, is because all questions in interest, belong to the rule of three. The first term is $\text{£} 100$ or $\text{£} 100$; the second term is the rate per cent., and the third term is the principal.

* Interest is so called from the Latin *inter* among, and *esse* to eat, because it eats among the property of the giver of money.

SIMPLE INTEREST.

Examples.

1. What is the interest of \$100 for 3 years, at 6 per cent. per annum? Ans. \$18.

\$100 = the principal.

6 = the rate per cent.

\$6,00 = the interest for 1 year.

\$18 = the interest for 3 years.

2. What is the interest of £120 10s. 6d. for one year, at 6 per cent. per annum?

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 120 \quad 10 \quad 6 \\
 \hline
 \text{£} 7, 21 \quad 19 \quad 0 \\
 \quad \quad 20 \\
 \hline
 \text{s.} 4, 39 \\
 \quad \quad 12 \\
 \hline
 \text{d.} 4, 68 \\
 \quad \quad 4 \\
 \hline
 \text{qrs.} 2, 72
 \end{array}$$

$$\text{Ans. } \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \quad \text{qrs.} \\ 7 \quad 4 \quad 4 \quad 2 \quad \frac{72}{100} \end{array}$$

Explanation.—Here the principal is multiplied by the rate, as in compound multiplication, and two figures are cut off in the pounds, which leaves £7 for interest. Then the £21 cut off are reduced to shillings, and the 19s. added in, and two figures cut off from the product as before, leaves 4s. for interest, &c.

If the interest had been wanted for several years, the interest for 1 year must have been multiplied by the number of years. If for 1 year and 6 months, half the interest for a year should be added to itself; if 4 months, $\frac{1}{3}$; if 3 months, $\frac{1}{4}$, if 2 months, $\frac{1}{6}$; if 1 month, $\frac{1}{12}$, must be added.

When you want the interest for weeks, remember that a week is the $\frac{1}{52}$ part of a year. Also, a day is the $\frac{1}{365}$ th part of a common year.

SIMPLE INTEREST.

<u>£.</u> <u>s.</u> <u>d.</u>					
What is the interest of 728 : 16 : 8 for three years, nine months at $4\frac{1}{4}$ per cent, per annum?					
<u>£.</u> <u>s.</u> <u>d.</u>					
728 : 16 : 8 = Principal					
$4\frac{1}{4}$ Rate per cent.					
			<u>£.</u> <u>s.</u> <u>d.</u>		
2915 : 6 : 8			M 34 : 12 : 4 $\frac{1}{2}$ = Int. for 1 year:		
364 : 8 : 4			6 $\frac{1}{2}$		
182 : 4 : 2			3		
34,61 : 19 : 2			103 : 17 : 1 $\frac{1}{2}$ = Int. for 3 years		
20			17 : 6 : 2 $\frac{1}{4}$ = do. 6 months		
			8 : 13 : 1 = do. 3 months		
12,39					
12			129 : 16 : 4 $\frac{1}{4}$ Ans.		
4,70					
4					
2,80					

Remark. — The 80 which remains after multiplying by 4, is $\frac{80}{100}$ of a farthing, = $\frac{4}{5}$.

11. What is the Interest of 254l. 17s. 6d. for five years, at 4 per cent. per annum? Ans. 50l. 19s. 5d. $\frac{1}{2}$
12. What is the Interest of 556l. 13s. 4d. at 5 per cent. per annum, for 5 years? Ans. 139l. 3s. 4d.
13. What is the Interest of 325l. 7s. 6d. at 6 per cent. per annum, for 3 years and a half? Ans. 68l. 6s. 6d. $\frac{1}{4}$
14. What is the Interest of 547l. 2s. 4d. for five years and a half at 4 per cent. per annum? Ans. 120l. 7s. 3d. $\frac{1}{4}$
15. What is the Interest of 257l. 5s. 1d. at 4 per cent. for a year and 3 quarters? Ans. 18l. 0s. 1d. $\frac{1}{2}$
16. What is the Interest of 479l. 5s. for 5 years, and one quarter, at 5 per cent. per annum? Ans. 125l. 16s. 1d. $\frac{1}{2}$

To find the interest of any sum for any number of months only, at 6 per cent. per annum.

Rule. — Multiply the principal by half the number of months, and proceed in other respects as in finding the interest for a year.

Reason. — Because the rate per cent. 6, is half the number of months in a year.

When the rate per cent. is more or less than 6, find the interest for 6, and then add or subtract a proportional part.

Examples.

1. What is the interest of £ 100 10s. for 8 months, at 6 per cent. per annum?

Ans. £ 4 0s. 9 $\frac{60}{100}$ d.

SINGLE INTEREST.

$\frac{\text{£}}{100} \quad \frac{\text{s.}}{10} = \text{the principal.}$
 $4 = \text{half the months.}$

$\text{£ } 4,040$
 $\quad 20$
 $\text{s. } 0,80$
 $\quad 12$
 $\text{d. } 9,60$

2. What is the interest of $\text{£ } 100$ for 9 months, at 8 per cent. per annum?

Ans. $\text{£ } 6$.

$\frac{\text{£}}{100} = \text{the principal.}$
 $4\frac{1}{2} = \text{half the months.}$

$400 = \text{the product by } 4.$

$50 = \frac{1}{2} \text{ the multiplicand.}$

$\text{£ } 4,50 = \text{the product by } 4\frac{1}{2}.$

$\quad 20$
 $\text{s. } 10,00$

$3) \frac{\text{£}}{4} \quad \frac{\text{s.}}{10} = \text{the int. at 6 per cent.}$

$\quad 1 \quad 10 = \text{the int. at 2 per cent.}$

$\quad 6 \quad 0 = \text{the int. at 8 per cent.}$

$\frac{1}{3}$ of 6 added to itself, is 8.

3. What is the interest of $\text{£ } 50 \text{ } 14\text{s. } 6\text{d.}$ for 15 months, at 6 per cent. per annum?

Ans. $\text{£ } 3 \text{ } 15\text{s. } 1\frac{5}{100}\text{d.}$

To find the interest of any sum for any number of days, at 6 per cent. per annum.

Rule.—As 6083 days are to the principal, so is the given number of days to the interest required.

Reason.—Because any principal will double, or produce a sum in interest equal to itself, in 6083 days, at 6 per cent. per annum.[†]

Examples.

1. What is the interest of $\text{£ } 100$ for 23 days, at 6 per cent. per annum?

[†] When the rate per cent is more or less than 6, add or subtract a proportional part.

S I M P L E

I N T E R E S T

Ans. 75. 6 $\frac{3}{4}$ d. nearly.

$$\begin{array}{r} \text{D.} \\ 6083 : 100 :: 23 \\ \hline \text{L.} \\ 6083 \overline{) 2300} \left(\frac{\text{L.}}{0} \right. \\ \underline{20} \\ 6083 \overline{) 46000} \left(\frac{3}{7} \right. \\ \underline{42581} \\ 3429 \\ \underline{12} \\ 6083 \overline{) 41148} \left(\frac{\text{d.}}{6} \right. \\ \underline{36498} \\ 4550 \\ \underline{4} \\ 6083 \overline{) 18200} \left(\frac{\text{grs.}}{3} \text{ nearly.} \right. \end{array}$$

2. What is the interest of \$150 for 28 days, at 7 per cent. per annum?

Ans. \$0.80 $\frac{1}{2}$ +.

$$\begin{array}{r} \text{D.} \\ 6083 : 150 :: 28 \\ \hline \text{L.} \\ 6083 \overline{) 1200} \\ \underline{90} \\ 6083 \overline{) 4200} \left(\frac{\text{L.}}{0} \right. \\ \hline \text{D.} \\ 6083 \overline{) 420000} \left(\frac{\text{cts.}}{69,061. +} \right. \\ \underline{36498} \\ 55120 \\ \underline{54747} \\ 37300 \\ \underline{36498} \\ 8020 \end{array}$$

6) $\frac{\text{cts.}}{69,061}$ = the interest at 6 per cent.
 $\frac{11,510}{80,571}$ = int. at 1 per cent.
 $\frac{80,571}{80,571}$ = the int. at 7 per cent.

$\frac{1}{6}$ of 6 added to itself, makes 7.

Remark. After finding the interest of any sum at 6 per cent., if you want it for 3, subtract $\frac{1}{2}$; if for 4, subtract $\frac{1}{3}$; if 5, subtract $\frac{1}{6}$. If you want the interest for 7 per cent., add $\frac{1}{6}$; if for 8, add $\frac{1}{3}$; if for 9 per cent., add $\frac{1}{2}$ of that of 6 per cent. to itself, &c.

SIMPLE INTEREST

3. What is the interest of £27 5s. 4½d. for 9 days, at 6 per cent. per annum? Ans. 9d. 2½ 208 grs.

SIMPLE INTEREST IN FEDERAL MONEY.

Simple Interest on small sums may be reckoned mentally, by knowing what proportion the interest bears to the principal. At 6 per cent. per annum, the interest on a dollar is 6 cents per year, or half a cent per month.

Questions to be solved mentally.

1. What is the interest of \$5 for a month? Ans. 2½ cents.
2. What is the interest of \$6 for 3 months? Ans. 9 cents.
3. What is the interest of \$10 for half a month? Ans. 2½ cents.

To compute Interest by Rule.

When the principal is Dollars only, to find the Interest for a year.

Rule.—Multiply the principal by the rate per cent., and cut off the two right hand figures, which will be cents; the figures on the left hand will be dollars.

Reason.—Because cutting off two figures on the right hand, is the same as dividing by 100; and 100 cents make a dollar.

Examples.

1. What is the interest of \$125 for a year, at 6 per cent. per annum? Ans. \$7, 50.

\$125 = the principal.

6 = the rate per cent.

\$7, 50 = the interest.

2. What is the interest of \$340, at 5 per cent.? Ans. \$17.
3. --- 115, at 7 per cent.? Ans. \$8, 05.
4. --- 27, at 9 per cent.? Ans. \$2, 43.
5. --- 76, at 8 per cent.? Ans. \$6, 08.
6. --- 144, at 6 per cent.? Ans. \$8, 64.

When the Principal is Dollars and Cents, to find the Interest for a year.

Rule.—Multiply the principal by the rate per cent., and cut off four figures to the right hand; the left hand figures will be dollars; the two first to the right of the comma will be cents; the third mills, and the fourth decimals

SIMPLE INTEREST.

An Example
For finding the interest of Dollars, Cents,
and Mills for years, months and days,
by taking aliquot parts.

What is the interest of \$ 58, 25, 5, for
 2 years, 5 months, and 17 days, at 6 per
 cent. per annum?

		\$ 58, 25, 5	
4 mon. = $\frac{1}{3}$	\$ 3, 49, 53	0 = interest for 1 year.	
		2	
	6, 99, 06	0 = int. for 2 yrs.	
1 mon. = $\frac{1}{4}$	1, 16, 57	0 = int. for 4 months.	
10 days = $\frac{1}{3}$	29, 12	7 = int. for 1 month	
5 days = $\frac{1}{2}$	9, 70	9 = int. for 10 days.	
1 day = $\frac{1}{5}$	4, 85	4 = int. for 5 days.	
1 day = $\frac{1}{5}$	9, 7	0 = int. for 1 day.	
		0 = int. for 1 day.	
	\$ 8, 61, 200	Answer.	

In dividing by the aliquot parts, the remainders after the decimal of a mill, may be neglected, as of small value.

A Shorter and Easier Rule
For finding the interest of any sum
in Federal Money, for years, months,
and days, at 6 per cent. per an-
num.

The Rule.—Multiply the principal by half the number of months, and the sixth part of the days. The $\frac{1}{6}$ of the days will be a decimal, and must be set to the right hand of $\frac{1}{2}$ the number of months. When the number of months is odd, take half the greatest even number, and reduce the odd month to days, and add to the given number of days. A month is here reckoned 30 days.

After multiplying, if the principal is

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dollars only, cut off one figure at the right hand of the product, which will be mills, the others will be cents.

If the principal is dollars and cents, cut off three figures; those on the left hand will be cents, the figure next to cents will be mills, and the others decimals of a mill.

If the principal is dollars, cents, and mills, cut off four figures for mills and decimals of a mill; the rest will be cents, as before, which reduce to dollars by cutting off the two right hand figures.*

When there is a remainder in dividing the days by 6, place it over the divisor, and set the vulgar ^{fraction} so formed, to the right of the decimal. Then, in multiplying by the vulgar fraction, if it be

$\frac{1}{6}$, take $\frac{1}{6}$ of the principal, or multiplicand.

$\frac{2}{6}$, take $\frac{1}{3}$.

$\frac{3}{6}$, take $\frac{1}{2}$.

$\frac{4}{6}$, take $\frac{1}{3}$ and set it down twice.

$\frac{5}{6}$, take $\frac{1}{2}$, and then $\frac{1}{3}$.

The quotient or quotients arising from these fractions respectively, being added to the partial product, will give the whole sum.

Reason. The reason why the sixth part of the days is taken, and expressed decimally, is because that is a short way of reducing days to the decimal of a month, and taking half of it.

Examples.

1. What is the interest of \$58, 25, 5, for 2 years, 5 months, and 17 days, at 6 per cent. per annum?

* This rule is sufficiently accurate for computing interest, except in very large sums. When the principal is very large, deduct the $\frac{1}{3}$ part of the product by the $\frac{1}{6}$ of the days from the interest, because 5 days is the $\frac{1}{73}$ part of 365.

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\$58, 25, 5 = the principal.

14, $7\frac{5}{6}$ = $\frac{1}{2}$ the months and $\frac{1}{6}$ of the days.

$$\begin{array}{r} 40 \ 7 \ 78 \ 5 \\ 233 \ 0 \ 20 \\ \hline 582 \ 9 \ 12 \end{array} 7 = \frac{1}{2} \text{ of } \$58, 25, 5$$

$$1 \ 9 \ 4 \ 1 \ 8 = \frac{1}{3} \text{ of } \$58, 25, 5$$

\$8, 61, 2030 = the answer, or interest sought.

Explanation.—Here the number of months in 2 years and 5 months, is 29, the greatest even number is 28, half of which is 14. Then calling the odd month 30 days, and adding them to the 17 given days, makes 47, one sixth of which are $7\frac{5}{6}$, because there are 7 times 6 in 47, and 5 remain, which 7 is a decimal, and $\frac{5}{6}$ a vulgar fraction. In taking $\frac{5}{6}$ of the principal, or multiplicand, $\frac{1}{2}$ is taken first, and then $\frac{1}{3}$, because $\frac{3}{6}$ is $\frac{1}{2}$, and $\frac{2}{6}$ is $\frac{1}{3}$, and $\frac{3}{6}$ and $\frac{2}{6}$ is $\frac{5}{6}$. After adding all the numbers, four figures are cut off to the right, the first 2 which 2, is mills, and the other three are decimals of a mill. The figures to the left hand of those cut off were cents, and by cutting off the two right hand ones, they become \$8, 61, and the whole is \$8, 61, $2\frac{3}{100}$, differing only the fraction from the answer by the other method. This last method for its conciseness and ease, is the best for practice, and by it the answers to the following questions were found.

When the rate per cent. is not 6, first find the interest at 6 per cent., and add or subtract a proportional part. Thus, for 5 per cent. subtract $\frac{1}{6}$ of the interest for 6 per cent.: for 7, add $\frac{1}{6}$, &c.

2. What is the interest of \$30, 25 for 1 year, 9 months, and 13 days, at 6 per cent. per annum? Ans. \$3, 24, 179, or \$3, 24, 2 nearly.

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3. What is the interest of \$140,44 for 3 years, 7 months, and 14 days, at 6 per cent. per annum? Ans. \$30,38,2 nearly; or, \$30,38,185.

4. What is the interest of \$75 for 11 months and 5 days? at 6 per cent. per annum?

Ans. \$4,18,7.

5. What is the interest of \$97,75 for 19 months and 2 days, at 7 per cent. per annum?

Ans. \$10,87,2.-

6. What is the interest of \$324 for 4 days, at 6 per cent. per annum? Ans. 54 cents.

$$\begin{array}{r} \$324\frac{2}{3} \\ \underline{103} \\ 324 \\ \frac{1}{3} = 108 \\ \frac{2}{3} = 108 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} 216$$

Ans. 54,0

$$\begin{array}{r} 3)324 \\ \underline{108} = \frac{1}{3} \text{ of } 324. \\ 216 = \frac{2}{3} \text{ of } 324. \end{array}$$

Remark.—Here the number of days 4, being less than 6, the quotient is 0, and the vulgar fraction $\frac{4}{6}$ is equivalent to $\frac{2}{3}$. The value of $\frac{2}{3}$ of \$324 is found on the right hand, by first taking $\frac{1}{3}$, and multiplying it by 2, which must give the same result as setting $\frac{1}{3}$ of 324 down twice, and adding them together. It is immaterial which method is used; some may prefer one; others, the other.

7. A gave his note to B, for \$100, payable in 13 months and 12 days with interest at 6 per cent. per annum, but the day he gave the note, he paid \$6 and had it endorsed; what was the interest due at the time the note became due? Ans. \$6,29,8.

Remark.—As \$6 were paid down, the principle was reduced to \$94.

Useful Problems in Simple Interest.

SIMPLE INTEREST.

I. The amount, time, and rate per cent.
given to find the principal.

Rule.—As the amount of \$100 or £100
for the given rate and time is to 100, so
is the amount given, to the principal re-
quired.

Examples.

1. What principal being put to interest
will amount to \$195 in 5 years, at
6 per cent. per annum? Ans. \$150.

$$\begin{array}{r} \$100 \\ 30 = \text{int. 5 yrs.} \\ \hline \text{Amt. } 130 : 100 :: \$195 \\ \hline 130 \overline{) 19500} \begin{array}{l} 100 \\ 130 \\ \hline 650 \\ 650 \\ \hline 0 \end{array} \end{array}$$

(150 Ans.)

2. What principal being put to interest
will amount to £42 10s. in 5 years, at
3 per cent. per annum?
Ans. £350.

II. The Principal, rate per cent., and a-
mount, given to find the time.

Rule.—As the interest of the princi-
pal for one year is to one year, so is the
whole interest to the time required.

Examples.

1. In what time will \$100 amount
to \$136, at 6 per cent. per annum?
Ans. 6 years.

2. In what time will £540 amount to
£734 8s., at 4 per cent. per annum?
Ans. 9 years.

III. The Principal, amount, and
time, given to find the rate per cent.

SIMPLE INTEREST

Rule.—As the principal is to the interest for the whole time, so is £100 or \$100 to the interest for the same time. Divide the interest by the time, and the quotient will be the rate per cent.

Examples.

1. At what rate per cent. will £350 amount to £402 10s. in 5 years?

Ans. 3 per cent.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \\
 402 \quad 10 = \text{amount.} \\
 350 \quad 00 = \text{principal.} \\
 \hline
 52 \quad 10 = \text{whole int.}
 \end{array}
 \quad
 \left.
 \begin{array}{l}
 \text{As } 350 : 52 \frac{10}{100} :: 100 \\
 \frac{20}{1050} \\
 \frac{100}{350} \overline{) 105000} (300 = \text{£ } 15. \\
 \underline{1050} \\
 \dots 00
 \end{array}
 \right\}$$

$\frac{\text{£}}{5} \overline{) 15} = \text{int. of £100 for 5 years.}$
 $3 = \text{int. for 1 year, } \underline{\text{Ans.}}$

2. At what rate per cent. will \$575 amount to \$695.75, in 3 years?

Ans. 7 per cent.

The Nature of Simple Interest on Notes and Bonds, on which Endorsements or Partial Payments are made.

There are many different opinions respecting the time when interest becomes due. It has been a popular idea that interest is not due till the note or obligation drawing interest is paid, and it is generally known that the Courts will not sustain an action for the payment of interest from year to year, unless it be plainly expressed in the obligation that the interest shall be paid annually. On this principle interest on notes and obligations, when enclosures have been made, has gen-

SIMPLE INTEREST.
usually been computed by the following
Rule. (I.)

Find the amount of the principal and interest from the time the obligation is given till it is all paid. Find also, the amount of each endorsement, by computing interest on it from the time of endorsement till the time of settlement. From the amount of the debt, deduct the amount of the endorsements, and the remainder is the balance due.

Remark.—This ^{rule} appears to be equitable when a note is given to be paid at a future time, and no agreement is made that the interest shall be paid annually, but when a note is given on demand, the interest as well as principal is legally due at any time.

Example.

If a note was given Jan. 1, 1828, for \$500, payable Jan. 1, 1830, with interest at 6 per cent. ~~per annum~~, and the following endorsements were made, viz. Received, Apr. 1, 1829, \$30.— Aug. 1, 1829, \$57.50, what was the balance due on the note Jan. 1, 1830? Ans. \$469, 71, 2½.

Calculation.

Principal, - - - - -	\$ 500, 00, 0
Interest for 2 years, - - - - -	60
Amount of the note, - - - - -	560, 00, 0
First payment, Apr. 1, 1829, \$30, 00, 0	
Interest of \$30 for 9 months, - -	1, 35, 0
Second payment, Aug. 1, 1829, - -	57, 50, 0
Interest of \$57.50 for 5 months, - -	1, 43, 7½
Amount of the payments, - - - - -	\$ 90, 28, 7½
Balance due on settlement, - - - - -	\$ 469, 71, 2½

☞ If the interest of a sum of money due several years hence, be paid annually, it is nothing more nor less than paying compound interest; for the effect is the same, both to the borrower and lender of money, whether the interest due at the end of a year be paid to the lender, and he lets it ^{on interest} to another person, or leaves it in the hands of the original borrower and renews the note and adds the interest to the sum first lent, to form a new

SIMPLE INTEREST.

Principal, since the sum of the interest of \$100 and of \$6 for any time is the same as the interest of one sum of \$106 for the same time. The borrower, also, may as well pay interest on interest to the original lender, as to hire the money on interest of a second person to pay the interest to the first.

If a person hires a house or a piece of ground of another, at a certain rent per year, and finds it convenient to pay the rent at the beginning, instead of the end of the year, the one who rents the premises ought to discount from the annual rent, or else pay interest for a year on the sum received, at the year's end, and the same reason will hold in hiring money as in hiring a house.

Rule II.

1. First find the interest of the given sum or principal from the time the interest commences to the time of the first payment which either by itself, or with any preceding payment or payments, if any, ^{equals or} exceeds the interest due at that time, and add it to the given principal.

2. From the amount found, subtract the payment made at that time, with the preceding payments if any, and the remainder will form a new principal, with which and the after payments, proceed as with the first, till the time of settlement.

Remark.—This rule is considered as the most equitable, because no part of the interest forms a part of the principal drawing interest. It is particularly adapted to endorsements on notes given on demand, but as all money paid ought to draw interest or stop interest, no debtor who understands the subject, will make a partial payment on a note unless he can pay ^{as much, or} more than the interest due at the time, for if he does, he loses the use of his money, because it stops interest on his note. For by this rule the whole principal draws interest till the sum of the endorsements, however many, is equal to the interest due when the last is paid. The debtor may therefore as well trade upon his money until he can pay at once, a sum exceeding the interest then due, and thereby reduce the principal.

¶ This second rule has been said to be the one established in Massachusetts, but it appears to have originated in New-York.

Examples.

1. A, by his note dated Jan. 1, 1780, promises to pay \$1000 in six months, with interest from the date, at 6 per cent. per annum.

Or

SIMPLE INTEREST.

On his note are the following endorsements:

Received, April 1, 1780, \$24. - August 1, 1780, \$4. -
 December 1, 1780, \$6. - Feb. 1, 1781, \$60. - July 1, 1781,
 \$40. - June 1, 1784, \$300. - Sept. 1, 1784, \$12. -
 Jan. 1, 1785, \$15. - Oct. 1, 1785, \$50. What was
 due on the note, Dec. 1, 1790?

Ans. \$1061, 95.

Calculation to the nearest Cent.

The principal carrying interest from Jan. 1, 1780, - - -	\$	1000	cts.	00
Interest to Apr. 1, 1780, (3 months) - - -		15	00	
Paid Apr. 1, 1780, a sum exceeding the interest, - - -	Amount	1015	00	
		24	00	
Remainder for a new principal, - - -		991	00	
Interest on \$991 from Apr. 1, 1780, to Feb. 1, 1781, (10 mo.) - -		49	55	
Paid Aug. 1, 1780, a sum less than the interest, \$4, 00	Amount	1040	55	
Paid Dec. 1, 1780, do. - - -		6	00	
Paid Feb. 1, 1781, do. greater than the interest, 60, 00				
Amount of the payments, - - -		70	00	
Remainder for a new principal, - - -		970	55	
Interest on \$970, 55 from Feb. 1, 1781, to July 1, 1781 (5 mo.) - -		24	26	
Paid July 1, 1781, a sum exceeding the interest, - - -	Amount	994	81	
		40	00	
Remainder for a new principal, - - -		954	81	
Interest on \$954, 81, from July 1, 1781, to June 1, 1784, } (2 years and 11 months) - - -		167	09	
Paid June 1, 1784, a sum exceeding the interest, - - -	Amount	1121	90	
		300	00	
Remainder for a new principal, - - -		821	90	
Interest on \$821, 90 from June 1, 1784, to Oct. 1, 1785 } (1 year and 4 months) - - -		65	75	
Sept. 1, 1784, paid a sum less than the interest, \$12, 00	Amount	887	65	
Jan. 1, 1785, paid do. - - -		15	00	
Oct. 1, 1785, paid do. - - -		50	00	
Remainder for a new principal, - - -		810	65	
Interest on \$810, 65 from Oct. 1, 1785, to Dec. 1, } 1790 (5 years and 2 months) - - -		251	30	
Required amount due, - - -		1061	95	

The interest of each principal is found by multiplying by half the number of months contained in any number of years and months. -
 When there are days as well as months, multiply by half the number of months and the sixth part of the days, &c.

IMPLE INTEREST.

To find the time from one date to another, set the less under the greater, and subtract, borrowing 30 in the days, and 12 in the months, when necessary; reckoning the number of the months as follows.

Jan. 1. Apr. 4. July 7. Oct. 10.
Feb. 2. May 5. Aug. 8. Nov. 11.
March 3. June 6. Sept. 9. Dec. 12.

Example.

If a note was given Jan. 5, 1795, and an endorsement was made on it Aug. 3, 1800, what was the time between the two dates? Ans. 5 yrs. 6 mo. 28 d.

yrs.	mo.	d.
1800	8	3
1795	1	5
Difference	5	6 28

In computing interest on notes and obligations by Rule II., the first step is to find the time between two dates; the second is to find the interest to the time of the first payment; then if the payment is more than the interest, subtract the payment from the sum of the principal and interest, which will leave the next principal, and so on; but when the payment is less than the interest due at the time, the payment is not subtracted, but the whole principal continues on interest till a second payment is made, and if the sum of the first and second payments is less than the interest due when the second payment is made, the same principal continues on interest till such a payment is made that by itself or with the preceding payments exceeds the interest due when the last mentioned payment is made. I have been thus particular here, that the scholar may have a clear idea of the method.

2. Supposing a note to be given Feb. 5, 1829, for \$100, on interest at 6 per cent. per annum, on which are the two following endorsements:

Received, Aug. 4, 1829, \$2. — Jan. 3, 1830, \$15,
how much is due on the note April 7, 1830?

Ans. \$90,32,6.

The Table explained.
Look at the left hand side of the Table for the month the time commences from, then draw your finger from that month upon a straight line till you come to the square that lies straight under the month the time terminates in, and the figures contained in that square will be the number of days from any one day of the former to the same day of the latter month. For instance, I want to know the number of days from May 9, to December 9, I carry my finger in a straight line from May till I come to the square under December, where I find 214, the number of days required.

A TABLE shewing the Number of Days from any Day in one Month to the same Day in any other Month.

From	to Jan	to Feb	to Mar	to Apr	to May	to Jun	to Jul	to Aug	to Sep	to Oct	to Nov	to Dec
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	244	275	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	334	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	242	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

SIMPLE INTEREST.

Simple Interest By Decimals.

A TABLE OF RATIOS.

Rate per cent.	Ratio.	Rate per cent.	Ratio.
3	,03	6	,06
3½	,035	6½	,065
4	,04	7	,07
4½	,045	7½	,075
5	,05	8	,08
5½	,055	8½	,08½

Explanation.—Ratio is the simple interest of £1 or \$1, for a year, expressed decimally. Thus, at 6 per cent. per annum, the interest of \$1 is 6 cents, or $\frac{6}{100}$, which reduced to a decimal, is ,06. The interest of £1 is ,06 of a £, at the same rate, &c.

To find the interest of any sum by Decimals.

Rule.—Multiply the principal, time, and ratio continually together, and the product will be ^{the} interest required, observing to cut off as many decimals in every product as there are decimals in the multiplier and multiplicand.

When there are months and days in the time, reduce them to the decimal of a year. Also reduce the principal, if lawful money, to the decimal of a pound, when shillings, &c. make a part of the principal. — Federal money needs no reduction, as cents and mills are decimals of a dollar.

Examples.

1. What is the interest of \$75,25 for 3 years, at 6 per cent. per annum?

Ans. \$13,63,5.

\$75,75 = the principal.

3 = the time.

227,25

1,06 = the ratio.

\$13,6350 = the interest.

2. What is the interest of £10 10s. for 4 years and 6 months, at 6 per cent. per annum?

Ans. £2 16s. 8½ d.

£10,5 = the principal.

4,5 = the time.

525

420

47,25

£2,5250 = the interest in pounds and decimals of a pound.

COMPOUND INTEREST.

3. What is the interest of \$50 for 7 months, at 6 per cent. per annum? Ans. \$1, 74, 9.

Q To solve this question decimally, reduce the 7 months to the decimal of a year.

4. What is the interest of \$320, 30 for 100 days, at 6 per cent. per annum? Ans. \$5, 26, 1.

Q Here 100 days are $\frac{100}{365}$ of a year, = .274+ expressed decimally.

I Questions for Examination.

1. What is interest? 2. What is simple interest? 3. What is the general rule for ^{finding} the interest of any sum for a year? 4. How do you find the interest of a sum for several years? What is the principal? The rate per cent.? The amount? When the rate is 6 per cent. how do you find the interest for any number of months? How do you find the interest for days, or months and days? How is interest by decimals computed?

COMPOUND INTEREST.

Compound Interest is that which arises from the sum of the principal and interest added together, when the interest is not paid as it becomes due, by which ~~both~~ the principal and interest are always increasing. Compound interest is therefore also called interest upon interest.

Rule.—Find the amount by simple interest for the first year, and that will be the principal for the second year, and so on from year to year for any number of years. Subtract the first principal from the last amount, and the remainder will be the compound interest.

Remark.—As all the difference between simple and compound interest is, that in simple interest there is only one principal, and in compound interest a new principal is formed every time the interest is added to the last, the method of operation and reason of the rule is the same as in simple interest.

COMPOUND INTEREST.

Examples.

1. What is the compound interest on \$100, at 6 per cent. per annum, for 4 years?

Calculation.

Ans. \$26,24,7 +.

\$100 = the principal for the 1st year.
 $\frac{6}{100} =$ the rate per cent.
 6,00 = interest for the 1st year.
 100,00 = 1st principal.
 $\frac{106}{100} =$ the prin. for the 2d year. (1st amount.)
 $\frac{6}{100} =$ the rate.
 6,36 = int. for 2d year.
 106,00 = 2d principal.
 $\frac{112,36}{100} =$ prin. for 3d year. (2d amt.)
 $\frac{6}{100} =$ the rate.
 6,7416 = int. for 3d year.
 $\frac{112,36}{100} =$ prin. for 3d year.
 119,1016 = prin. for 4th year. (3d amt.)
 $\frac{6}{100} =$ the rate.
 7,146096 = int. for 4th year.
 119,1016 = prin. for 4th year.
 \$126,247696 = the amount for 4 years.
 \$100..... = the 1st principal.
 826,247696 = the compound interest required. ‡

Explanation. When the principal is dollars only, the scholar will recollect that only two figures are to be cut off from the product after multiplying by the rate, which figures cut off are cents, and the others are dollars. But when the principal is dollars and cents, four figures are cut off; two, so as to divide by 100, and two more for cents. Four places for decimals of a dollar are sufficient, viz. two for cents, one for mills, and one for decimals of a mill, but in this work all the decimals produced are set down, to show the nature of it. As dividing by 100 cuts off two figures, two more than are contained in the principal and rate must be cut off from every product.

‡ The same found by dividing, instead of multiplying.

\$100 = 1st prin.
 $\frac{1}{20} = \frac{1}{5}$ } 1st year's int.
 5,3 }
 1,06 } 2d int.
 112,36 = 3d prin.
 $\frac{1}{20} = \frac{1}{5}$ } 3d int.
 5,618 }
 1,1236 }
 119,1016 = 4th prin.
 $\frac{1}{20} = \frac{1}{5}$ } 4th int.
 5,95508 }
 1,191016 }
 \$126,247696 = the amount, as before, from which subtract the first principal.

Explanation.

As 5 is the 20th part of 100, dividing the principal by 20, gives the interest at 5 per cent.; therefore, for 6 per cent. add $\frac{1}{5}$ of the interest at 5 per cent. to itself. This method, for practice, is generally preferred to the first or general rule, though that is rather easier for students, and answers for any rate.

COMPOUND INTEREST.

When there are months and days in the time, find the interest on the last amount for even years, for the odd months, or months and days, and add it to the amount for the years, and the sum will be the whole amount.

2. What is the compound interest of \$876.90, for $3\frac{1}{2}$ years, at 6 per cent. per annum?

Ans. \$198.79 nearly.

3. What is the amount of 550l. 10s. for 3 years and 6 months, at 6 per cent. per annum, compound interest?

Ans. 675l. 6s. 5d.

4. What is the compound interest of 764l. for 4 years and 9 months, at 6 per cent. per annum?

Ans. 243l. 18s. 8d.

5. What is the compound interest of 57l. 10s. 6d. for 5 years, 7 months, and 15 days, at 5 per cent. per annum?

Ans. 18l. 3s. 8d. $\frac{1}{4}$

COMPOUND INTEREST BY DECIMALS.

Rule.

Multiply the given principal (reducing the less denominations if any, to the decimal of the greater) continually by the amount of \$1 or £1, at the given rate per cent. until the number of multiplications are equal to the given number of years, and the product will be the amount, from which subtract the principal, and the remainder will be the compound interest; recollecting that as many decimals must be pointed off in the product, as are contained in the multiplier and multiplicand.

Or, find the amount of \$1 or £1 for the given number of years and rate per cent., in the following table, by which multiply the given principal, and the product will be the amount.

Note. When the time is years, months, &c., find the interest of the odd months, &c., by the rules of simple interest, and add it to the amount of the number of years.

The amount of \$1 or £1 for one year, at 5 per cent., is \$1.05, or £1.05. At 6 per cent., it is \$1.06, or £1.06, &c.

Examples.

1. What is the amount and compound interest of \$51.25 for 3 years, at 6 per cent. per annum?

\$51.25 = the principal.

1.06 = amount of \$1 for 1 year.

30750

51250

533250 = 1 multiplication.

106

3259500

5432500

57585500 = 2 multiplications. — (carried over.)

COMPOUND INTEREST.

$57,5845 \dots$ (brought over, omitting the ciphers.)
 $\begin{array}{r} 1,06 \\ 3555070 \\ 5758450 \\ \hline \end{array}$
 $\$61,039570 =$ the amount, or 3d product.
 $51,25 \dots =$ the principal.
 $\$9,78,9\frac{1}{2}, \&c. =$ the compound interest.

Operation by the following Table.

$\$1,19101 =$ the amount of $\$1$ for 3 years.
 $51,25 =$ the principal.
 $\begin{array}{r} 595505 \\ 238202 \\ 119101 \\ \hline 595505 \end{array}$
 $\$61,0392625 =$ the amount, as before.
 This method by the table is the most concise, and the best for practice. *

or $\$1$

A. TABLE II. Shewing the amount of 1l. for any number of years under 31, at 5 and 6 per cent. per ann.

Years.	5 Rates	6	Years.	5 Rates	6
1	1,05	1,06	16	2,18287	2,54035
2	1,1025	1,1236	17	2,29201	2,69277
3	1,15762	1,19101	18	2,40662	2,85434
4	1,2155	1,26247	19	2,52695	3,0256
5	1,27628	1,33822	20	2,65329	3,20713
6	1,34009	1,41852	21	2,78596	3,39956
7	1,4071	1,50363	22	2,92526	3,60352
8	1,47745	1,59384	23	3,07152	3,81975
9	1,55132	1,68948	24	3,2251	4,04893
10	1,62889	1,79084	25	3,38635	4,29187
11	1,71034	1,89829	26	3,55567	4,54938
12	1,79585	2,01219	27	3,73345	4,82234
13	1,88565	2,13292	28	3,92013	5,11168
14	1,97993	2,2609	29	4,11613	5,41838
15	2,07892	2,39655	30	4,32194	5,74349

The above table is thus made: As 100 : 105 ::
 1 : 1,05 for the first year; then, As 100 : 105 :: 1,05 :
 1,1025, second year, &c, it is therefore easy to make
 such tables for any rate per cent.

2. What will $\underline{\pounds}225$ amount to in 3 years, at 5 per cent. per annum, compound interest?

Ans. $\underline{\pounds}260\ 9s. \ 3\frac{3}{4}d.$

3. What is the amount of $\$100$ for 4 years, at 6 per cent. per annum, compound interest?

Ans. $\$126,24,7.$

This last question is the same as it respects the amount, as the first set in Compound Interest. Let the scholar solve it by decimals, by the help of the preceding table, and he will quickly see the superior excellency of the last method for despatch.

Questions for Examination.

1. What is compound interest?
2. What is the general rule for calculating compound interest?

* The method of finding compound interest by logarithms is the shortest of any, but is more complicated than this last method by decimals.

DOUBLE RULE OF THREE E.

3. How is compound interest by Decimals found?

DOUBLE RULE OF THREE E, OR COMPOUND PROPORTION.

The Double Rule of Three teaches how to solve such questions as require two or more statements in the single rule of three; therefore, as one statement in the double rule is compounded of two or more in the single, the double rule is also called compound proportion. In this rule there is always an odd number, ^{and} the given numbers are never less than five, and are sometimes seven, nine, &c. When there are but five numbers given to find the sixth, or answer,

Rule.—Place three of the given numbers or terms in such order that the one which gains, loses, or gives, &c., a certain number of things, shall be the first term; the number which denotes time, space, distance, rate, &c., must be the second term; and the number that signifies gain, loss, action, &c., must be the third term. Then place the other two terms under those of the same name, and if the blank, or empty space fall under the third term, the question is direct, and the answer is found by multiplying the three last terms together, and dividing their product by the product of the first and second.

But, if the blank fall under the first or second term, the proportion is inverse, and then the first, second, and fifth (or last) terms must be multiplied together for a dividend, and the third and fourth for a divisor, and the quotient will be the answer.

Proof.—If the multiplications and division in the work are right, the whole is right, if the statement is according to rule. The work is also proved by two separate statements in the single rule of three direct. **

Reason of the Rule.—The reason why the three last terms must

** Although a question in the double rule of three may be in inverse proportion, if solved by two statements in the single rule, both statements will be in direct proportion. This is a little curious.

DOUBLE RULE OF THREE.

be multiplied together for a dividend, and the two first for a divisor in the double rule of three direct, is because that if the two last terms are set in the same order to the right hand of the three first, the middle term will be of the same name as the answer, and the product of the first and second terms will be of the same name as the product of the fourth and fifth, which reduces the five terms in the double rule to three terms, which constitute the single rule. Thus, suppose the question to be, if \$100 gain \$6 in 12 months, what will \$50 gain in 2 months?

Statements.

$$\begin{array}{l} \text{1st.} \\ \$ \\ 100 : 12 : \$6 \\ 50 : \end{array}$$

$$\begin{array}{l} \text{2d.} \\ \$ \\ 100 : 12 : \$6 : 50 : \end{array}$$

Here in the 1st statement the blank falls under the 3d term, therefore the proportion is direct, and if the continued product of the three last terms be divided by the product of the 1st and 2d, the quotient or answer will be \$2. Now if the two last terms are placed on the right hand of the three first, as in the 2d statement, it is plain that the five terms are equal to two statements in the single rule of three; for as the 1st term is to the 3rd, so is the 4th (of the same name as the 1st) to \$3; and as 12 months are to \$3, so are 8 months to \$2, as before. But by multiplying the two first terms together for the 1st term, and the two last for the 3d term, the whole will form one statement in the single rule of three, which will be as \$1200 : \$6 :: 400 : \$2 gain, which is the very same thing in another form, as the operation by the first statement.

But, suppose the question to be, if \$100 gain \$6 in 12 months, in how long a time will \$50 gain \$2? Here, according to the rule for stating, the statement will be as follows, and as the blank falls under the 2d term, the proportion is inverse, and the answer 8 months.

$$\begin{array}{l} \$ \\ 100 : 12 : \$6 \\ 50 : \end{array}$$

Now the reason why the 1st, 2d, and 5th must be multiplied together for a dividend, and 3d and 4th for a divisor, when the proportion is inverse, is because that in this case when the blank falls under the 1st term, the answer will be of the same name as the 1st, and the compound statement is reduced to one in the single rule of three inverse, by saying, as the product of the 2d and last terms is to the 1st term, so is the product of the 3d and 4th term to the answer.

But, when the blank falls under the 2d term, the answer will be of the same name as the 2d term, and the statement is reduced to one statement

DOUBLE RULE OF THREE.

in the single rule, by saying, as the product of the first and last terms is to the 2d term, so is the product of the 3d and 4th terms inversely, to the answer.

Examples.

1. If 7 men can build 36 rods of wall in 3 days, how many rods can 10 men build in 14 days.

Statement.

$$\begin{array}{l} \text{M.} \quad \text{D.} \quad \text{R.} \\ 7 : 3 :: 36 \\ 10 : 14 \end{array} \left. \begin{array}{l} \text{Here, as the blank falls under the 3d} \\ \text{term, the proportion is direct.} \end{array} \right\}$$

The work.

$$\begin{array}{r} 36 = \text{the 3d term.} \\ 10 = \text{the 4th term.} \\ \hline 360 \\ 14 \quad = \text{the last term.} \\ \hline 1350 \\ 36 \\ \hline 5040 \quad (240 \text{ rods, answer.}) \\ 42 \\ \hline 84 \\ 84 \\ \hline 0 \end{array}$$

1st term = 7
2d term = 3
Divisor = 21

By two statements in the Single Rule of Three.

First.

$$\text{As } \frac{\text{M.}}{7} : \frac{\text{D.}}{36} :: \frac{\text{M.}}{10}$$

$$7 \overline{) 360} \quad 51 \frac{3}{7} \text{ rods.}$$

Second.

$$\text{As } \frac{\text{D.}}{3} : 51 \frac{3}{7} :: \frac{\text{D.}}{14}$$

$$\begin{array}{r} 14 \\ \hline 204 \\ 516 = 14 \text{ times } \frac{3}{7} \\ \hline 3 \overline{) 720} \\ 240 \text{ men, answer.} \end{array}$$

2. If \$100 principal gain \$6 interest in a year, in what time will \$200 gain \$14?

Ans. 14 months.

Statement.

$$\begin{array}{l} \$ \quad \text{M.} \quad \$ \text{ int.} \\ 100 : 12 :: 6 \\ 200 : 14 \end{array} \left. \begin{array}{l} \text{Here the blank falls under the 2d} \\ \text{term, and the proportion is inverse.} \end{array} \right\}$$

3. What Principal at 6 per cent. per annum will gain \$14 in 14 months? Ans. \$200.

Statement.

$$\text{As } \$100 : 12 \text{ mo.} :: \$6 \\ 14 :: 14 \quad \left. \begin{array}{l} \text{Here the proportion is also} \\ \text{inverse.} \end{array} \right\}$$

4. If 6 persons receive £120 as wages for 21 weeks, how much will be the wages of 14 persons for 46 weeks? Ans. £613 6s. 8d.

5. If a person travel 130 miles in 3 days, when the days are 12 hours long, in how many days of 10 hours long, can he travel 360 miles?

Ans. $9 \frac{63}{65}$ days.

6. A wall to be built 27 feet high, was raised 9 feet by 12 men, in 6 days; how many men can finish the wall in 4 days, working at the same rate?

Ans. 36 men.

DOUBLE RULE OF THREE.

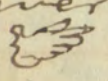
RULE II.

1. Place that number which is of the same name as the answer, for the middle term; then take any two others of the given numbers which are of the same kind, or name, and if the answer, from the nature of the question, should be greater than the second term, place the greater of the two on the right hand, and the other on the left; but if the fourth term resulting from the comparison of any two numbers with the middle term should be less than the middle term, place the greater of the two numbers on the left hand, and thus proceed till all the numbers are stated.

2. Multiply all the numbers on the right hand continually together, and the last product will be the third term.

3. Multiply all the left hand numbers together for the first term.

4. Multiply the second and third terms together, and divide the product by the first, and the quotient will be the answer.

 This rule is general for solving any question in comproportio, proportion, and is peculiarly useful when there are more than five given numbers.

The method of proof is by as many statements in the single rule of three direct, as there are first or third terms in the general arrangement. The reason is the same as in the single rule of three.

Examples.

1. If 18 persons consume £30 worth of flour, in 9 months, when the price is 16s. per quarter, how many persons will consume £54 value in 6 months; when the price is 12s. per quarter? Ans. 64 $\frac{2}{3}$.

Statement.

$$\begin{array}{l} 30\text{£.} \left\{ \begin{array}{l} 18 \\ 9\text{mo.} \end{array} \right\} 54\text{£.} \\ 12\text{s.} \left\{ \begin{array}{l} 18 \\ 6\text{mo.} \end{array} \right\} 16\text{s.} \end{array}$$

Explanation. - Here the answer required is persons, therefore the 18 persons is the middle term, and as 54£. worth will suffice for

P R A C T I C E.

more persons than 30^l., the greater number is placed on the right hand; again, the same value or quantity of flour will last more persons 6 months than it will 9 months, therefore 9 months instead of 6, is placed on the right hand, and for the same reason the greater number of shillings, 16, is placed on the right likewise.

Now if the continued product of all the 3d terms be multiplied by the 2d term, and that product be divided by the continued product of the 1st terms, the quotient will be the answer.

Remark.—The operation by this rule may generally be abridged by striking out equal numbers from both sides of the statement, or by dividing any two terms, one on each side, by any number that will divide both without a remainder, and then using the remaining numbers and quotients. Thus, in this statement 3 will divide 30 and 54; 3 will also divide 6 and 9; and 4 will divide 12 and 16; and the quotients may also be often further divided.

2. If 248 men, in 5 days of 11 hours long, dig a trench 230 rods in length, 3 feet wide, and 2 feet deep; in how many days each 9 hours long, will 24 men dig a trench 420 rods long, 5 feet wide, and three feet deep?

Ans. $288 \frac{59}{267}$.

I Questions for Examination.

1. Which is the Double Rule of Three?
2. What is the common method of stating and working questions in the double rule of three?
3. How are operations in the double rule of three proved?
4. What is the best method of stating and solving questions in the double rule of three when the given numbers are more than five?

P R A C T I C E.

Practice is a short method of solving questions which

PRACTICE.

belong to the rule of three direct, when the first term is 1, that is, when the price of one thing is given; and the price of several things is required.

CASE I.
When the price of 1 yard, lb., &c. is given to find the price of any number of yards, pounds, &c.

Rule. - 1. Suppose the price of the given quantity to be 1s., 1l., &c. for each yard, lb., &c.; then the quantity itself will be the answer at the supposed price.

2. Divide the given or real price into aliquot parts, either of the supposed price or of one another, as the nature of the question requires, and find the quotients of the several aliquot parts, and their sum will be the answer.

CASE II.
When the price of one hundred weight is given, in several denominations, as pounds, shillings and pence, or dollars and cents, &c., to find the value of a quantity of several denominations, as hundredweights, quarters, lbs., &c.

Rule. - Multiply the price of a cwt. by the number of hundredweights, and take aliquot parts for the rest, and the sum of the product and quotients will be the answer.

These two rules will solve any question in practice, and are easily remembered. - When fractions or mixed numbers are to be divided, proceed as in compound division.

Proof. By the Single Rule of Three Direct. ‡

TABLE OF ALIQUOT OR EVEN PARTS.

Parts of a Pound.	Parts of a Shilling.	Parts of a Ton.	Parts of a Cwt.	Parts of a Dollar.
$\frac{1}{10}$ d. is $\frac{1}{20}$ s.	$\frac{1}{6}$ s. is $\frac{1}{12}$ l.	$\frac{1}{10}$ cwt. is $\frac{1}{20}$ tons.	$\frac{1}{56}$ cwt. is $\frac{1}{112}$ tons.	$\frac{1}{50}$ dollar is $\frac{1}{100}$ cents.
6 d. - $\frac{1}{3}$ s.	4 s. - $\frac{1}{3}$ l.	5 cwt. - $\frac{1}{4}$ tons.	28 cwt. - $\frac{1}{4}$ tons.	$33\frac{1}{3}$ cents - $\frac{1}{3}$ dollar.
5 d. - $\frac{1}{4}$ s.	3 s. - $\frac{1}{4}$ l.	4 cwt. - $\frac{1}{5}$ tons.	16 cwt. - $\frac{1}{7}$ tons.	25 cents - $\frac{1}{4}$ dollar.
4 d. - $\frac{1}{5}$ s.	2 s. - $\frac{1}{5}$ l.	2 2 cwt. - $\frac{1}{5}$ tons.	14 cwt. - $\frac{1}{5}$ tons.	$16\frac{2}{3}$ cents - $\frac{1}{6}$ dollar.
3 d. 4 - $\frac{1}{6}$ s.	$1\frac{1}{2}$ s. - $\frac{1}{8}$ l.	2 cwt. - $\frac{1}{10}$ tons.	8 cwt. - $\frac{1}{14}$ tons.	$12\frac{1}{2}$ cents - $\frac{1}{8}$ dollar.
2 d. 6 - $\frac{1}{8}$ s.	1 s. - $\frac{1}{12}$ l.	1 cwt. - $\frac{1}{20}$ tons.	7 cwt. - $\frac{1}{16}$ tons.	$8\frac{1}{3}$ cents - $\frac{1}{12}$ dollar.
2 d. - $\frac{1}{10}$ s.				$6\frac{1}{4}$ cents - $\frac{1}{16}$ dollar.
1 d. 8 - $\frac{1}{12}$ s.				$3\frac{1}{3}$ cents - $\frac{1}{30}$ dollar.
1 d. - $\frac{1}{20}$ s.				

§ Although, as President Webber observes in his Mathematics, "the rule of practice is nearly superseded by the use of Federal Money," yet as merchants in England who trade with Americans, make out their invoices in sterling money, an acquaintance with this useful rule is necessary to all who are employed in mercantile pursuits.
+ Thus, to divide $\frac{1}{2}$ by 5, $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$; and $1\frac{1}{2} = \frac{3}{2}$ divided by 5 will be $\frac{3}{2} \times \frac{1}{5} = \frac{3}{10}$, &c., which is only multiplying the denominator of the fraction by the divisor, when the divisor is a whole number, which it always is in practice.

Examples.

Questions Solved.

1. What will 2758 yards of cloth come to, at $5\frac{3}{4}$ d. per yard?

$$\begin{array}{r} 5d. = \frac{1}{3} \quad 2753s. \text{ at } 1s. \text{ per yd.} \\ 1 = \frac{1}{4} \quad 917s. 8d. \text{ at } 4d. \text{ a yd.} \\ \frac{1}{2} = \frac{1}{2} \quad 229s. 5d. \text{ at } 1d. \text{ a yd.} \\ \frac{1}{4} = \frac{1}{2} \quad 114s. 8\frac{1}{2}d. \text{ at } \frac{1}{2}d. \text{ do.} \\ \quad \quad 57s. 4\frac{1}{2}d. \text{ at } \frac{1}{4}d. \text{ do.} \\ 2|0)131|9 \quad 1\frac{3}{4} \\ \underline{\quad} \\ \text{£ } 65 \quad 19 \quad 1\frac{3}{4} \text{ Ans.} \end{array}$$

Explanation.—1. at 1s. a yard the whole must cost as many shillings as there are yards.

2. As 4d. is $\frac{1}{3}$ of a shilling, the price at 4d. a yard must be $\frac{1}{3}$ of the value at 1s. a yard. Then 1d. a yard must be $\frac{1}{4}$ of the price at 4d.; and $\frac{1}{2}$ d. will give $\frac{1}{2}$ the price at 1d.; and $\frac{1}{4}$ d. is $\frac{1}{2}$ as much as $\frac{1}{2}$ d.

Another Method.

$$\begin{array}{r} \frac{1}{2}d. = \frac{1}{2} \quad 2753d. \text{ at } 1d. \text{ a yard.} \\ \quad \quad 5 \\ \frac{1}{4}d. = \frac{1}{2} \quad 1376s. 5d. \text{ at } 5d. \text{ a yd.} \\ \quad \quad 1376s. 6\frac{1}{2}d. \text{ at } \frac{1}{2}d. \text{ do.} \\ \quad \quad 688s. 8\frac{1}{2}d. \text{ at } \frac{1}{4}d. \text{ do.} \\ 12)15829\frac{3}{4} \text{ at } 5\frac{3}{4}d. \text{ do.} \\ 2|0)131|9s. 1\frac{3}{4}d. \\ \underline{\quad} \\ \text{£ } 65 \quad 19s. \quad 1\frac{3}{4} \text{ Ans.} \end{array}$$

Here the number of yards is called so many pence, instead of so many shillings.

Exam. 2. What will 37 cwt. 3 qrs. 7 lb. of sugar come to at $\text{£ } 14, 40$ per cwt.?

$$\begin{array}{r} 2qrs. = \frac{1}{2} \quad \text{£ } 14, 40 \text{ per cwt.} \\ \quad \quad 37 \\ \quad \quad 100 \quad 80 \\ \quad \quad 43 \quad 2 \quad 0 \\ \quad \quad 53 \quad 2, 80 \text{ price of 37 cwt.} \\ 1qr. = \frac{1}{4} \quad 7, 20 \text{ do. of } \frac{1}{2} \text{ cwt.} \\ 7lb. = \frac{1}{4} \quad 3, 60 \text{ do. of } \frac{1}{4} \text{ cwt.} \\ \quad \quad 90 \text{ do. of } 7lb. \\ \underline{\quad} \\ \text{£ } 54 \quad 4 \quad 150 \text{ Ans.} \end{array}$$

Here $\frac{1}{2}$ of $\text{£ } 14, 50$ is $\text{£ } 7, 20$; and $\frac{1}{4}$ the value of 2 qrs. is the value of 1 qr., &c., 7 lb. being $\frac{1}{4}$ of 28 lb. or 1 qr.

If the price were pounds, shillings, and pence, suppose $\text{£ } 3 \quad 5s. \quad 6d.$, the work would be as follows, and on the same principles, and by the same reasoning any other questions may be solved.

Questions for Exercise.

$$\begin{array}{r} \text{£ } \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{6} \text{ per cwt.} \\ 6 \times 6 = 36 \\ 19 \quad 13 \quad 0 \text{ price of 6 cwt.} \\ \underline{\quad} \\ 117 \quad 18 \quad 0 \text{ price of 36 do.} \\ 3 \quad 5 \quad 6 \text{ price of 1 cwt.} \\ 1 \quad 12 \quad 9 \text{ do. of 2 qrs.} \\ \quad \quad 16 \quad 4\frac{1}{2} \text{ do. of 1 qr.} \\ \underline{\quad} \\ \text{£ } 123 \quad 13s. \quad 7\frac{1}{2}d. \text{ Ans.} \end{array}$$

What is the value of

3215 lb. at 1s. $1\frac{1}{4}$ d. per lb.?

Ans. $\text{£ } 177 \quad 9s. \quad 10\frac{3}{4}d.$

9254 lb. at 1s. $2\frac{1}{2}$ d. per lb.?

Ans. $\text{£ } 559 \quad 1s. \quad 11d.$

7925 lb. at 1s. $6\frac{3}{4}$ d. per lb.?

Ans. $\text{£ } 619 \quad 2s. \quad 9\frac{3}{4}d.$

1572 gallons, at 8s.?

Ans. $\text{£ } 628 \quad 16s.$

2102 gallons, at 10s.?

Ans. $\text{£ } 1051.$

2101 cwt. at 12s. per cwt.?

Ans. $\text{£ } 1260 \quad 12s.$

3123 cwt. at 16s. per cwt.?

Ans. $\text{£ } 2498 \quad 8s.$

1621 cwt. at 18s. per cwt.?

Ans. $\text{£ } 1458 \quad 18s.$

When the price is 10s. take half the quantity, and if any thing remain, it is 10s. the rest are pounds.

5763 gallons, at 10s. per gallon?

$$2)5763$$

$\text{£ } 2881 \quad 10s. \text{ Ans.}$

When the price is an even number of shillings, multiply the given quantity by half price, and double the right hand figure of the product for shillings; the rest will be pounds.

Reason. Because the right hand figure is but $\frac{1}{2}$ what it would be multiplied by the whole price, and the rest are what they would be if multiplied by the whole price and divided by 2, or 20 with the cipher cut off.

7563 cwt. at 22s. per cwt.?

Ans. $\text{£ } 8319 \quad 6s.$

367 tons, at 28s. per ton?

Ans. $\text{£ } 513 \quad 16s.$

An Example in which a fraction is divided,
and the work is proved.

What will $17\frac{1}{2}$ lb. of sugar come to, at 3s. 7d. a lb. ? Ans. £ 3 12 12 $\frac{1}{8}$ s.

Operation.

$$\begin{array}{r} \text{d.} \\ 6 = \frac{1}{2} \end{array} \left| \begin{array}{r} 17\frac{1}{2} \\ 3 \end{array} \right.$$

$$\begin{array}{r} \text{d.} \\ 1 = \frac{1}{6} \end{array} \left| \begin{array}{r} 523\frac{1}{2} \text{ s. at } 3\text{s.} \\ 872\frac{3}{4} \text{ s. at } 6\text{d.} \\ 145\frac{19}{48} \text{ at } 1\text{d.} \end{array} \right.$$

$$210 \overline{) 6252\frac{1}{2} \text{ s. at } 3\text{s. } 7\text{d.}}$$

£ 3 12 12 $\frac{1}{8}$ s.

Proof by the Rule of Three.

As $\frac{1}{2}$ lb. : $\frac{3}{12}$ s. :: $17\frac{1}{2}$ lb. : $\frac{3}{4}$ s.

$$\begin{array}{r} 6959 \\ 33 \\ \hline 20937 \\ 27916 \\ \hline 4) 300097 \\ 12) 75024\frac{1}{2} \text{ d.} \\ 210) 6252\frac{1}{2} \text{ s.} \\ \hline \text{£ } 3 \text{ } 12 \text{ } 12\frac{1}{8} \text{ s.} \end{array}$$

Explanation. In the operation by practice, the quantity, $17\frac{1}{2}$ lb. is multiplied by the number of shillings. Next, 6 of the 7d. is taken, which is $\frac{1}{2}$ of a shilling, and in dividing by 2, $\frac{3}{4}$ remain, and $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$, either by reason, or by multiplying the numerators of $\frac{1}{2}$ and $\frac{3}{4}$ together, and also their denominators, 2 and 4. Then the 1d. is taken, which is $\frac{1}{6}$ of 6d., therefore the value at 6d. is divided by 6, and $2\frac{3}{4}$ remain, which is $\frac{19}{8}$, and $\frac{1}{6}$ of $\frac{19}{8}$ ($\frac{1}{6} \times \frac{19}{8}$) is $\frac{19}{48}$. Now to add the three fractions, $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{19}{48}$ together, they must first all be reduced to decimals, which is easy, or reduced to one denomination in vulgar fractions, which for the sake of explaining that method, we will do, thus; first, $\frac{1}{2}$ is equal to $\frac{2}{4}$; then $\frac{2}{4}$ and $\frac{3}{8}$ is $\frac{5}{8}$; now find some number by which if 8 is multiplied, the product will be 48, and 6 will be that number, therefore multiply both the numerator and denominator of $\frac{5}{8}$ by 6, and you will get $\frac{30}{48}$, which added to $\frac{19}{48}$ is $\frac{49}{48}$, or 1 and $\frac{1}{48}$, so $\frac{1}{48}$ is set down, and one is carried to 5, &c. In the same manner all other questions of the same nature are solved.

The method by the rule of three gives the same answer, and therefore proves all the work to be right. — In dividing by 12, $\frac{1}{2}$ remained, and $\frac{1}{2}$ of a penny is $\frac{1}{48}$ of a shilling, as before.

¶ What is practice?

What are the general rules for working practice?

How is practice proved?

DISCOUNT.

Discount is the allowance made for the payment of money, bills &c. before such payments become due.†
 Note. As the interest is generally paid, instead of the discount, upon all bills said to be discounted, and as few people understand the difference between discount and interest, I shall endeavour to explain the nature and reason of such difference, before I proceed to the rule.

Now, the discount of any sum is so much deducted from it as will leave a sum which if put to interest for the given rate and time will amount to the given sum.

The sum left after deducting the discount, is called the present worth. The interest of any sum is more than the discount. Thus, the interest of \$106 for a year at 6 per cent. per annum, is \$6.36; but the discount is only \$6; for if a debt of \$106 be due a year hence, without interest, it is plain that \$100 at 6 per cent. per annum will amount to \$106 in a year, therefore \$100 paid down, will pay a debt of \$106 due a year hence.

R U L E.

1. As the amount of 100l. for the given rate and time, is to 100l. : so is the given sum to the present worth; which subtracted from the given sum leaves the discount, OR,
2. As the amount of 100l. for the given rate and time, is to the interest of 100l. for that time : so is the given sum to the discount.

E X A M P L E S.

1. What is the discount of 100l. payable a year hence at 5 per cent.?

Ans. 4l. 15s. 3d.

£.	£.			
100	100	Principal		
5	5	Interest		
5.00	As 105	: 100	: 100	
		100		
£.		£. s. d.		
100		105)10000(95:4:9	present worth	
		945		
		95:4:9 +		
		550		
		4:15:3 Discount	525	
			25	
			30	
		105)500(4		
		420		
		80		
		120		
		105)960(9		
		945		
		15		
		4		
		60		
		\$ 105	= 12	= 4
			21	7
				qr.

2. What is the discount of \$75 due a year hence, at 6 per cent. per annum?

Ans. \$70.75.4+.

* \$100, in Federal Money

† Discount derives its name from the Latin *dis*, Deduction, &c., and the French *compte*, an account.

DISCOUNT

3. What is the present worth of £357 10s., due 9 months hence, at 5 per cent. per annum? Ans. £344 11s. 7d.

4. What is the discount of \$500 due 10 months hence, at 6 per cent. per annum? Ans. \$23.81.

Remark. When discount is made without regard to time, it is found by multiplying the sum or price by the rate per cent., and dividing the product by 100, or cutting off the two right hand figures, the same as in interest.

Examples.

1. Bought a quantity of books, which at the retail prices cost \$65, but the bookseller agreed to make 20 per cent. discount, if he could have the money paid when the books were delivered. How much was the discount, and what did the books cost after making the discount?

Ans. Discount \$13, and books cost \$52.

2. What must be given for a due bill for \$32, 75, at 7 per cent. discount?

Ans. \$30.45, 2½.

3. A, has a note against B, for \$50, on demand with interest, but doubting whether B will be able to pay the whole, A offers C, B's note, at 15 per cent. discount on the face of it. How much must C give for the note?

Ans. \$42.50.

I Questions for Examination

1. What is Discount?

2. What is the Present Worth of any sum, discounting at a certain rate per cent. per annum?

3. How is the present worth, and discount, of any sum found?

BANK DISCOUNT

BANK DISCOUNT

Bank Discount is a deduction made by bankers, out of a note payable to them in a certain number of days.

Nature of Bank Discount.—If a note for any sum be discounted at a bank, for 30 days, the interest of that sum for 33 days, is deducted for the discount, because custom has allowed the borrower three days of grace—that is, he is not obliged to pay the note till three days after the time when it becomes due, or is out as it is called.

If a note be discounted at a bank for 60 days, the interest of it for 63 days is deducted as discount, for the same reason.

If the payment of a note at the proper time be not convenient, a new note must be presented on the day of discount immediately preceding the expiration of the time of payment, paying the same discount or interest for the time as before. When discount is made for 30 days, a discount month is 28 days: that is, though the payment may be withheld 33 days, including the days of grace, yet if it is not convenient to pay the note at the time, it must be renewed, or supposed to be so, in 28 days from the date, so that the giver of the note loses the interest of five days by renewing; that is, he pays the interest for five days twice, instead of once. On the same principle, when a note is discounted for 60 days, and payment is not made at the time, it must be renewed in 56 days, which is a discount month when discount is made for 60 days, so that in this last case he who gives the note loses the interest of seven days by renewing. When a note is renewed, the interest for the time must be paid when the note is given.

When discount is made for 30 days, that is, when a note is renewable every 28 days, and the number of days before payment is ^{made} ~~required~~ exceeds a month, the number of months is found by dividing the days by 28. When discount is renewed for 60 days, the number of months is found by dividing the number of days by 56. — These are the principles on which banking business is generally conducted.

BANK DISCOUNT.

When a person offers a note to be discounted at a bank, two endorsers are generally required, to the first of whom it is said to be payable. Thus, for example, A, having occasion for a sum of money from the bank, procures B and C as endorsers to his note, which he offers for discount in the form following, varying the sum, and date, and number of days, according to circumstances.

\$100.

Boston, Jan. 1, 1831.

For value received, I promise to pay B, or order, at the Boston Bank, one hundred dollars, ^{on demand,} with interest after sixty days. A.

When bank shares, state notes, &c. are lodged in a bank as security for money, a note is offered in this form.

\$100.

Boston, Jan. 1, 1831.

For value received, I promise to pay the President, Directors, and Company of the Boston Bank, or their order, at said Bank, on demand, one hundred dollars, with interest after sixty days. A.

Calculation of Bank Discount.

To find the Bank Discount of any sum for 30 or 60 days.

Rule. Multiply the sum by one sixth of the days, setting down the right hand figure of $\frac{1}{6}$ of the days as a decimal, and cut off three decimals from the right hand of the product; the figures not cut off will be dollars: the next two cents, and the third mills.

Examples.

1. How much is the interest of \$120, discounted for 30 days? Ans. 66 cents.

\$120

$5\frac{1}{2} = \frac{1}{6}$ of 33* days.

600

60

660 Ans.

2. What is the interest or discount of \$120, discounted for 60 days?

\$120

$10\frac{1}{2} = \frac{1}{6}$ of 63* days.

1200

60

1260 Ans.

3. What is the discount of \$161, discounted for 30 days?

Ans. 88½ cents.

4. What is the discount of \$161, discounted for 60 days?

Ans. \$1.69.

* The 3 days of grace are added to 30, and 60.

COMMON AND BROKERAGE.

When the time is several months and days.

Rule. Find the discount or interest for one month, which multiply by the number of months, and take parts for the odd days.

Examples.

1. How much is the discount of \$225 payable in 250 days, discounting for 30 days?

$$\begin{array}{r} 28 \overline{) 250} \quad (8 \text{ months.} \\ \underline{224} \\ 26 \text{ days.} \end{array}$$

$$\begin{array}{r} \$225 \\ \underline{.5\frac{1}{2}} = \frac{1}{6} \text{ of } 33 \text{ days.} \\ 1125 \\ \underline{112} \\ \$1,23,7 \text{ for 1 month.} \end{array}$$

$$14 \text{ d.} = \frac{1}{2} \text{ mo.}$$

$$7 \text{ d.} = \frac{1}{2} \text{ of } 14$$

$$4 \text{ d.} = \frac{1}{2} \text{ of } 8 \text{ mo.}$$

$$1 \text{ d.} = \frac{1}{5} \text{ of } 5 \text{ d.}$$

$$9,89,6 \text{ for 8 months.}$$

$$61,8 \text{ for 14 days.}$$

$$30,9 \text{ for 7 days.}$$

$$17,6 \text{ for 4 days.}$$

$$5,4 \text{ for 1 day.}$$

$$\$11,04,3 \text{ Ans.}$$

2. How much is the discount of the preceding sum, discounting for 60 days? Ans. \$10,54,3.

☞ The two last answers show the difference between discounting for 30 and 60 days.

Observe, in this last question, as 56 days are a discount month, the 250 days must be divided by 56 to get the number of months: then multiply the discount for 1 month by the number of months, and take parts for the odd days. The sixth part of 63 days will always be a constant number, viz., $1,0\frac{1}{2}$; and that for 33 days is always $.5\frac{1}{2}$, for 6 per cent. When the per cent. is not 6, take a proportional part.

I Questions for Examination.

1. What is Bank Discount?
2. How is Bank Discount found for any sum for 30 and 60 days, at 6 per cent.?
3. Which discount is the most, for 30 or 60 days, when the time is several months? Ans. For 30 days.

COMMON AND BROKERAGE.

Commission and Brokerage are premiums, at a rate per cent. agreed on, allowed to a person called an Agent, a Correspondent, Factor, or Broker, for services in assisting Merchants and others in buying and selling goods.

BUYING AND SELLING STOCKS.

Rule.—Multiply the given sum by the rate per cent., and divide by 100, or cut off the two right hand figures, as in simple interest.

Examples.

1. What does the commission of £ 287 10s. come to, at $3\frac{1}{2}$ per cent.? Ans. £ 10 3s. 3d.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \\
 287 \quad 10 \\
 \times 3\frac{1}{2} = \text{the rate per cent.} \\
 \hline
 862 \quad 10 \\
 153 \quad 15 = \frac{1}{2} \text{ of } £ 287 \text{ 10s.} \\
 \hline
 \text{£ } 10 \mid 0 \quad 6 \quad 5 \\
 \quad \quad 20 \\
 \quad \quad \hline
 \quad \quad 3.1 \mid 2 \quad 5 \\
 \quad \quad \quad 12 \\
 \quad \quad \hline
 \quad \quad \text{cl. } 3 \mid 0 \quad 0
 \end{array}$$

2. My agent informs me that he has collected debts for me to the amount of \$ 575, for which I am to pay him 5 per cent. for collecting. What must I pay him at that rate? Ans. \$ 28, 75.

3. A correspondent has in his hands \$ 900, which he is to lay out in goods, reserving to himself 3 per cent. on the purchase. How much will his commission come to, and how much will he lay out in goods?

Ans. { His commission comes to \$ 26, 21, 4,
and he will lay out \$ 873, 78, 6.
So much on the purchase means so much for every \$ 100 laid out: Thus, if \$ 105 is the whole sum, and \$ 5 are to be reserved on the purchase, there will be \$ 100 to lay out. — Questions of this kind are solved by the rules of common discount, and not by the rule of simple interest, which Mr. Walsh has given, and the author of the "Columbian Arithmetician" has fallen into the same error.

Questions for Examination!

1. What is Commission and Brokerage?
2. How is the commission on any sum found?
3. When the commission is so much per cent. on the purchase, how is it found?

BUYING AND SELLING STOCKS.

BUYING AND SELLING STOCKS.

Stock, considered as a money capital, is a fund established by government, or a corporate body, the value of which varies according to the demand for money, and other circumstances. The practice of purchasing and selling sums of money in these funds, is called buying and selling stocks.

per cent.	RULE. Multiply the sum to be purchased by the excess above 100; divide the product by 100, the produce of which added to the given sum, is the purchase required.
	If under par (that is under 100) multiply by the rate per cent. that product, divided by 100, gives the purchase thereof.
5. What is the purchase of 575l. 10s. bank stock, at 131 $\frac{1}{4}$ per cent.	6. What is the purchase of 254l. 17s. bank annuities, at 97 $\frac{1}{4}$ per cent.?
$6 \times 5 + 1 = 31$	$12 \times 8 + 1 = 97$
575 : 10	254 : 17
6	12
3453 : 0	3058 : 4
5	8
17265 : 0 : 0 = 30	24465 : 12 : 0 = 96
$\frac{1}{2}$) 575 : 10 : 0 = 1	$\frac{1}{4}$) 254 : 17 : 0 = 1
$\frac{1}{4}$ = $\frac{1}{2}$) 287 : 15 : 0 = $\frac{1}{4}$	63 : 14 : 3 = $\frac{1}{4}$
143 : 17 : 0 = $\frac{1}{4}$	24784 : 3 : 3 = 97 $\frac{1}{4}$
182172 : 2 : 6 = 31 $\frac{1}{4}$	20
20	1683
14143	12
12	9199
510	4
575 : 10 : 0	3196
182 : 14 : 5	Ans. £247 : 16 : 9 $\frac{1}{4}$
Ans. £758 : 4 : 5	

3. What is the purchase or value of \$1500 bank stock of the United States, at 127 $\frac{1}{2}$ per cent.?

Here the excess above 100 per cent. is 27 $\frac{1}{2}$.

\$1500

27 $\frac{1}{2}$

10500

30

750 = $\frac{1}{2}$ of \$1500.

412,50 = value at 27 $\frac{1}{2}$ per cent.

1500,00 = do. at 100 per cent.

\$1912,50 Ans.

4. What is the value of \$675 national bank stock, at 134 per cent. ? Ans. \$904,50.

5. What is the purchase or value of \$689 bank stock, at 87 $\frac{1}{2}$ per cent. ? Ans. 602,87,5.

Remark. The last part of the rule is general for any rate per cent., and is the most convenient in Federal Money. Thus, in the third example, if \$1500 had been multiplied by 127 $\frac{1}{2}$, or 127,5, and the product divided by 100, the answer would be the same.

AMERICAN DUTIES.

I Questions for Examination

1. What is buying and selling stocks, as the practice relates to money capitals.
2. What is the rule for finding the value of any sum of bank or funded stock.

AMERICAN DUTIES.

Duty, as it relates to commerce, is a tax or custom paid on goods imported into a country.

Rule.—Multiply the quantity by the duty on lb. &c. of the quantity, and the product will be the answer.

Examples.

1. What is the duty on 2318 of Coffee at $2\frac{1}{2}$ cents p. lb.
 cwt. gr. lb.
 4 Ans. 8 dolls. 15 cents.

—

11

28

—

106

22

—

326 lbs.

$2\frac{1}{2}$ duty

—

652

163

815 cents = 8 dolls. 15 cents.

2. Calculate the duty on 827 dozen of porter at 20 cents per dozen.
 Ans. 165 dolls. 40 cents.

cwt. gr. lb.

3. What is the duty on 324 of Cheese at 4 cents per pound?
 Ans. 15 dolls. 84 cents.

cwt. gr. lb.

4. What is the duty on 5317 of Soap at 2 cents per pound?
 Ans. 13 dolls. 22 cents.

5. What is the duty on 572 pair of silk shoes at 10 cents per pair?
 Ans. 57 dolls. 20 cents.

6. What is the duty on 256 pair of boots at 50 cents per pair?
 Ans. 128 dolls.

7. What is the duty on 8276 pair of leather shoes at 7 cents per pair?
 Ans. 579 dolls. 32 cents.

8. What duty must be paid on 39 hhds. of distilled spirits at 8 cents per gall.?
 Ans. 196 dolls. 56 cents.

9. What duty must be paid on 125 hhds of Madeira at 18 cents per gallon?
 Ans. 1417 dolls. 50 cents.

Method of estimating ad valorem* rates of duty.

Rule.—Add 20 per cent. to the actual cost of the goods, &c. when imported from or beyond the Cape of Good Hope, and 10 per cent. when imported from any other place. [Insurance, commission, &c. do not belong to the actual cost.]

After adding the 20 or 10 per cent. to the actual cost, find the duty for the rate per cent. by the rule of simple interest.

* Ad valorem signifies the real value.

I N S U R A N C E.

It is most convenient to reduce the actual cost to Federal Money in the first place, and then to add the per cent. and find the duty.

Examples.

1. What will be the duty on an invoice of goods, which cost £393 7s. 6d. sterling, at 15 per cent. ad valorem when imported in an American vessel, or at 1½ per cent. more when imported in a foreign vessel from England?

Remark. £1 sterling is equal to \$4.44, 4.

£393 7s. 6d. = \$1748.15, 8 the actual cost.

Add 10 per cent. = 174.81, 5

10 = $\frac{1}{10}$ 1922.97, 3

5 = $\frac{1}{2}$ 192.29, 5 = duty at 10 per cent.

15 = 96.14, 8 = do. at 5 do.

At 15 per cent. \$288.44, 5 Ans. in Am. vessel.

At 16½ per ct. \$317.28, 9 Ans. in For. ship.

Explanation. At 10 per cent. the value is the tenth part of the actual cost. Instead of dividing by 10 and 5, to get the duty, \$1922.97, 3 might have been multiplied by 15, the whole rate, and the right hand figures of the product cut off as in interest.

2. What is the duty on goods which cost in India, \$1390.25, imported in an American ship, at 12½ per cent. ad valorem? Ans. \$208.53, 7½.

I Questions for Examination.

1. What is duty, as it relates to commerce?
2. What is the rule for calculating the duty on goods, without regard to the value of them?
3. What is the mode of computing ad valorem duties, that is, when the duty is so much per cent.? the actual cost of the goods being considered?

I N S U R A N C E.

Insurance or assurance is an exemption from hazard, or security given by contract, to make good the loss of ships, houses, goods, &c., which may happen by fire, storms, or other causes.

The sum insured is called the principal.

The sum paid for insuring is called the premium.

The average loss is variable, and if the owner of the property insured suffer any loss or damage not exceeding the average loss, he bears it himself, and the underwriter or insurer is free.

The Policy is the instrument or obligation by which the insurers oblige themselves to make good the property insured.

I N T R A N S I T.

Case I.

To find the premium at any rate per cent. for insuring any sum.

Rule.—The same as for simple interest for a year.

Examples.

1. What is the premium for insuring \$564 at 12 per cent.?

$$\begin{array}{r} \$564 \\ 12 \end{array}$$

\$67.68 Ans.

2. What is the premium for (or of) insuring £600.10s. at 9 per cent.?

Ans. £54 0s. 9 $\frac{6}{10}$ d.

3. If a house worth \$2500 be insured at 4 per cent. on the value, what is the premium for insuring? Ans. \$100.

Case II.

When it is required to find the sum for which a policy should be taken out to cover a given sum.

Rule.—Subtract the premium per cent. from £100 or \$100: then say by the Rule of Three,

As the remainder is to £100 or \$100,

So is the sum adventured to the policy.

Or, which is the same thing, annex two ciphers to the sum adventured, or to be covered, and divide by the remainder, and the quotient will be the answer.

A sum is said to be covered, when the value of the goods insured, and the premium and other charges are insured. Thus, in order to recover \$92 a policy must be taken for \$100, if the rate of insurance be 8 per cent.

Examples.

1. What must a policy be taken out for to cover or secure \$900; the premium being 7 per cent.? Ans. \$967.74.

2. A merchant sent a vessel and cargo to sea, worth \$4675; what sum must a policy be taken out for, to cover or secure his

property, at 20 per cent. premium?

Ans. \$5843, 75.

II Questions for Examination.

1. What is insurance?
2. What is a policy?
3. How do you find the whole premium for insuring a sum, when the rate per cent. is given?
4. How do you find how much a policy must be taken for to cover or secure a given sum?

TARE AND TRET.*

When coarse goods, such as Groceries &c. are purchased in large quantities; the casks, boxes, &c. must generally be weighed with the commodities; but as it would be unjust to pay for them at the same rate as for the goods, certain allowances are made to the buyer.

These allowances are called TARE, TRET, CLOFF, &c.

TARE, is for the weight of the box, cask, wrapper, &c. and is either at a certain rate per box, cask, &c. or, at a certain rate per cent; or, at so much in the whole.

TRET, is an allowance of 4lb. in every 104 lbs. for waste, dust, &c.

CLOFF, is an allowance of 2lb. to the citizens of LONDON, on every draught above 3 cwt. on some sort of goods; as, beaver, galls, madder, argol, &c.

GROSS WEIGHT, is the whole weight, including the box, bag, &c. before any allowance is made.

SUTTLE, is the weight, after one or two allowances have been deducted.

NEAT WEIGHT, is the pure weight, after all allowances are deducted.†

GENERAL RULE.

1. Find the TARE, which subtract from the GROSS, and call the remainder SUTTLE.

2. Find the TRET, if any, which subtract from
T

* Tare takes its name from the Italian word tare, to subtract.

Tret (formerly and now often spelled trett) comes from the Latin tritrus or atritus, worn or wasted.

Cloff is so called because the box, bag, wrapper, &c. which contained the merchandise, were sometimes so called.

† Neat, when applied to weight, is sometimes spelled nett, from the Italian netto. Sometimes, also, the French word net is written for neat.

AND

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the SUTTLE; the remainder, if there is any further allowance, will still be called SUTTLE.

3. Find the CLOFF, if any, and subtract from the last SUTTLE; and so on, till every allowance is deducted.

4. The last remainder will be the NEAT WEIGHT; which may be valued by the rules of PRACTICE or by the RULE of THREE.

RULES for finding the TARE, ~~TRET~~, CLOFF, &c.

~~Rule 1.~~ When the TARE is at so much in the whole, subtract the TARE from the GROSS.

~~Rule 2.~~ When the TARE is at so much per cask, bag, hhd. &c. multiply the number of casks, bags, hhd. &c. by the TARE; which subtract from the GROSS.

~~NOTE.~~ To reduce pounds into gallons, multiply by 2 and divide by 15.

aliquot ~~Rule 3.~~ When the TARE is at a certain rate per cwt. take ~~the~~ parts for the tare, out of 112 lbs. as in practice; and subtract the tare from the GROSS.

~~Rule 4.~~ When the question is an invoice; add all the GROSS weights into one sum, and all the TARES into another;---subtract the total TARE from the total GROSS.

NOTE 1. The remainders in all the above cases are either the SUTTLE or the NEAT WEIGHT.

NOTE also, 7 lbs. is $\frac{1}{16}$; 8 lbs. is $\frac{1}{14}$; 14 lbs. is $\frac{1}{8}$; 16 lbs. is $\frac{1}{4}$ of 1 cwt.

RULE for finding the TRET.

~~Rule 5.~~ After having subtracted the TARE, divide the remainder, which is called SUTTLE, by 26 for the TRET, which subtract from the SUTTLE, the remainder is NEAT; unless there is some other allowance; if so, it is still called SUTTLE.---This is the TRET allowed in London.

NOTE. The reason of dividing by 26 is, because 4lb. is the $\frac{1}{26}$ of 104 lb. but if the TRET is at any other rate, other parts must be taken, according to the rate proposed.

AND

TARE & TRET

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In Cork they allow 11 lb. out of every cwt. of the Gross wt. of all goods, except beef, butter, hides, and wool, on which they make the following allowances:

BUTTER 2 lb. on every large cask; BEEF 8 lb. per carcass; RAW HIDES 4 lb. each; WOOL 8 lbs. for every 20 stone which is $\frac{1}{5}$; (16 lb. make a stone of wool in Ireland) but in Dublin and parts adjacent they allow 8 lb. for every 3 cwt. or 21 stone; which is $\frac{1}{2}$ after the Tare is deducted.

RULE for finding the CLOFF.

~~RULE 6.~~ Multiply the cwt. SUTTLE by 2, neglecting the odd qrs. & lbs. and divide the product by 3; the quotient will be the CLOFF in lbs. which subtract from the SUTTLE; the remainder will be the neat weight.

OR,

~~RULE 7.~~ Reduce the Suttle weight into lbs. and divide by 168 (because 2 lbs. is the $\frac{1}{84}$ of 3 cwt.) the quotient is the Cloff, which subtract from the SUTTLE.

most exact

NOTE, The latter is the ~~truest~~ method; but the former is the easiest; and true enough for all common purposes.

EXAMPLES.

1. In 7 frails of raisins, each weighing 5 cwt. 2 qrs. 5 lb. gross; tare at 23 lb. per frail how much neat weight?

Ans. 37 cwt. 1 qr. 14 lbs.

OPERATION.

or thus:

23	5 : 2 : 5	5 : 2 : 5
7	7	23
4)		
28)161(5	38 : 3 : 7 = gross.	5 : 1 : 10
140 1 : 1	1 : 1 : 21 = tare.	7
21	37 : 1 : 14 neat.	37 : 1 : 14
21	37 : 1 : 14	37 : 1 : 14

2. In 241 barrels of figs, each 3 qrs. 19 lb. gross; tare 10 lb. per barrel--how many pounds neat?

Ans. 22413.

3. What is the neat weight of 25 hhds. of tobacco,

AND
TARE & TRET

weighing gross 163 cwt. 2 qrs. 15 lb.; tare 100 lb. per hoghead? *Ans.* 141 cwt. 1 qr. 7 lb.

4. In 16 bags of pepper, each 85 lb. 4 oz. gross; tare, per bag, 3 lb. 5 oz.--how many pounds neat? *Ans.* 1311.

5. What is the neat weight of 5 hhds. of tobacco, weighing gross 75 cwt. 1 qr. 14 lb.; tare in the whole 752 lb.? *Ans.* 68 cwt. 2 qrs. 18 lb.

6. In 75 barrels of figs, each 2 qrs. 27 lb. gross; tare in the whole 597 lb.--how much neat weight? *Ans.* 50 cwt. 1 qr.

7. What is the neat weight of 18 butts of currants, each 8 cwt. 2 qrs. 5 lb.; tare at 14 lb. per cwt?

$$\begin{array}{r} 8 : 2 : 5 \\ 9 \times 2 = 18 \end{array}$$

$$\begin{array}{r} 76 : 3 : 17 \\ 2 \end{array}$$

See Rule 3.

$$\begin{array}{r} 14 = \frac{1}{8} \text{) } 153 : 3 : 6 \\ 19 : 0 : 25\frac{1}{4} \end{array}$$

$$134 : 2 : 8\frac{1}{4}$$

8. In 25 barrels of figs, each 2 cwt. 1 qr. gross; tare per cent. 16 lb.--how much neat weight? *Ans.* 48 cwt. 24 lb.

9. What is the neat weight of 9 hhds. of nutmegs, each weighing gross 8 cwt. 3 qrs. 14 lb.; tare 16 lb. per cent? *Ans.* 68 cwt. 1 qr. 24 lb.

10. What is the neat weight of 12 casks of argol, gross 84 cwt. 2 qrs. 14 lb.; tare per cent. 14 lb.? *Ans.* 74 cwt. 54 lb.

11. What is the neat weight and value of 4 barrels of indigo; numbers, weights and allowances as follow; at 3s. 4d. per lb. *Ans.* 169 lbs. neat: value 281l. 16s. 8d.

AND
TARE & TRET.

	Cwt.	qr.	lb.	lb.
No 1	Gross	4	: 1 : 10	Tare 36
2	—	3	: 3 : 2	— 29
3	—	4	: 0 : 19	— 32
4	—	4	: 0 : 0	— 35
				Total —

Total Gross 16 : 1 : 3 Tare 132 lbs.

4

65

28

See Rule 4.

523

130

From 1823 lbs. Gross
Subtract 132 lbs. Tare

3s. 4d. $\frac{1}{8}$ 1691 lbs. neat

L281 : 16 : 8 Value

NOTE, In this question the price of a lb. being given, the value is easiest found by reducing the Gross into lbs.

12. What is the value of 2 hhds. of sugar, weight and allowance as under at 35s. per cwt. ?

Ans. 38l. 18s. 9d

	Cwt.	qr.	lb.	lb.
No 1	Gross	11	: 1 : 17	Tare 112
2	—	12	: 2 : 0	— 73

NOTE, As the price of a cwt. is given in this question, the value will be soonest found by bringing the Tare into Cwts.

13. In one butt of currants, weighing 12 cwt. 2 qrs.

A. N. D.

TARE & TRETT.

24 lb. gross; tare 14 lb. per cent. trett 4 lb. per 104 lb.
—how many pounds neat?

12 : 2 : 24

4

50

28

See Rule 5.

14 = $\frac{1}{8}$) 1424 gross.

178 tare,

26) 1246 futtle.

47 trett.

1199 neat.

14. In 7 cwt. 3 qrs. 27 lb. gross; tare 36 lb. trett
4 lb. per 104 lb.—how many lb. neat? *Ans.* 826 lb.

15. In 152 cwt. 1 qr. 3 lb. gross; tare 10 lb. per
cent. trett 4 lb. per 104 lb.—how much neat weight?
Ans. 133 cwt. 1 qr. 11 lb.

16 In 15 chests of sugar, weighing 117 cwt. 21 lb.
gross; tare 173 lb. trett 4 lb. per 104 lb.—how many
cwt. neat? *Ans.* 111 cwt. 22 lb.

17. What cost 2 hhds of tallow; gross wt, tare and
trett as under; at 30s. per cwt.?

cwt. qrs. lbs.

Cwt. qrs. lbs.

No. 1 Gross 10 : 1 : 11 Tare 0 : 3 : 20 } Tret 1 lb.

2 — 11 : 0 : 17 — 0 : 3 : 14 } p. cwt.

Ans. 29l. 5s. 7d. the value: and the trett 19 lb. 11 oz.

18. Sold 4 hhds. of sugar at 1l. 15s. 4d. per cwt.
weight and allowance as under; what do they
amount to?

Cwt. qrs. lbs.

Cwt. qrs. lbs.

No. 1 Gross 7 : 3 : 16 Tare 0 : 3 : 20 } Tret 1 lb.

2 — 8 : 0 : 10 — 0 : 3 : 24 } Tret 1 lb.

3 — 7 : 2 : 20 — 0 : 3 : 27 } per cwt.

4 — 8 : 2 : 18 — 0 : 3 : 25 }

Ans. 49l. 16s. 9d. the trett 1 qr. 0 lb. 7 oz.

F E L L O W S H I P

19. Bought 3 packs of wool, No. weight, and allowance as under; what is their value at 10s. 6d. per stone? (16 lbs. to the stone.)

		Cwt. qr. lbs.	
No. 1	Gross	3 : 1 : 12	} Tare 30lb. } Tre 8lb. for per pack. } 20 Stone.
2	—	3 : 3 : 7	
3	—	3 : 2 : 15	
Ans. 35l. 16s. 7d. $\frac{1}{2}$			

NOTE, In this question the tret is found by dividing by 40; see the note annexed to the Rule for finding the tret.

20. What is the neat weight of 3 hhds. of tobacco, weighing 15 cwt. 3 qrs. 20 lb. gross; tare 7 lb. per cent. tret 4 lb. per 104 lb. cloff 2 lb. for 3 cwt.?

Ans. 14 cwt. 1 qr. 2 lb. 10 oz.

		Cwt. qrs. lb. oz.	
7	—	15 : 3 : 20 : 0	gross.
		3 : 27 : 8	tare.

See Rule 6. $\frac{1}{20}$ 14 : 3 : 20 : 8 futtle.
2 : 8 : 5 tret.

14 : 1 : 12 : 2 futtle.
9 : 8 cloff.

Ans. 14 : 1 : 2 : 10 neat.

21. In 7 hhds. of tobacco, each weighing gross 5 cwt. 2 qrs. 7 lb.; tare 8 lb. per cent. tret 4 lb. per 104 lb. cloff 2 lb. per 3 cwt.—how much neat weight?

Ans. 34 cwt. 2 qrs. 8 lb.

F E L L O W S H I P

Fellowship is a rule by which merchants and others trading in partnership so adjust their accounts that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the capital employed in the joint stock, and the time in which he has traded in partnership.

Fellowship is either single or double. Single Fellowship is when stocks are employed for an equal term of time; that is, when all the partners begin and end trade together.— This is called Fellowship without time, because no regard is paid to the time. Double Fellowship is when the stocks or capitals are employed for unequal times; that is, when some of the partners trade longer than others.— This is called Fellowship with time.

S I N G L E F E L L O W S H I P

Rule.— As the whole stock is to the whole gain or loss, so is each one's share of the stock to his share of the gain or loss.*

F E L L O W S H I P

all the shares of the gain or loss together, and if the work is right, the sum will be equal to the whole. — But though this is the method given by Mr. Pike, and all others whose arithmetics have been mostly used, it is fallible for this reason: Supposing that in adding all the stocks together for the first term in working the question, A's stock by mistake, should be marked B's, or B's marked C's, &c., or if one of the stocks should be set down wrong, as \$50 for \$500, &c., in proving the work by adding all the gains or losses together, the sum of them all will be equal to the whole sum, the same as if the right numbers were used in working the question at first.

The surest method of proof is this.

As the whole gain or loss is to the whole stock, so is each person's share of the gain or loss to his share of the stock.

Reason of the Rule. — As Fellowship is performed by the Rule of Three Direct, the reason is the same as for that rule.

Examples.

1. Three merchants traded in company; A, put in \$500; B, \$700; and C, \$900, and they gained in the whole, \$800. — How much is each man's share of the gain?

Ans. { A's share \$190, 47 $\frac{13}{21}$.
B's — — — 266, 66 $\frac{14}{21}$.
C's — — — 342, 85 $\frac{15}{21}$.
Proof \$800, 00.

	<u>Stocks.</u>	
A's	\$500	
B's	700	
C's	900	
	<u>2100</u>	
		\$
	2100	: 800 :: 500
		500
	2100	) 400000 (190, 47 $\frac{13}{21}$
		21
	190	
	189	
	100	
	84	
	160	
	147	
	13	

2d method of proof.

As \$800 : \$2100 :: \$190, 47 $\frac{13}{21}$: \$500.
(A's stock.)

Explanation. — The whole stock which all put in is \$2100, and the whole gain is \$800. Then, as \$2100 is to \$800, so is \$500 A's stock, to \$190, 47 $\frac{13}{21}$, A's gain. In the

F E L L O W S H I P

same manner B's and C's gains were found, by making \$700 the third term in finding B's share of the gain, and \$900, in finding C's share: the two first terms being the same for all.

In proving the work by the common method, the fractions make 2 cents; for the numerators 13, 14, and 15, make 42, and $\frac{42}{21}$ is equal to 2 units, and the sum of all the gains is equal to the whole gain. By the 2d method, as the whole gain is to the whole stock, so is A's gain to A's stock, and B's and C's stocks are found from their gains in the same manner.

In proving this work by the 2d method, the first and third terms must be brought into $\frac{1}{2}$ the of a cent, because that is the lowest denomination in the third terms, and the first and third terms must always be of a name in working the rule of three, whether applied to Fellowship, or any other branch of the rule of three.

2. A, B, C, and D, entered into partnership. A put into the common stock, \$1000; B, \$2000; C, \$3000; and D, \$4000; they gained among them all \$1000; what was the share of each?

Ans. A's \$100; B's \$200; C's \$300; and D's \$400.

3. A and B agreed to do a piece of work for \$150; to complete which A worked 70 days, and B 50 days; what share of the money did each receive? Ans. A had \$87,50, and B \$62,50.

4. A, B, and C, freighted a ship with wine; A put in 150 pipes, B 200, and C 100; in a storm the seamen threw overboard 80 pipes; what was the loss of each?

Ans. A's, $26\frac{2}{3}$; B's, $35\frac{5}{9}$, and C's, $17\frac{2}{9}$ pipes.

5. Divide \$120 between three persons, so that their shares shall be to each other as the numbers 1, 2, and 3.

Ans. A will have \$20; B \$40; and C \$60.

6. A bankrupt is indebted to A \$277,33; to B \$305,17; to C \$152, and to D \$105. But his estate is worth only \$677,50; how much will each of his creditors receive?

Ans. A \$223, $81\frac{2580}{8395}$; B \$246, $28\frac{615}{8395}$; C \$122, $66\frac{6930}{8395}$; and D \$84, $73\frac{6665}{8395}$.

D O U B L E F E L L O W S H I P

Rule. Multiply each man's stock and time together; add the several products together; then as the sum of the products is to the whole gain or loss, so is each one's product to his share of the gain or loss.

Proof. Add the several gains or losses together, and if the work is right, the sum will be equal to the whole gain or loss. — This is the method given by all our authors on arithmetic, but they ought to tell you, if they know it, that the work by this method, will appear to be right when it is wrong.

A better method of Proof.
Multiply each man's gain and time together; as the sum of the products is to the whole stock, so is each one's product to his stock.

Reason of the Rule. — This is because that when the stocks are equal, the gain or loss must be in proportion to the time, and when the times are equal, the gain or loss must be in proportion to the stock. But when neither are equal, the gain or loss must be in proportion to the product of the stock and time. Therefore the gain or loss in Double Fellowship, increases in proportion as the stock and time increase, the same as simple interest increases in proportion to the principal and time.

EXAMPLES.

1. Two persons, A and B, let out money to C at 6 per cent. per annum, for which each was to receive, as in reason he ought to, such a part of the interest as his money gained. A let out \$100 for 12 months, and B \$200 for 6 months, and the interest both gained was \$12: what share of the gain or interest belonged to each?

Now it is plain from the rule of simple interest, and from reason, that \$100 will \$6 in a year, at 6 per cent., and that \$200 will gain the same in 6 months, so that A and B will each have \$6 of the gain: we

DUODECIMALS
will now solve the question by the rule of Double Fellowship.

B's stock \$200
B's time -- 6 months.
B's product 1200

A's Prod. 1200
 B's do. 1200
 Sum of do. 2400 : 12 :: 1200
 2400) 1200 $\frac{12}{12}$ { the gain of each,
 as their pro-
 ducts are equal.

2. A and B hired a pasture in partnership, for which they paid together, \$ 36. A put in 23 oxen for 27 days, and B 21 oxen for 35 days: what part of the \$36 did each pay?

Ans. { A \$ 16.48 $\frac{912}{1356}$.
B \$ 19.51 $\frac{544}{1356}$.

3. Two merchants traded together for 18 months: A at first put into stock \$200, and at the end of 5 months he put in \$100 more. B put in at first \$550, and at 4 months' end took out \$140. At the end of the 18 months they found they had gained \$526: what was each one's share of the gain?

Ans. { A's \$192, 95 $\frac{70}{1254}$.
B's \$333, 04 $\frac{1184}{1254}$.

Questions for Examination.

1. What is the rule called Fellowship?
2. What is Single Fellowship?
3. What is the rule for working Single Fellowship?
4. How is the work proved?
5. What is Double Fellowship?
6. What is the rule for working Double Fellowship?
7. How is Double Fellowship proved?

Quodecimals, or Cross Multiplication, is a rule
by which workmen and artificers compute

DUODECIMALS.

or cast up the contents of their work. Duodecimals are so called from duodecem, twelve, because they ^{generally} increase by twelves, or in a twelvefold proportion.

Dimensions are taken in feet, inches, seconds, thirds, &c.

Feet are marked Ft.; inches (sometimes called primes) are marked thus, $\prime$; ^{or $\frac{1}{12}$} seconds are marked $\prime\prime$; thirds, $\prime\prime\prime$; &c.

Table.

12 Seconds	{	make 1	{	Inch, or Prime...	$\frac{1}{12}$ in. or
12 Inches				Foot, ...	Ft.

Explanation. The meaning of the dimensions is, that a piece of any surface which is an inch wide and a foot long, is the twelfth part of a square foot; and a piece one second wide and a foot long, is the twelfth part of an inch, or the 144th part of a square foot, &c. So that in Duodecimals, every dimension or denomination is supposed to be a foot long, let the width be what it may.

Glaziers' and Masons' work is measured by the square foot.

Painting, plastering[†], and paving, are measured by the square yard.

Partitioning, flooring, roofing, slating, and rough-boarding, are measured by the square of 100 feet.

Stone and brick work are measured by the square rod.

RULE for MULTIPLYING DUODECIMALLY.

1. Under the multiplicand write the corresponding denominations of the multiplier.
2. Multiply each term in the multiplicand (beginning at the lowest) by the feet in the multiplier; write each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior.
3. In the same manner multiply the multiplicand by the primes in the multiplier, and write the result of each term one place more to the right-hand of those in the multiplicand.

[†] Sometimes spelled plastering.

DUODECIMALS.

4 Work in the same manner with the seconds in the multiplier, setting the result of each term two places to the right-hand of those in the multiplicand, and so on for thirds, fourths, &c.

Reason of the Rule. The reason is because feet multiplied by feet, give feet; feet multiplied by inches, give inches; inches by inches give seconds; &c.

Examples.

1. Multiply 7 feet 9 inches, by 3 feet 6 inches.

By Cross Multipl.	By Practice.	By Duodecimals.	By Decimals.
$\begin{array}{r} 7 \times 9 \\ 3 \times 6 \\ \hline 21:0:0 = 7 \times 3 \\ 2:3:0 = 9 \times 3 \\ 3:6:0 = 7 \times 6 \\ 0:4:6 = 9 \times 6 \\ \hline 27:1:6 \end{array}$	$\begin{array}{r} 7:9 \\ 3:6 \\ \hline 23:3 \\ 3:10:6 \\ \hline 27:1:6 \end{array}$	$\begin{array}{r} 7:9 \\ 3:6 \\ \hline 23:3 \times 3 \\ 3:10:6 \times 6 \\ \hline 27:1:6 \end{array}$	$\begin{array}{r} 7.75 \\ 3.5 \\ \hline 23.875 \\ 23.25 \\ \hline 27.125 \end{array}$

Explanation. By Cross Multiplication the 7 ft. is first multiplied by the 3 ft.; then the 9 inches by 3 feet, &c.

By Practice (or compound multiplication) 7 ft. 9 in. is multiplied by 3, and half the multiplicand is added to the product, because 6 inches is half of a foot.

By Duodecimals, the operation is according to the rule.

By Decimals, 9 inches is .75 of a foot, and 6 inches is .5 of a foot. The answer is the same by all the methods. — The third is the one to be used in the following questions.

	f. in.	by	f. in.	Ans.
2. Mult.	8:5	by 4:7		38:6:11
3. Mult.	9:8	by 7:6	A.	72:6
4. Mult.	8:1	by 3:5	A.	27:7:5
5. Mult.	7:6	by 5:9	A.	43:1:6
6. Mult.	4:7	by 3:10	A.	17:6:10
7. Mult.	7:5:9	by 3:5:3	A.	25:8:6:2:3
8. Mult.	10:4:5	by 7:8:6	A.	79:11:0:6:6
9. Mult.	75:7	by 9:8	A.	730:7:8
10. Mult.	97:8	by 8:9	A.	854:7:
11. Mult.	57:9	by 9:5	A.	543:9:9
12. Mult.	75:9	by 17:7	A.	1331:11:3

13. There is a house with three tiers of windows, 3 in a tier. the height of the first tier is 7 feet 10 inches; the second, 6 feet 8 inches; the third, 5 feet 4 inches. The width of each window is 3 feet 11 inches. How many square feet are contained in all nine of

B A R T E R.

by pence, are 20ths of a penny; and pence multiplied by pence, are 240ths of a penny.

Questions for Examination.

1. What are duodecimals?
2. What is the rule for multiplying duodecimals?

B A R T E R.

Barter is a rule which teaches merchants and traders so to proportion the prices and quantities of their commodities in exchanging one for another, that neither may sustain loss.*

Case I.

When the price and quantity of one article are given, and the price of another, to find how much of the last must be given for the first.

Rule. As the price per lb., yard, &c. of the quantity required,

Is to 1 lb., 1 yard, &c.;

So is the price of the whole given quantity,

To the quantity of the commodity required.

Proof and Reason of the Rule. - The same as of the rule of three.

Examples.

1. How much chocolate at 4s. per lb. must be given in barter for 2 lb. of tea, at 8s. per lb.? Ans. 4 lb.

2. How much sugar at 10 cents a pound must be given in barter for 10 lb. of coffee, at 20 cents a pound? Ans. 20 lb.

These two questions being easy, may be solved mentally, and will show the nature of the rule.

3. A, has 2 pipes of wine, at \$1.25 a gallon, and B, has brandy at \$1.50 a gallon. How much

* Barter is performed by the rule of three, and is called by another name because all questions in it relate to trade, whereas the rule of three applies to questions of any nature whatever.

B A R T E R.

brandy must B give deliver to A for his 2 pipes of wine? Ans. $143\frac{1}{3}$ gallons.

63 gallons = 1 hhd.

$\frac{126}{2} = 2$ hogsheads.

2 hhd. = 1 pipe.

252 do. in 2 pipes.

125 cents a gallon.

1260

504

152

8215, 00 price of the wine.

As $\frac{\text{cts.}}{150} : 1 :: 21500$

$15 \overline{) 21500} \quad 143\frac{1}{3}$
 $\underline{150}$
 65
 $\underline{60}$
 50
 $\underline{45}$
 5

3. How much tea, at 9s. per lb. can I have in barter for 4 cwt. 2 qrs. of chocolate, at 4s. per lb.?

Ans. 2 cwt.

4. Two merchants barter; A hath 20 cwt. of cheese, at 21s. 6d. per cwt. B hath eight pieces of Irish cloth, at 3l. 14s. per piece—I desire to know who must receive the difference, and how much?

Ans. B must receive of A 8l. 2s.

5. A and B barter; A hath $3\frac{1}{2}$ lb. of pepper, at $13\frac{1}{4}$ d. per lb. B hath ginger at $15\frac{1}{4}$ d. per lb.—how much ginger must he deliver in barter for the pepper?

Ans. 3lb. 10z. $\frac{3}{4}$

6. How many dozen of candles, at 5s. 2d. per dozen, must be delivered in barter for 3 cwt. 2 qrs. 16lb. of tallow, at 37s. 4d. per cwt. Ans. 26 doz. 3lb.

7. A hath 608 yards of cloth, worth 14s. per yard, for which B gives him 125l. 12s. in ready money, and 85 cwt. 2 qrs. 24 lb. of bees-wax. The question is—what did B reckon his bees-wax at per cwt.?

Ans. 3l. 10s.

CASE II.

When two traders have goods at a certain cash price in ready money, and one of them advances his price in barter, to find the other's bartering price.

Rule.—As the ready money price of the one is to his bartering price;

So is the ready money price of the other to his bartering price.

EXAMPLES.

1. Two traders, A and B, agree to exchange goods; A has sugar worth $\pounds 9$ per cwt. in ready money, but in barter he will have $\pounds 11.25$ per cwt. B has coffee at $\pounds 15$ per cwt. in ready money; what ought it to be in barter, in proportion to A's bartering price. Ans. $\pounds 18.75$.

To solve this question on general principles,

LOSS AND GAIN.

That is, by reason only, either mentally or on the slate, first find what part of \$9 \$2.25 is (\$11.25 less \$9 is \$2.25) and you will find it to be the 4th part; therefore add the 4th part of \$15 to itself and the sum \$18.75 must be B's bartering price.

By Rule, thus:-

$$\begin{array}{r}
 \text{Ba} \quad \$ \quad \$ \quad \$ \\
 \text{tra} \quad 9 : 11.25 :: 15 \\
 \text{tit} \quad \quad \quad 15 \\
 \text{on} \quad \quad \quad 56.25 \\
 \text{for} \quad \quad \quad 1125 \\
 \quad \quad 9) 16875 \\
 \quad \quad \$18.75 \text{ the answer.}
 \end{array}$$

Remark. The quantity of one kind of goods which must be given for another different in price and kind, may be found as in Case I, either by the ready money or bartering price.

2. A has raisins at 20 cts. per lb. ready money, or 25 cts. in barter; B has tea at \$1.25 per lb. in ready money; how must B rate his tea in barter, and what quantity of tea must he give A for 10 boxes of his raisins, each weighing 30 lb.?

Ans. { B's bartering price \$1.56 $\frac{1}{4}$, and 48 lb. of tea must be given in barter.

First.

$$\text{As } 20 : 25 :: 1.25 : \$1.56 \frac{1}{4}.$$

Secondly.

1. In 10 boxes of raisins, each 30 lb., there are 300 lb., which at 20 cts. a lb. come to \$60, or 600 cts; and at 25 cts. a lb. they come to \$75 or 7500 cts. Then,

As \$1.25 : 1 lb. :: \$60 : 48 lb. of tea, the answer.

Or,

$$\text{As } \$1.56 \frac{1}{4} : 1 \text{ lb.} :: \$75 : 48 \text{ lb. as before.}$$

Remark. In the first statement the first and third terms are the ready money prices, and in the second, the bartering prices. The first is the most convenient in this question, as it contains no fraction of a cent.

LOSS AND GAIN.

3. A, has nutmegs worth 7s. 6d. per lb. ready money, but in barter he will have 8s. and D has leaf tobacco worth 9d. per lb. ready money: how must D value his tobacco per lb. in barter, so that his profit may be equal to A's? Ans. 9 $\frac{3}{5}$ d.

I Questions for Examination.

1. What is Barter?
2. Having the price of different articles and the quantity of one of them, how do you ^{know} what quantity of one is equal in value to the given quantity of the other?
3. Having the ready money price of two different articles, and the bartering price of one of them given, how do you find the bartering price of the other?

LOSS AND GAIN.

or lower the prime cost price, so as to gain or lose so much per cent. or otherwise.

The questions in this rule are performed by the Rule of Three.

1. To find what is gained or lost per cent.

RULE.—First find the gain or loss by subtraction; then, As the prime cost, is to the gain or loss; so is 100. to the gain or loss per cent.*

2. To find at what rate goods must be sold, to gain or lose so much per cent.

RULE.—As 100. is to the prime cost; so is 100. with the profit added, or the loss subtracted; to the gaining or losing price.

3. The gain or loss per cent. being known, to find what the goods cost.

RULE.—As 100. with the gain per cent added; or the loss per cent subtracted; is to the selling price, so is 100. to the prime cost.

4. When so much is gained or lost by selling goods at a certain rate, to find the gain or loss, if sold at any other rate.

RULE.—As the first price, is to 100. adding the gain per cent, or subtracting the loss per cent; so is the other price to the gain or loss per cent.

If the answer exceed 100. the excess is the gain

gained
teaches to raise

* In Federal Money, \$100 is used the same as £100 in English or Lawful money.

LOSS AND GAIN.

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PROFIT and LOSS.

per cent; if less than 100l. the deficiency is the loss per cent.

EXAMPLES.

1. If a yard of cloth is bought for 11s. and sold for 12s. 6d.—what is the gain per cent? 2. If 60 ells of Holland cost 18l.—what must 1 ell be sold for to gain 8 per cent?

As 11 : 1 : 6 :: 100 :

12 20

18 2000

12 18

11 : 0 11)36000

1 : 6 12)3272

20(2712 : 8

Ans. L 13 : 12 : 8 $\frac{3}{4}$.

As 100 : 18 :: 108

108

1100)19144

20)12)19 : 8 : 9 $\frac{1}{2}$

3180 5)1 : 12 : 4 $\frac{1}{2}$

12 6 : 5 $\frac{1}{2}$

9160 4

2140

Ans. 6s. 5d. $\frac{1}{2}$.

3. If 1lb. of tobacco cost 16d. and is sold for 20d.—what is the gain per cent? Ans. 25l.

4. If a parcel of cloth be sold for 560l. and at 12s. per cent. gain—what was the prime cost? Ans. 500l.

5. If a yard of cloth is bought for 13s. 4d. and sold again for 16s.—what is the gain per cent? Ans. 20l.

6. If 112 lb. of iron cost 27s. 6d.—what must 1 cwt. be sold for to gain 15l. per cent? Ans. 1l. 11s. 7d. $\frac{1}{2}$.

7. If 375 yards of broad cloth be sold for 490l. and 20 per cent. profit—what did it cost per yard? Ans. 1l. 1s. 9d. $\frac{1}{2}$.

8. Sold 1 cwt. of hops for 6l. 15s. at the rate of 25 per cent. profit—what would have been the gain per cent. If I had sold them for 8l. per cwt.? Ans. 48l. 2s. 11d. $\frac{1}{2}$.

9. If 90 ells of cambric cost 60l.—how must I sell it at per yard to gain 18l. per cent? Ans. 12s. 7d.

Explanations.—In the first statement, the first term is 11s. and the third is £100. The second, 12s. 6d. is what 11s. gains.

In the work of the second question, after working the statement, the cost of 60 ells to gain 8 per cent. in the whole (£19 8s. 9d. $\frac{1}{2}$) is divided by 12, and the quotient by 5, because 5 times 12 are 60, the number of ells.

EQUATION OF PAYMENTS.

I Questions for Examination.

1. What is Loss and Gain?
2. How do you find what is gained or lost per cent.?
3. How do you find how goods must be sold, to gain or lose so much per cent.?

EQUATION OF PAYMENTS.

Equation of Payments is a rule which teaches to find a mean time for paying a whole debt, ^{at once,} when several sums are due at different times.

Common Rule.*

Multiply each payment by its time, and divide the sum of the products by the whole debt, and the quotient will be the mean or equated time.

Proof. Prove the multiplication and division of the work.

Reason of the Rule. — Because as much is supposed to be gained by paying one sum after it is due, as is lost by paying another before it is due.

* This rule supposes the interest and discount of any sum to be equal; but as there is a small difference, when perfect accuracy is required, the equated time is when the whole discount lost by paying sums before they are due, is equal to the interest of what is kept after it is due from that time to the equated time.

Some find the present worth of each sum due at a future time, by Discount, and then find how long a time is required for the sum of all the present worths to amount to the sum total of all the sums due at different times, and that is considered to be the true equated time. — The common rule is the easiest, and sufficiently exact for practical purposes.

EQUATION OF PAYMENTS

EXAMPLES.

1. A owes B 200l. whereof 40l. is to be paid at 3 months, 60l. at 5 months, and 100l. at 10 months; at what time may the whole debt be paid together, without prejudice to either? *essential loss to either?*

£.	mo.	
40	X 3	= 120
60	X 5	= 300
100	X 10	= 1000

2100) 14120

7 months, $\frac{1}{10}$.

2. B owes C 800l. whereof 200l. is to be paid at 3 months, 100l. at 4 months, 300l. at 5 months, and 200l. at 6 months; but they agreeing to make but one payment of the whole—I demand what that time must be?

Ans. 4 months, 18 days.

3. I bought of K a quantity of goods to the value of 360l. which was to have been paid as follows: 120l. at 2 months, 200l. at 4 months, and the rest at 5 months; but they afterwards agreed to have it paid at one mean time—the time is demanded?

Ans. 3 months, 13 days.

4. A merchant bought goods to the value of 500l. to pay 100l. at the end of 3 months, 150l. at the end of 6 months, and 250l. at the end of 12 months; but afterwards they agreed to discharge the debt at one payment; at what time was this payment made?

Ans. 8 months, 12 days.

5. A merchant has owing to him \$1200, to be paid $\frac{1}{6}$ in 2 months, $\frac{1}{3}$ in 3 months, and the remainder in 6 months; but the creditor agrees to pay $\frac{1}{2}$ in advance. In how long a time after 6 months ought the remainder to be paid? *Ans. $2\frac{2}{3}$ months nearly.*

Explanatory Remark.—As \$200 are paid 2 months before it is due, and \$400 is paid 3 months before it is due, it is plain that the creditor loses the discount of those sums for the time paid in advance, and that he ought to keep the \$600 due in 6 months so much longer than 6 months, as to gain as much interest as is lost in discount: therefore find the discount of \$200 for 2 months, and of \$400 for 3 months, and add them together; then find the interest of \$600 for a year; next, say as that interest is to 12 months, so is the sum of the discount to the time required.

I Questions for Examination.

1. What is Equation of Payments?

2. What is the common rule for finding the mean or equated time for paying at once, several sums due at different times?

VULGAR FRACTIONS.

Calculus of Vulgar Fractions.

The definition, origin, and forms of vulgar fractions have been treated of under the General View of Fractions. The rules for reducing, adding, subtracting, multiplying, ^{and} dividing vulgar fractions, ^{the method of} and applying them to practical uses are given and explained as follows.

PROBLEM I.

To find the GREATEST COMMON MEASURE of two or more numbers.

When there are only two numbers.

R U L E.

Divide the greater by the less; and that divisor by the remainder; and so on, always dividing the last divisor by the last remainder, till nothing remains; the last divisor is the GREATEST COMMON MEASURE.

When there are more than two numbers.

R U L E.

1. Find the greatest common measure of two of them, as before.
2. Find the greatest common measure of the last found common measure and another of the given numbers.
3. Repeat the operation as often as required; always using the last found common measure, and one of the given numbers.

NOTE, If the last found common measure is 1, the numbers are PRIME to each other.

VULGAR FRACTIONS.

E X A M P L E S.

1. What is the greatest common measure of 522, 918, and 1998? Ans. 18.

<i>First.</i> $\begin{array}{r} 522 \overline{) 918} 1 \\ \underline{522} \\ 396 \end{array}$	<i>last found com. meas.</i> $\begin{array}{r} 396 \overline{) 522} 1 \\ \underline{396} \\ 126 \end{array}$	<i>Again.</i> $\begin{array}{r} 18 \overline{) 1998} 111 \\ \underline{18} \\ 18 \\ \underline{18} \\ 0 \end{array}$
---	--	---

Which shews that 18 is the greatest common measure of 522 and 918.*

NOTE, The first found common measure (18) exactly dividing the third given number (1998) still remains the greatest measure of all the three.

2. What is the greatest common measure of 372 and 246? Ans. 6.
3. What is the greatest common measure of 612 and 540? Ans. 36.
4. What is the greatest common measure of 1224 and 1080? Ans. 72.
5. What is the greatest common measure of 1736, 720, and 336? Ans. 8.
6. What is the greatest common measure of 3996, 1836, and 1044? Ans. 36.

PROBLEM II.

To find the least COMMON MULTIPLE of two or more numbers.

R U L E.

1. Divide the given numbers, by any number that will divide two, or more of them without a remainder, and set the quotients, and the undivided numbers, underneath.
2. Divide these quotients, and undivided numbers,

* That is, though 2, or 3, will divide either 522, or 918, without a remainder, yet 18 is the greatest number which will so divide them.

VULGAR FRACTIONS.

by any number that will divide two, or more of them, as before; and continue the operation till no two numbers are left, capable of being so divided.

3. Multiply the divisors, and last quotients together, and the product will be the least COMMON MULTIPLE required.

EXAMPLES.

1. What is the least common multiple of 24, 36, 48 and 60? *Ans. 720.*

$$12)24, 36, 48, 60$$

$$2)2, 3, 4, 5$$

$$1, 3, 2, 5$$

$$12 \times 2 \times 3 \times 2 \times 5 = 720 \text{ Ans. } ^+$$

2. What is the least common multiple of 3, 5, 8, and 10? *Ans. 120.*

3. What is the least number that 20, 16, 10, and 6 will divide without a remainder? *Ans. 240.*

4. What is the least common multiple of 3, 4, 8, and 12? *Ans. 24.*

5. What is the least number that can be divided by 1, 2, 3, 4, 5, 6, 7, 8, 9, without a remainder? *Ans. 2520.*

REDUCTION of VULGAR FRACTIONS

Teaches to bring them out of one denomination into any other denomination.

NOTE, The pupil is recommended to be careful always to work with his fractions in their lowest terms; which will save him much labour, and render his operations easy.

CASE I.

To reduce Fractions to their lowest terms.

GENERAL RULE.

1. Find their greatest common measure as before taught.

2. Divide both parts of the fraction by this common measure; the quotients will be the new fraction in its lowest terms.

+ That is, 720 is the least number that can be divided by all the numbers 24, 36, 48, and 60, separately, without a remainder.

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OR, Divide both parts of the fraction by any number that will divide them both, without a remainder; divide the quotients in a similar manner, and so on, till no number will divide them both: the last quotients will be the fraction in its lowest terms.

E X A M P L E S.

1. Reduce $\frac{295}{1770}$ to its lowest terms. *Ans.* $\frac{1}{6}$.

By the General Rule.

$$\begin{array}{r} \text{greatest} \quad 295 \overline{) 1770} 6 \\ \text{com. meas.} \quad 1770 \end{array} \quad 295 \overline{) \frac{295}{1770}} \left(\frac{1}{6} \right) \text{Ans.}$$

By the Second Rule,

$$5 \overline{) \frac{295}{1770}} \left(\frac{59}{354} \right) \quad 59 \overline{) \frac{59}{354}} \left(\frac{1}{6} \right) \text{Ans.}$$

2. Reduce $\frac{144}{216}$ to its lowest terms. *Ans.* $\frac{2}{3}$.
3. Reduce $\frac{112}{144}$ to its lowest terms. *Ans.* $\frac{7}{9}$.
4. Reduce $\frac{3619}{6251}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
5. Reduce $\frac{208}{684}$ to its lowest terms. *Ans.* $\frac{52}{171}$.
6. Reduce $\frac{828}{960}$ to its lowest terms. *Ans.* $\frac{55}{64}$.
7. Reduce $\frac{5184}{6912}$ to its lowest terms. *Ans.* $\frac{3}{4}$.

C A S E II.

To reduce a mixed number to a Vulgar Fraction of the same value.

R U L E.

Multiply the whole number, by the denominator of the fraction, adding the numerator to the product, for a new numerator; which placed over the denominator gives the required fraction.

To express a whole number fraction-wise, set 1 for the denominator ~~given~~ under the given number.

E X A M P L E S.

1. Reduce $18\frac{3}{7}$ to an improper fraction. *Ans.* $\frac{129}{7}$.
 $18 \times 7 + 3 = 129$ new numerator.
2. Reduce $56\frac{13}{25}$ to an improper fraction. *Ans.* $\frac{1423}{25}$.
3. Reduce $18\frac{5}{21}$ to an improper fraction. *Ans.* $\frac{385}{21}$.
4. Reduce $13\frac{1}{5}$ to an improper fraction. *Ans.* $\frac{66}{5}$.

† Here 5 will go 59 times in 295, and 354 times in 1770. Next, 59 is contained once in 59, and 6 times in 354. The work might be arranged thus.

$$\begin{array}{r|l} 5 & 59 \\ \hline 295 & 59 \quad 1 \\ 1770 & 354 \quad 6 \end{array}$$

Here the divisors are placed over the fractions, which are divided as in short division. — This is generally the easiest way to reduce fractions to their lowest terms.

Any number ending with an even number or a cipher, is divisible by 2, without a remainder.

Any number ending with 0 or 5, is divisible by 5: — ending with 0, is divisible by 10.

If the two right hand figures of any number, taken together, are divisible by 4, the whole is divisible by 4.

VULGAR FRACTIONS.

5. Reduce $2\frac{2}{3}$ to an improper fraction. Ans. $\frac{24}{9}$.
6. Reduce $5\frac{14}{16}$ to an improper fraction. Ans. $\frac{82}{16}$.

4. To reduce an improper fraction to its proper terms.

RULE. Divide the upper term by the lower.

EXAMPLES.

1. Reduce $\frac{129}{7}$ to its proper terms. Ans. $18\frac{3}{7}$.
2. Reduce $\frac{129}{7}$ to its proper terms. Ans. $18\frac{3}{7}$.
3. Reduce $\frac{129}{7}$ to its proper terms. Ans. $18\frac{3}{7}$.
4. Reduce $\frac{129}{7}$ to its proper terms. Ans. $18\frac{3}{7}$.
5. Reduce $\frac{129}{7}$ to its proper terms. Ans. $18\frac{3}{7}$.
6. Reduce $\frac{129}{7}$ to its proper terms. Ans. $18\frac{3}{7}$.

Ans. $18\frac{3}{7}$.

Ans. $56\frac{13}{24}$.

Ans. $18\frac{3}{7}$.

Ans. $13\frac{1}{4}$.

Ans. $2\frac{1}{2}$.

Ans. $5\frac{14}{16}$.

a whole or mixed number.

a whole or mixed number.

a whole or mixed number.

a whole or mixed number.

a whole or mixed number.

a whole or mixed number.

To reduce a COMPOUND FRACTION to a SINGLE ONE.

GENERAL RULE.

Multiply all the numerators for a new numerator; and all the denominators for a new denominator; and reduce the new fraction to its lowest terms.

NOTE, Mixed Fractions must be always reduced to Simple Fractions before they can be multiplied or divided.

A BETTER RULE.

1. Place all the numerators in a row, with the multiplication ($\times$) sign between them.
2. Place all the denominators in the same manner under the numerators.
3. Cancel all the upper and under figures that are alike; and divide the upper and under figures by any number that will measure them, or divide them.
4. Multiply the remaining numerators for a new numerator; and the remaining denominators for a new denominator; the new fraction will be the answer in the lowest terms.

without a remainder.

EXAMPLE.

1. Reduce $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{1}{2}$ to a simple fraction.

If the three right hand figures of any number are divisible by 8, the whole is divisible by 8.

If the sum of the digits or figures of any number is divisible by 3 or 9, the whole is divisible by 3 or 9.

VULGAR FRACTIONS.

OPERATION by the CONTRACTED RULE.

2) 2, 4, 8

2) 1, 2, 4 $2 \times 2 \times 2 = 8$ least com. multiple, or
[denominator.]

1, 1, 2

~~$\frac{1}{2} \times \frac{4}{4} = \frac{4}{4}$~~ ~~$\frac{2}{4} \times \frac{2}{2} = \frac{4}{4}$~~ ~~$\frac{4}{8} \times \frac{1}{1} = \frac{4}{8}$~~

these numerators placed over the denominator gives the answer $\frac{4}{8}, \frac{6}{8}, \frac{5}{8}$ equal in value to $\frac{32}{64}, \frac{48}{64}, \frac{40}{64}$.

2. Reduce $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \frac{7}{8}$ to their least common denominator. *Ans.* $\frac{12}{24}, \frac{16}{24}, \frac{20}{24}, \frac{21}{24}$.

3. Reduce $\frac{3}{8}, \frac{5}{9}, \frac{7}{12}$ to their least common denominator. *Ans.* $\frac{27}{72}, \frac{40}{72}, \frac{42}{72}$.

4. Reduce $\frac{2}{5}, \frac{4}{6}, \frac{5}{9}, \frac{7}{10}$ to their least common denominator. *Ans.* $\frac{36}{90}, \frac{60}{90}, \frac{50}{90}, \frac{63}{90}$.

5. Reduce $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{16}, \frac{17}{24}$ to their least common denominator. *Ans.* $\frac{16}{48}, \frac{36}{48}, \frac{40}{48}, \frac{42}{48}, \frac{11}{16}, \frac{17}{24}$.

To reduce fractions of one denomination to the fraction of another, but greater, retaining the same value.

RULE. Reduce the given fraction to a compound one, by comparing it with all the denominations between it, and that denomination which you would reduce it to; and reduce that compound fraction to a single one.

E X A M P L E S.

1. Reduce $\frac{7}{8}$ of a penny to the fraction of a pound.

Ans. $\frac{7}{8}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{7}{1920}$.

2. Reduce $\frac{1}{4}$ of a penny to the fraction of a pound.

Ans. $\frac{1}{960}$.

3. Reduce $\frac{1}{2}$ of a dwt. to the fraction of a lb. Troy.

Ans. $\frac{1}{384}$.

4. Reduce $\frac{1}{2}$ of a lb. Avoirdupoise to the fraction of an cwt.

Ans. $\frac{1}{192}$.

To reduce fractions of one denomination to the fraction of another, but less, retaining the same value.

RULE. Multiply the numerator by the parts con-

$\left. \begin{array}{l} 2 \times 4 = 8 \\ 3 \times 3 = 9 \\ 5 \times 5 = 25 \end{array} \right\} \begin{array}{l} \text{com. denom.} \\ \text{Numerators.} \end{array}$

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tained in the several denominations between it, and that you would reduce it to, for a new numerator, and place it over the given denominator.

NOTE, Before you multiply, reduce the multipliers and divisor as low as possible.

EXAMPLES.

1. Reduce $\frac{7}{1920}$ of a pound to the fraction of a penny. Ans. $\frac{7}{8}$.

~~7 x 96 x 12 = 7 x 1152 = 7920~~
~~1920~~
~~96~~
~~12~~
~~1~~

When two or more figures are marked with a dash above and below, the dash is supposed to go through all the figures.

$$\frac{7}{1920} \text{ of } \frac{20}{1} \text{ of } \frac{12}{1} = \frac{7}{8}^*$$

2. Reduce $\frac{1}{960}$ of a pound to the fraction of a penny. Ans. $\frac{1}{4}$.

3. Reduce $\frac{4}{1200}$ of a lb. Troy to the fraction of a penny weight. Ans. $\frac{1}{3}$.

4. Reduce $\frac{1}{784}$ of an cwt. to the fraction of a lb. Ans. $\frac{1}{4}$.

To reduce fractions of one denomination to another of the same value, having the numerator given of the required fraction.

RULE. As the numerator of the given fraction is to its denominator, so is the numerator of the intended fraction to its denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a fraction of the same value, whose numerator shall be 12. As 2:3::12:18. Ans. $\frac{12}{18}$.

2. Reduce $\frac{5}{8}$ to a fraction of the same value, whose numerator shall be 25. Ans. $\frac{25}{40}$.

3. Reduce $\frac{1}{2}$ to a fraction of the same value, whose numerator shall be 47. Ans. $\frac{47}{94}$.

To reduce fractions of one denomination to another of the same value, having the denominator given of the fraction required.

RULE. As the denominator of the given fraction is to its numerator, so is the denominator of the intended fraction to its numerator.

$$* \frac{7}{1920} \times \frac{20}{1} = \frac{140}{1920} \times \frac{12}{1} = \frac{1680}{1920} = \frac{7}{8}, \text{ found by reducing } \frac{1680}{1920} \text{ to its lowest terms.}$$

VULGAR FRACTIONS.

E X A M P L E S.

1. Reduce $\frac{2}{3}$ to a fraction of the same value, whose denominator shall be 18. As $3:2::18:12$. Ans. $\frac{12}{18}$.
2. Reduce $\frac{2}{3}$ to a fraction of the same value, whose denominator shall be 35. Ans. $\frac{23}{35}$.
3. Reduce $\frac{2}{3}$ to a fraction of the same value, whose denominator shall be $65\frac{1}{2}$. Ans. $\frac{47}{65\frac{1}{2}}$.

To reduce a mixed fraction to a single one.

RULE. When the numerator is ~~the integral part~~, a mixed number, multiply it by the denominator of the fractional part, adding in the numerator of the fractional part for a new numerator; then multiply the denominator of the fraction by the denominator of the fractional part for a new denominator.

E X A M P L E S.

1. Reduce $\frac{36\frac{2}{3}}{48}$ to a simple fraction. Ans. $\frac{110}{144} = \frac{55}{72}$.
 $36 \times 3 + 2 = 110$ numerator.
 $48 \times 3 = 144$ denominator.
2. Reduce $\frac{23\frac{5}{8}}{38}$ to a simple fraction. Ans. $\frac{166}{152} = \frac{83}{76}$.

When the denominator is ~~the integral part~~, a mixed number, multiply it by the denominator of the fractional part, adding the numerator of the fractional part for a new denominator: then multiply the numerator of the fraction by the denominator of the fractional part for a new numerator.

E X A M P L E S.

1. Reduce $\frac{47}{65\frac{1}{2}}$ to a simple fraction. Ans. $\frac{94}{131}$.
2. Reduce $\frac{19}{44\frac{1}{2}}$ to a simple fraction. Ans. $\frac{38}{89}$.

To find the proper value of a fraction in the known parts of an integer.

RULE. Multiply the numerator by the ~~common~~ parts in the next *Vulgar Fraction.* ~~parts of the integer~~, and divide by the denominator, *less denomination,* ~~contracting the operation as you proceed.~~ and thus proceed with every *denomina-*
 tion to the lowest.

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EXAMPLES.

1. Reduce $\frac{1}{4}$ of a pound sterling to its proper quantity. ‡

$$3 \times 20 = 60 \div 4 = 15 \text{ s. } \text{Ans.}$$

2. Reduce $\frac{2}{3}$ of a shilling to its proper quantity. *Ans. 4d. $3\frac{1}{3}$ qrs.*

3. Reduce $\frac{1}{2}$ of a lb. Avoirdupoise to its proper quantity. *Ans. 9 oz. $2\frac{1}{2}$ dr.*

4. Reduce $\frac{1}{3}$ of a cwt. to its proper quantity. *Ans. 3 qrs. 3 lb. 1 oz. $12\frac{1}{2}$ dr.*

5. Reduce $\frac{1}{2}$ of a lb. Troy to its proper quantity. *Ans. 7 oz. 4 dwt.*

6. Reduce $\frac{1}{3}$ of an ell English to its proper quantity. *Ans. 2 qrs. $3\frac{1}{3}$ nails.*

7. Reduce $\frac{1}{2}$ of a mile to its proper quantity. *Ans. 6 furl. 16 poles.*

8. Reduce $\frac{1}{3}$ of an acre to its proper quantity. *Ans. 2 roods, 20 poles.*

9. Reduce $\frac{1}{4}$ of a hoghead of wine to its proper quantity. *Ans. 54 gallons.*

10. Reduce $\frac{1}{3}$ of a barrel of beer to its proper quantity. *Ans. 12 gallons.*

11. Reduce $\frac{1}{2}$ of a chaldron of coals to its proper quantity. *Ans. 15 bushels.*

12. Reduce $\frac{1}{3}$ of a month to its proper time. *Ans. 2 weeks, 2 days, $19\frac{1}{3}$ hours.*

To reduce any given quantity to the fraction of any greater denomination, retaining the same value.

RULE. Reduce the given quantity to the lowest denomination mentioned for a numerator, under which set the integral part reduced to the same for a denominator, and it will give the fraction required.

EXAMPLES.

1. Reduce 15s. to the fraction of a pound sterling. *Ans. $\frac{15}{20} = \frac{3}{4}$.*

2. Reduce 4d. $3\frac{1}{3}$ qrs. to the fraction of a shilling. *Ans. $\frac{2}{5}$.*

$$\begin{array}{r} 4\frac{2}{3} \text{ qrs.} \\ \frac{4}{3} \times \frac{5}{5} = \frac{20}{15} \\ \frac{19}{15} \end{array}$$

$$\frac{19}{15} \text{ num. } \frac{12}{48} \text{ denom.}$$

$$\frac{96}{240} = \frac{2}{5} \text{ of a shilling, the answer.}$$

‡ Reducing a fraction to its proper quantity, means finding the value of it in less denominations than the given fraction is of. Thus, in the fourth question, the meaning is, what is the value of $\frac{1}{3}$ of a cwt. in quarters, pounds, ounces, and drams?

VULGAR FRACTIONS.

3. ~~38~~ Reduce 9 oz. $2\frac{2}{3}$ dr. to the fraction of a lb. Avoirdupoise. *Ans.* $\frac{4}{7}$.
4. ~~39~~ Reduce 3 qrs. 3 lb. 1 oz. $12\frac{2}{3}$ dr. to the fraction of an cwt. *Ans.* $\frac{7}{9}$.
5. ~~40~~ Reduce 7 oz. 4 dwt. to the fraction of a lb. Troy. *Ans.* $\frac{3}{5}$.
6. ~~41~~ Reduce 2 qrs. $3\frac{1}{2}$ nails, to the fraction of an English ell. *Ans.* $\frac{5}{9}$.
7. ~~42~~ Reduce 6 furlongs 16 poles to the fraction of a mile. *Ans.* $\frac{4}{7}$.
8. ~~43~~ Reduce 2 roods 20 poles to the fraction of an acre. *Ans.* $\frac{5}{8}$.
9. ~~44~~ Reduce 54 gallons to the fraction of a hoghead of wine. *Ans.* $\frac{6}{7}$.
10. ~~45~~ Reduce 12 gallons to the fraction of a barrel of beer. *Ans.* $\frac{1}{2}$.
11. ~~46~~ Reduce 15 bushels to the fraction of a chaldron of coals. *Ans.* $\frac{5}{12}$.
12. ~~47~~ Reduce 2 weeks, 2 days, $19\frac{1}{2}$ hours, to the fraction of a month. *Ans.* $\frac{1}{4}$.

ADDITION of VULGAR FRACTIONS. R U L E.

Reduce compound fractions to single ones, mixed numbers to improper fractions, and fractions of different denominations to those of the same, and add them to a common denominator: then the

sum of the numerators placed over the common denominator, will be the sum of the fractions required.

When there are mixed numbers, use the fractional parts, and add the whole numbers to their sum.

E X A M P L E S.

1. Add $\frac{2}{3}$ and $\frac{5}{7}$ together. *Ans.* $\frac{22}{21} = 1\frac{1}{21}$ *
2. Add $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{3}{8}$ together. *Ans.* $1\frac{17}{80}$.
3. Add $\frac{1}{2}$, $4\frac{1}{2}$ and $\frac{3}{4}$ together. *Ans.* $4\frac{11}{4}$.
4. Add $7\frac{2}{3}$ and $\frac{2}{3}$ together. *Ans.* $8\frac{1}{3}$.
5. Add $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{1}{2}$ together. *Ans.* $\frac{1}{2}$.
6. Add $5\frac{2}{3}$, $6\frac{2}{3}$ and $4\frac{1}{2}$ together. *Ans.* $17\frac{1}{2}$.
7. When the fractions are of several denominations, reduce them to the proper quantities and add as before.
7. Add $\frac{1}{4}$ of a pound to $\frac{1}{2}$ of a shilling. *Ans.* 15s. 10d.
8. Add $\frac{1}{2}$ of a penny to $\frac{2}{3}$ of a pound. *Ans.* 13s. $4\frac{1}{2}$ d.
9. Add $\frac{1}{4}$ of a pound Troy to $\frac{1}{2}$ of an ounce. *Ans.* 9 oz. 3 dwt. 8 gr.
10. Add $\frac{1}{4}$ of a ton to $\frac{1}{2}$ of a lb. *Ans.* 16 cwt. 13 oz. $5\frac{1}{2}$ dr.

* Found thus, by the rule.

$$2 \times 7 = 14$$

$$5 \times 3 = 15$$

$\frac{29}{21}$ = the sum of the numerators.

$3 \times 7 = 21$, the common denominator.

By the short and easier method thus.

$\frac{2}{3}, \frac{5}{7}$ - - - - - } Here the numerator and denominator of each fraction is multiplied by the denominator of the other.

$$\frac{14}{21}, \frac{15}{21} = \frac{29}{21} = 1\frac{8}{21}$$

To add the fractions $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{5}{6}$, by this method, first add the $\frac{2}{3}$ and $\frac{2}{7}$, and then the three fractions will be reduced to two, which add in the same manner, and thus any number of fractions may be added, in the plainest manner possible. But by this method the sum of the fractions sometimes comes out in high terms, but they may be easily reduced to their lowest.

VULGAR FRACTIONS.

11. Add $\frac{1}{3}$ of a chaldron to $\frac{1}{4}$ of a bushel.
Ans. $2\frac{1}{2}$ bush. 3 pecks.
12. Add $\frac{1}{6}$ of a yard to $\frac{1}{3}$ of an inch.
Ans. 6 inch. 2 bar. c.

SUBTRACTION OF VULGAR FRACTIONS. RULE.

Prepare the fractions as in addition: then the difference of the numerators placed over the common denominator, is the difference of the two fractions.†

EXAMPLES.

1. From $\frac{3}{4}$ take $\frac{1}{2}$. Ans. $\frac{1}{4}$.
2. From $\frac{5}{6}$ take $\frac{1}{3}$ of $\frac{5}{6}$. Ans. $\frac{1}{2}$.
3. From $5\frac{2}{3}$ take $\frac{1}{6}$. Ans. $4\frac{1}{2}$.
4. From $\frac{3}{4}$ take $\frac{1}{2}$. Ans. $\frac{1}{4}$.
5. From $\frac{1}{2}$ take $\frac{1}{3}$ of $\frac{1}{2}$. Ans. $\frac{1}{6}$.
6. From $6\frac{1}{4}$ take $\frac{2}{3}$ of $\frac{1}{4}$. Ans. $6\frac{1}{6}$.
3. When the fractions are of several denominations reduce them to their proper quantities, and subtract as before.
7. From $\frac{1}{4}$ of a pound take $\frac{1}{4}$ of a shilling.
Ans. 14s. 3d.
8. From $\frac{2}{3}$ of a shilling take $\frac{1}{2}$ of a penny. Ans. $7\frac{1}{2}$ d.
9. From $\frac{1}{4}$ of a lb. Troy take $\frac{1}{6}$ of an ounce.
Ans. 8 oz. 16 dwts. 16 gr.
10. From $\frac{1}{2}$ of a ton take $\frac{1}{2}$ of a lb.
Ans. 15 cwt. 3 grs. 27 lb. 2 oz. 10 $\frac{1}{2}$ dr.
11. From $\frac{1}{3}$ of a chaldron take $\frac{1}{4}$ of a bushel.
Ans. 23 bush. 1 pk.
12. From $\frac{1}{6}$ of a yard take $\frac{2}{3}$ of an inch.
Ans. 1 in. 1 b. c.

or values,

† All that is necessary to be done in subtracting vulgar fractions, is to multiply them crosswise, and the difference of the products will be the difference of the numerators, and the product of the two denominators will be the common denominator. Thus, in the first example,

$$\begin{array}{r} \frac{3}{4} \times \frac{5}{7} = 28 \text{ the com. denominator} \\ \frac{5}{20} \quad \frac{3}{21} \\ \hline \frac{20}{1} \text{ diff. of the numerators.} \end{array}$$

Or, Here by multiplying the 3 and 4, the $\frac{3}{4}$, by 7, the denominator of $\frac{5}{7}$, the $\frac{3}{4}$ becomes $\frac{21}{28}$, and by multiplying the 5 and 7, (the $\frac{5}{7}$) by 4, the denominator of $\frac{3}{4}$, the $\frac{5}{7}$ becomes $\frac{20}{28}$, therefore 20 twenty eighths subtracted from 21 twenty eighths, leaves 1 twenty eighth, as before.

VULGAR FRACTIONS.

MULTIPLICATION OF VULGAR FRACTIONS.

R U L E.

1. Reduce Compound Fractions to Single ones; and mixed numbers to improper fractions.
2. Multiply the numerators together for a new numerator, and the denominators for a new denominator.

Examples.

1. Multiply $\frac{3}{5}$, $5\frac{1}{3}$, $\frac{8}{11}$, and $4\frac{1}{2}$ of $\frac{5}{16}$. Ans. $\frac{2}{3}$.

$$\frac{3 \times 16 \times 8 \times 11 \times 5}{5 \times 3 \times 11 \times 12 \times 16} = \frac{2}{3}$$

To work the above; first place the $\frac{2}{3}$ which is a simple fraction; secondly reduce $5\frac{1}{3}$ to a single fraction ($\frac{16}{3}$) and place it next the $\frac{2}{3}$; then the $\frac{8}{11}$ and lastly the compound fraction $4\frac{1}{2}$ of $\frac{5}{16}$ which may be placed like two single fractions, the word of being equivalent to the multiplication sign ($\times$).

2. Multiply $\frac{4}{7}$ by $\frac{5}{8}$. Ans. $\frac{5}{14}$.
3. Multiply $\frac{4}{5}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{1}{3}$. Ans. $\frac{16}{27}$.
4. Multiply 24 by $\frac{2}{3}$. Ans. 16.
5. Multiply $9\frac{1}{2}$ by $\frac{2}{3}$. Ans. $3\frac{1}{3}$.
6. Multiply $4\frac{1}{2}$, $\frac{4}{5}$ of $\frac{2}{3}$, and $18\frac{1}{2}$. Ans. $9\frac{1}{2}$.
7. Multiply $\frac{2}{3}$, $3\frac{1}{4}$, 5, and $\frac{1}{2}$ of $\frac{3}{4}$. Ans. $4\frac{1}{8}$.
8. Multiply $\frac{4}{5}$ of $\frac{2}{3}$ of 5 by $\frac{1}{2}$ of 7. Ans. 3.

D I V I S I O N.

R U L E.

Prepare the fractions, if necessary, invert the divisor, and proceed as in Multiplication.

E X A M P L E S.

1. Divide $\frac{4}{7}$ by $\frac{2}{17}$. Ans. $1\frac{1}{2}$.

$$\frac{4 \times 17}{7 \times 2} = 1\frac{1}{2}$$

By the common method thus, $\frac{3}{5} \times \frac{16}{3} = \frac{48}{15} \times \frac{8}{11} = \frac{384}{165} \times \frac{11}{12} = \frac{4224}{1980} \times \frac{5}{16} = \frac{21120}{31680}$, which reduced to the lowest terms is $\frac{2}{3}$, as by the abridged method.

N.B. The other answers are in their lowest terms, and are reduced after finding them by the common or given rule.

The reason why the divisor is inverted, is because that if 1 be divided by any fraction whose numerator is 1, the quotient will be the denominator of the fraction, the same as if the divisor was inverted and the fractions multiplied together; therefore, since the numerators and denominators may be any numbers, as well as 1, the shortest way of finding the quotient is by inverting the divisor, and multiplying the fractions together.

To find what part of a given whole number or fraction any fraction is, invert the given number, and then multiply both together, and the product is the answer.

By the common method thus; $\frac{21}{8} \times \frac{1}{7} = \frac{21}{56} = \frac{3}{8}$ or $1\frac{1}{2}$.

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2. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{2}{7}$ of $\frac{1}{9}$. Ans. 9.

$$\frac{1' \times 3' \times 1' \times 9}{3' \times 1' \times 2' \times 1} = 9^*$$
3. Divide $\frac{1}{17}$ by $\frac{2}{3}$. Ans. $\frac{3}{34}$.
4. Divide $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{4}$ of $\frac{5}{6}$. Ans. $\frac{2}{5}$.
5. Divide $\frac{2}{16}$ by $4\frac{1}{2}$. Ans. $\frac{1}{36}$.
6. Divide 16 by 24. Ans. $\frac{2}{3}$.
7. Divide $3\frac{1}{2}$ by $9\frac{1}{2}$. Ans. $\frac{1}{3}$.
8. Divide $672\frac{3}{4}$ by $13\frac{1}{2}$. Ans. $48\frac{1}{2}$.
9. Divide 5 by $\frac{7}{10}$. Ans. $7\frac{1}{7}$.
10. Divide $\frac{5}{8}$ of 19 by $\frac{2}{3}$ of $\frac{1}{4}$. Ans. $7\frac{3}{5}$.

RULE of THREE DIRECT in VULGAR FRACTIONS.

RULE.

1. State the question, and prepare the terms as taught in Reduction, whole numbers.
2. Invert the first term, and place it so inverted, with the second and third terms, like a multiplication question; and proceed according to the multiplication rule; the answer will be of the same name with the second term.

multiply the three terms together, and the product will be the answer.

EXAMPLES.

1. If $\frac{5}{8}$ of a gallon cost $\frac{5}{8}$ of a pound what will $\frac{5}{8}$ of a ton cost? Ans. 140 $\frac{1}{2}$.

OPERATION.

gal.	£.	Tons	ton.
Stating $As \frac{5}{8}$	$;$	$\frac{5}{8}$	$;$
		$;$	$\frac{5}{8}$
			$;$
			$\frac{5}{8}$

$$\frac{8' \times 5' \times 5 \times 4 \times 63}{5' \times 8' \times 9 \times 1 \times 1} = 140\frac{1}{2}^+$$

NOTE. After stating the question, as above, draw a line; and invert the first term; place it so inverted, $\frac{8}{5}$ and then place the second and third terms, next to it, viz. $\frac{5}{8}$ and $\frac{5}{8}$; the third term being tons it is necessary to reduce it to gallons, that it may be of the same name with the first term; it must therefore be multiplied by 4 and 63, because 4 hhd. make a ton, and 63 gallons a hog-

* Without abbreviating, thus; $\frac{2}{3} \times \frac{3}{7} = \frac{6}{21}$; and $\frac{2}{7} \times \frac{1}{9} = \frac{2}{63}$; then invert the divisor, $\frac{2}{63}$, and multiply the fractions together, thus; $\frac{6}{21} \times \frac{63}{2} = \frac{378}{42}$, and the numerator 378 divided by the denominator 42, gives 9 for the answer. — The other answers are in the lowest terms.

Division of vulgar fractions is performed in a very plain and simple manner, by first reducing the divisor and dividend to a common denominator, thus; to divide $\frac{6}{21}$ by $\frac{2}{63}$, multiply $\frac{6}{21}$ by 3, and it is $\frac{18}{63}$; then nothing more is required to be done than to divide 18 by 2, and the quotient 9 is the answer; and so of any other fraction.

+ By the common rule thus; $\frac{5}{9} \times \frac{63}{1} = \frac{315}{9}$ of a gallon, if 63 gallons were a ton, but as 4 hogsheads make a ton, $\frac{315}{9} \times \frac{4}{1} = \frac{1260}{9}$ of a gallon is the 3d term; therefore, $\frac{8}{5} \times \frac{5}{8} = \frac{40}{40} = 1$, and $\frac{1260}{9} \times 1 = \frac{1260}{9}$ of a £, that is £ 140.

head; 4 and 63 must therefore be placed as multipliers after the third term; the upper and under figures being cut as before taught, gives the answer 140l. with very little trouble.

ANOTHER EXAMPLE.

2. If $\frac{3}{4}$ of a yard cost $\frac{2}{3}$ of a pound, what cost $\frac{8}{9}$ of an English ell. *Ans.* $\frac{2}{3}$ l. or 13s. 4d.

$$\begin{array}{r} \text{If } \frac{3}{4} \text{ of a yard cost } \frac{2}{3} \text{ of a pound, what cost } \frac{8}{9} \text{ of an English ell.} \\ \frac{3}{4} \text{ of a yard} \quad \frac{2}{3} \text{ of a pound} \quad \frac{8}{9} \text{ of an English ell.} \\ \frac{3}{4} \times \frac{2}{3} = \frac{2}{3} \times \frac{20}{3} = \frac{1 \times 12}{3} = 4 \end{array}$$

In the above example English ells must be reduced to yards; placing therefore, 5 above the line and 4 below it, which is the same as multiplying by 5 and dividing by 4; reduces the first and third terms into one name, viz. yards.

3. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a shilling, what will $\frac{7}{8}$ of a yard cost? *Ans.* 2s. 4d.

4. If $\frac{3}{8}$ of a yard cost $\frac{1}{2}$ of a pound what cost $\frac{2}{3}$ of a yard? *Ans.* 16s. 8d.

5. If $2\frac{1}{2}$ yards cost 3l. what cost $4\frac{1}{4}$ yards? *Ans.* 6l. 6s. 8d.

6. If 1 cwt. cost $\frac{3}{4}$ l. what will $\frac{1}{4}$ cwt. cost? *Ans.* 13s. 4d.

7. If $2\frac{1}{4}$ yards cost $2\frac{1}{2}$ shillings what will $3\frac{3}{4}$ yards cost? *Ans.* 4s.

8. If $\frac{3}{8}$ of an ell cost $\frac{2}{3}$ l. what will $\frac{5}{8}$ of an ell cost? *Ans.* 6s. 8d.

9. If $\frac{3}{4}$ of a yard cost $\frac{7}{8}$ l. what will $\frac{6}{7}$ of an ell English cost? *Ans.* 9s. 8d. $\frac{2}{3}$.

10. What is the value of an ounce when $\frac{2}{3}$ of an ounce cost $\frac{1}{4}$ l.? *Ans.* 1l. 5s. 8d.

11. A person having $\frac{3}{4}$ of a coal mine, sells $\frac{1}{4}$ of his share for 171l.; what is the value of the whole mine? *Ans.* 380l.

12. Bought $3\frac{1}{2}$ pieces of cloth each piece $24\frac{1}{2}$ yards, what is the value of the whole at 6s. od. $\frac{1}{2}$ per yard? *Ans.* 25l. 14s. 6d. $\frac{1}{2}$.

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13. If $\frac{3}{10}$ of a ship cost 273l. 2s. 6d. what is $\frac{5}{12}$ of her worth? *Ans.* 227l. 12s. 1d.
14. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a pound, what will $\frac{2}{3}$ of an ell English come to? *Ans.* 2l.
15. If 1 yard cost 6s. 6d. $\frac{2}{3}$ what cost $3\frac{1}{2}$ pieces each 24 $\frac{2}{3}$ yards? *Ans.* 25l. 17s. 2d. $\frac{1}{4}$ — $\frac{5}{16}$.
16. If $\frac{1}{6}$ of a ship be worth 73l. 1s. 3d. what share of her may I buy for 250l. 10s.? *Ans.* $\frac{3}{7}$ of her.

RULE of THREE INVERSE in VULGAR FRACTIONS.

R U L E.

1. State the question and prepare the terms, as in the rule of three direct.
2. Invert the third term (*instead of the first*) and proceed as in the last rule.

E X A M P L E S.

1. How much shalloon $\frac{2}{3}$ yard wide, will line 9 $\frac{1}{2}$ yards of cloth $2\frac{1}{2}$ yards wide? *Ans.* 31 $\frac{2}{3}$ yards.

First $9\frac{1}{2} = \frac{19}{2}$; and $2\frac{1}{2} = \frac{5}{2}$;
yds. wide. yds. long. yds. wide.

Then—If $\frac{5}{2} : \frac{19}{2} :: \frac{2}{3} : x$

$$\frac{5 \times 19 \times \frac{2}{3}}{2 \times \frac{19}{2} \times 3} = \frac{19}{3} = 3\frac{1}{3} \text{ yards } Ans.$$

2. How many yards of cloth $\frac{3}{8}$ yard wide, are equal to 3 $\frac{3}{4}$ yards of cloth $1\frac{1}{2}$ yard wide? *Ans.* 9 yards.

3. How many yards of baize $\frac{3}{4}$ yards wide will line a cloak containing 18 $\frac{7}{8}$ yards of cloth $\frac{1}{2}$ yard wide? *Ans.* 11 yds. 1 qr. 1 $\frac{1}{2}$ na.

4. What quantity of shalloon $\frac{1}{4}$ yards wide, will line 7 $\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yards wide? *Ans.* 15 yards.

5. If 48 men can build a wall in 24 $\frac{1}{2}$ days—how many men can do the same in 192 days? *Ans.* 6 $\frac{1}{6}$ men.

6. If 25 $\frac{2}{3}$ s. will pay for the carriage of a cwt. 145 $\frac{1}{4}$ miles—how far may 6 $\frac{1}{4}$ cwt. be carried for the same money? *Ans.* 22 $\frac{2}{3}$ miles.

7. If 3 $\frac{1}{4}$ yards of cloth, that is 1 $\frac{1}{4}$ yard wide, be sufficient to make a cloak—how much must I have of

By the Common
method.

$$\frac{5}{2} \times \frac{19}{2} = \frac{95}{2} \times \frac{2}{3} = \frac{95}{3} = 31\frac{2}{3},$$

or 31 $\frac{2}{3}$ yards, *Ans.*

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that sort which is $\frac{4}{5}$ yard wide, to make another of the same bigness? *Ans.* $4\frac{7}{8}$ yards.

8. If 3 men can do a piece of work in $4\frac{1}{2}$ hours—in how many hours will ten men do the same work? *Ans.* $1\frac{2}{5}$ hour.

9. If a penny ~~white~~ loaf weigh 7 oz. when a bushel of wheat cost 5s. 6d.—what is the bushel worth when the penny ~~white~~ loaf weighs but 2 oz. $\frac{1}{2}$? *Ans.* 15s. $4\frac{1}{2}$ d.

COMPOUND PROPORTION IN VULGAR FRACTIONS.

R U L E.

Prepare the Fractions, and then proceed according to the GENERAL RULE (page 116.)

(Three. II. is the Double Rule of

E X A M P L E S.

1. If 600l. principal gain 33 $\frac{1}{2}$ l. interest in 10 $\frac{2}{3}$ months; in what time will 100l. gain 6 $\frac{1}{4}$ l? *Ans.* 1 year. *

2. If 15 men earn 2l. 10s. in 6 days; how much will 12 men earn in 18 $\frac{1}{8}$ days? *Ans.* 6l. 2s. 2 $\frac{1}{2}$ d.— $\frac{3}{4}$

3. If a carrier receives 2 $\frac{1}{16}$ l for the carriage of 3 cwt. 150 miles—how much ought he to receive for the carriage of 7 cwt. 3 $\frac{1}{2}$ qrs. 50 miles? *Ans.* 1l. 16s. 9d.

4. If 100l. in 12 months gain 6l. interest—what principal will 3 $\frac{1}{2}$ l. in 9 months? *Ans.* 75.

5. If 9 students spend 10 $\frac{1}{2}$ l. in 18 days—how much will 20 students spend in 30 days? *Ans.* 39l. 18s. 4d. $\frac{20}{9}$.

6. A man and his wife having laboured one day, earned 4 $\frac{3}{4}$ s.—how much must they have for 10 $\frac{1}{2}$ days, when their two sons helped them? *Ans.* 4l. 17s. 1 $\frac{1}{2}$ d.

7. If 50l. in 5 months gain 2 $\frac{3}{4}$ l.—what time will 13 $\frac{1}{2}$ l. require to gain 1 $\frac{1}{2}$ l.? *Ans.* 9 months.

8. If the carriage of 60 cwt. 20 miles cost 14 $\frac{1}{2}$ l.—what weight can I have carried 30 miles for 5 $\frac{1}{4}$ l? *Ans.* 15 cwt.

* The Operation.

First. $\pounds 600 = \pounds \frac{600}{1}$; and $\pounds 33\frac{1}{2} = \pounds \frac{100}{3}$; and $10\frac{2}{3}$ mo. = $\frac{32}{3}$ mo.; and $\pounds 100 = \pounds \frac{100}{1}$; and $\pounds 6\frac{1}{4} = \pounds \frac{25}{4}$; all being expressed in fractions.

Secondly. The statement and work are as follows.

$$\left. \begin{array}{l} \frac{\pounds}{100} \\ \frac{1}{1} \end{array} \right\} \begin{array}{l} M. \\ \frac{32}{3} \end{array} \left\{ \begin{array}{l} \frac{\pounds}{600} \\ \frac{1}{1} \\ \frac{25}{4} \end{array} \right.$$

Explanation.—The answer being required in months, or time, the $10\frac{2}{3}$ mo. = $\frac{32}{3}$ is the middle term; then as it will require more time for $\pounds 100$ to gain $\pounds \frac{100}{3}$ than it will for $\pounds 600$, the $\pounds \frac{100}{1}$ is put for the 1st term, and the $\pounds \frac{600}{1}$ for the 3d. Again, as $\pounds 6\frac{1}{4}$ requires less time to be gained in than $\pounds 33\frac{1}{2}$, the less number is put for the 3d term, and

S Q U A R E R O O T.

the greater for the 1st, in the 2d statement. Next, as the product of the terms at the left hand of the middle term is the divisor, insert those terms, that is, the numerators and denominators, and multiply all the terms continually together, and the product will be the answer in a fraction of the same name as the middle term.

Questions for Examination.

1. What is a vulgar fraction?
2. What is the greatest common ^{measure} of two or more numbers?
3. What is the rule for finding the greatest common measure?
4. What is the least common multiple of two or more numbers?
5. How do you find the least common multiple?
6. How is a vulgar fraction reduced to its lowest terms?
7. How do you reduce a mixed number to an improper fraction?
8. How do you reduce a compound fraction to a single one?
9. How are fractions reduced to a common denominator?
10. How do you find the value of a vulgar fraction?
11. How do you add? — Subtract? — Multiply? — And divide, vulgar fractions?
12. How is the rule of three direct performed in vulgar fractions? — The rule of three inverse?
13. How is the double rule of three or compound proportion in vulgar fractions performed?

T H E S Q U A R E R O O T.

The Square Root is a number which multiplied by itself produces a square.

To extract the Square Root of a given number, is to find such a number as being multiplied by itself produces the given number, as follows.

Table.	{	Roots,	1.	2.	3.	4.	5.	6.	7.	8.	9.
	{	Squares,	1.	4.	9.	16.	25.	36.	49.	64.	81.*

To extract the Square Root.

Rule. — 1. Separate the given number whose root is to be extracted, into periods of two figures each, by placing a point over units, and every other figure towards the left hand, and also over every other figure to the right hand, when there are decimals in the number.

2. Find a root whose square is either the whole of the

* Raising powers, that is multiplying numbers by themselves, and the products again by those numbers, is called involution. Extracting the roots of numbers is called evolution. When the root of a number can be exactly found, the number is called a complete power. But when the root cannot be exactly found, by reason of remainders, the number is called a surd.

SQUARE ROOT.

First or left hand period, or the nearest less number,
and place the root in the quotient. Z

3. Subtract the square of the root, that is, ^{the first} figure of ^{it}, from the first period, and to the remainder bring down the next period, for another dividend.

4. Double the root, and place it on the left hand of the dividend for a divisor.

5. Find how often the divisor is contained in the dividend excepting the right hand figure, and place the number in the quotient for another figure of the root, and likewise place the same figure to the right hand of the divisor; Multiply the divisor with the figure annexed, by the last figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend

6. Double the figures of the root already found, for the next divisor, and proceed as before, and so on for every period, till the whole root is extracted. ‡

Note.—When the number of figures divided into periods is odd, the left hand period will consist of but one figure. — Also, when the number of decimal figures is odd, they must be made even by annexing a cipher.

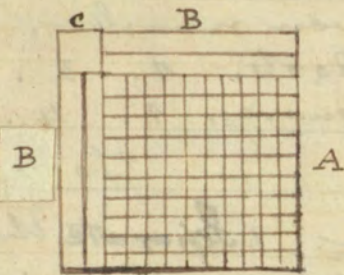
Proof.—Square the whole root, that is, multiply it by itself, and the square or product will be equal to the number whose root is extracted, if the work is right.

‡ When the square root is to be extracted to a great number of places, the work may be greatly abbreviated by dividing the last remainder by the last divisor, annexing a cipher each time you divide, as in decimals: the last divisor with the last figure of the root annexed, answering for the whole. This method should not be used till at least one more than half the number of figures required in the whole root, are extracted by the common rule.

Reason of the Rule.

The reason why a number has a point over the first or right hand figure of each period, is to direct what figures to bring down, without counting them. The reason why two figures make a period, and must be brought down at a time, instead of one, is because that the square of any figure of the root except 3, consists of two figures, and two figures cannot be subtracted from one. But when the left hand period is but one figure, which is often the case, then the square of its greatest root will be but one figure, and can therefore be subtracted from its period.

The reason why we point from the unit's place towards the left hand, in whole numbers, instead of beginning at the left hand and pointing towards the right, is because that when the number of figures is even, to begin to point at the left hand, would be making the number whose root is wanted, ten times less, and would cause a point to fall on the first place of decimals, which it should not. But, when the number of figures is odd, it is immaterial at which hand you begin to point.



The reason why the root is doubled for a divisor, is because that if it were not doubled, the quotient figure or root would express a number of some denomination, which if placed all on one side of a square, would make ~~make~~ the piece longer than it would be wide, but by dividing by double the root, the quotient being half what it would be if the root were the divisor, is reckoned twice, that is, it is placed both on the end and side of a square, which keeps the length and breadth equal. For example, suppose there is a piece of board 144 inches long and 1 inch wide, which is to be brought into a piece exactly square, if the square root of 144 be extracted, it is 12 which shows that the required square piece of board is 12 inches long, and 12 inches wide. The annexed figure represents such a piece. The figure 1, of the root 12, is one 10, because the square root of 100 is 10, therefore extracting the first figure of the root reaches 100 square inches to a square form, 10 inches long and 10 wide, represented by the large square A. Now after subtracting the root figure 1 from its period, there remain 44 square inches to be added to the square A, in such a proportion as to make the whole square at last. To do this, the figure 1 of the root is doubled for a divisor, and it makes 2, which is contained twice in 44, omitting the right hand figure: this figure of the root 2, is a piece 2 inches wide, and 10 long, making 20 square inches, represented by the pieces B B in the figure, both making 40 square inches, B being taken twice because the quotient is but half as much as if the root had not been doubled for a divisor. There now remains only the small square c to be filled up, and as this is 2 inches long and 2 inches wide, equal to 4 square inches, the square of the last figure of the root 2, exactly completes the whole. For placing the square of the last quotient figure under the unit's place, and multiplying it when placed to the right hand of the divisor, is exactly the same thing; observing to carry one for every ten to the divisor in multiplying that, when the square of the last quotient figure is more than ten.

SQUARE ROOT.

Examples.

The three first are easy questions to be solved mentally, and will show the nature of the rule.

1. What is the square root of 9? Ans. 3.

Solution.—Find a number which multiplied by itself, the product will be 9, and the number is 3.

2. What is the square root of 64? Ans. 8, because 8 times 8 are 64.

3. What is the square root of 144? Ans. 12, because the square of 12, or 12 multiplied by 12, is 144.

4. What is the square root of 152399025?

The Operation. Ans. 12345.

$$\begin{array}{r}
 15\ 2\ 3\ 9\ 9\ 0\ 2\ 5\ (12345^* \\
 2\ 2\ \overline{)5\ 2} \\
 \underline{4\ 4} \\
 2\ 4\ 3\ \overline{)8\ 3\ 9} \\
 \underline{7\ 2\ 4} \\
 2\ 4\ 6\ 4\ \overline{)1\ 1\ 0\ 9\ 0} \\
 \underline{1\ 0\ 8\ 8} \\
 2\ 5\ 6\ 8\ 5\ \overline{)1\ 2\ 3\ 4\ 2\ 5} \\
 \underline{1\ 2\ 3\ 4\ 2\ 5} \\
 \dots\dots\dots
 \end{array}$$

Explanation.—Here, after pointing the given number into periods, the root of the left hand period 1, is 1, which is placed in the quotient. Then $1 \times 1 = 1$, which subtracted from the first period leaves nothing; the next period 52 is next brought down, and the root doubled which makes 2 for a divisor; then 2 will go in 5 twice, therefore place 2 in the quotient for the next figure of the root, and place the same to the right hand of the divisor; which makes 22, which multiplied by 2, the root or quotient figure, produces 44, which subtracted from 52, leaves 8, to which the next period 39, is brought down, and 839 is a new dividend to be divided by double the root found, (12) which makes 24; then the next figure of the root, 3 is found in the same manner as the 2, and so on to the last. Having found the whole root, multiply it by itself, and the product will be 152399025, which proves the work to be right.

* As there is no remainder in this example, the number whose root is extracted is a complete power. If there had been a remainder, pairs of ciphers must have been annexed, and the operation continued would have given the root in whole numbers and decimals. The number of decimals would be equal to the periods of ciphers added. Or, when there is a remainder, double the root placed under it will be the true denominator of the fraction nearly.

SQUARE ROOT.

5. What is the square root of 184,2? Ans. 13,57+.

Operation.

$$\begin{array}{r}
 184,20(13,57 \\
 23 \overline{)84} \\
 \underline{69} \\
 265 \overline{)1520} \\
 \underline{1325} \\
 2707 \overline{)19500} \\
 \underline{18949} \\
 551 \text{ remainder.}
 \end{array}$$

In working this sum, as it is called, one cipher is added to the decimal 2, at the first step, because, in decimals, two figures are always required to make a period. After extracting three figures of the root, two more ciphers are added, so as to obtain another decimal in the root, and the operation might be thus continued to any length, if necessary.

6. What is the square root of 106929? Ans. 327
7. What is the square root of 2268741? Ans. 1506,23+.
8. What is the square root of 7596796? Ans. 2756,228+.
9. What is the square root of 36372961? Ans. 6031.
10. What is the square root of 22071204? Ans. 4698.

When the given number consists of a whole number, and decimals together, make the number of decimals even, by adding cyphers to them; so that there

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may be a point fall on the units place of the whole number.

11. $\rightarrow$ What is the square root of 3271,4207? Ans. 57,19+.
12. $\rightarrow$ What is the square root of 4795,2573? Ans. 69,247+.
13. $\rightarrow$ What is the square root of 4,372594? Ans. 2,091+.
14. $\rightarrow$ What is the square root of 2,2710957? Ans. 1,50701+.
15. $\rightarrow$ What is the square root of ,00032754? Ans. ,01809+.
16. $\rightarrow$ What is the square root of 1,270054? Ans. 1,1269+.

To extract the SQUARE ROOT of a VULGAR FRACTION.
R U L E.

Reduce the fraction to its lowest terms; then extract the square root of the numerator for a new numerator; and the square root of the denominator for a new denominator.

If the fraction be a surd (i. e.) a number where a root can never be exactly found, reduce it to a decimal, and extract the root of it.

E X A M P L E S.

1. $\rightarrow$ What is the square root of $\frac{2104}{3784}$? Ans. $\frac{2}{3}$.
2. $\rightarrow$ What is the square root of $\frac{2704}{4225}$? Ans. $\frac{2}{3}$.
3. $\rightarrow$ What is the square root of $\frac{9216}{12544}$? Ans. $\frac{6}{7}$.

S U R D S.

4. $\rightarrow$ What is the square root of $\frac{275}{346}$? Ans. ,89931+.
5. $\rightarrow$ What is the square root of $\frac{352}{475}$? Ans. ,86602+.
6. $\rightarrow$ What is the square root of $\frac{75}{346}$? Ans. ,93308+.

To extract the square root of a mixed number.

R U L E.

1. Reduce the fractional part of the mixed number to its lowest term, and then the mixed number to an improper fraction.
2. Extract the roots of the numerator and denominator for a new numerator, and denominator.

S Q U A R E R O O T .

If the mixed number given be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the square root therefrom.

E X A M P L E S .

1. ~~19.~~ What is the square root of $51\frac{2}{3}$? *Ans.* $7\frac{1}{2}$.
2. ~~20.~~ What is the square root of $2\frac{9}{16}$? *Ans.* $1\frac{3}{4}$.
3. ~~21.~~ What is the square root of $9\frac{1}{9}$? *Ans.* $3\frac{1}{3}$.

S U R D S .

4. ~~22.~~ What is the square root of $85\frac{1}{4}$? *Ans.* $9,27\frac{1}{2}$.
5. ~~23.~~ What is the square root of $8\frac{1}{2}$? *Ans.* $2,9519\frac{1}{2}$.
6. ~~24.~~ What is the square root of $6\frac{2}{3}$? *Ans.* $2,5819\frac{1}{2}$.

T H E A P P L I C A T I O N .

1. There is an army consisting of a certain number of men, who are placed rank and file (that is, in the form of a square, each side having 576 men) I desire to know how many the whole square contains?

Ans. 331776.

2. A certain pavement is made exactly square, each side of which contains 97 feet — I demand how many square feet are contained therein? *Ans.* 9409.

To find the mean proportional between any two given numbers.

R U L E .

The square root of the product of the given numbers is the mean proportional sought.

E X A M P L E S .

1. ~~25.~~ What is the mean proportional between 3 and 12? *Ans.* 6.
2. ~~26.~~ $3 \times 12 = 36$ then $\sqrt{36} = 6$ the mean proportional.*
What is the mean proportional between 4276 and 842? *Ans.* 1897,44.

To find the side of a square equal in area to any given superficies.

R U L E .

The square root of the content of any given superficies, is equal to the side of the square sought.

E X A M P L E S .

1. ~~27.~~ If the content of a given circle be 160 — what is the side of a square? *Ans.* 12,64911.

*Square Root.
use of the*

* A mean proportional between two numbers is a number as many times greater than the least of the two given numbers as it is times less than the greater. Thus, 3 is a mean proportional between 1 and 9, because it is 3 times greater than 1, and 3 times less than 9.

SQUARE ROOT. 255

30. If the area of a circle is 750—what is the side of the square equal to? *Ans. 27,38612.*

The area of a circle given to find the diameter.

R U L E.

As 355:452, or, as 1:1,273239:: the area to the square of the diameter:—or, multiply the square root of the area, by 1,12837 and the product will be the diameter.

E X A M P L E.

31. What length of cord ~~will be fit to tie to an~~ *animal's* cow's tail, the other end fixed in the ground, ~~to let~~ *will permit the animal* her have liberty of eating an acre of grass, and no more, supposing the ~~and~~ and tail to be $5\frac{1}{2}$ yards? *Ans. 6,136 perches.*

The area of a circle given to find the periphery or circumference.

R U L E.

As 113:1420, or, as 1:12,56637:: the area to the square of the periphery:—or, multiply the square root of the area by 3,5449 and the product is the circumference.

E X A M P L E S.

32. When the area is 12—what is the circumference? *Ans. 12,2798.*

33. When the area is 160—what is the periphery? *Ans. 44,839.*

Any two sides of a right-angled triangle given to find the third side.†

1. The base and perpendicular given to find the hypotenuse.

R U L E.

The square root of the sum of the squares of the base and perpendicular is the length of the hypotenuse.

E X A M P L E S.

34. The top of the castle from the ground is 45 yards high and surrounded with a ditch 60 yards broad; what length must a ladder be to reach from the outside of the ditch to the top of the castle? *Ans. 75 yards.*

† A right-angled triangle is a three sided figure, having three angles or corners, one of which is a square corner. The longest side, or that opposite to the right angle or square corner, is called the hypotenuse. The next longest side is called the base; and the shortest is called the perpendicular.

S Q U A R E R O O T

Ditch.

Base 60 yards.

35. The wall of a town is 25 feet high, which is surrounded by a moat of 30 feet in breadth—I desire to know the length of a ladder that will reach from the outside of the moat to the top of the wall?

Ans. 39,05 feet.

2. The hypotenuse and perpendicular given to find the base.

R U L E.

The square root of the difference of the squares of the hypotenuse and perpendicular is the length of the base.

3. The base and hypotenuse given to find the perpendicular.

R U L E.

The square root of the difference of the square of the hypotenuse and base is the height of the perpendicular.

~~The~~ The two last questions may be varied for examples to the two last propositions.

Any number of men being given to form them into a square ~~battalion~~, or to find the number of rank and file.

R U L E.

The square root of the number of men given, is the number of men either in rank or file.

36. An army consisting of 331776 men—I desire to know how many rank and file? *Ans.* 576.

battalion,

I Questions for Examination.

1. What is the square root?
2. What is the square of a number?
3. What is the rule for extracting the square root?
4. How is the extraction of the square root proved?

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6. Multiply all the figures of the root or quotient, except the last, by the square of the last, and that product by 30, and place the product under the last; then under these two products place the cube of the last quotient figure, and add all three numbers together, and call their sum the subtrahend.

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend; then find a divisor and proceed in the same manner as before till the whole root be extracted, annexing periods of ciphers to the remainder, for finding decimals, when necessary.*

If the subtrahend happens to be greater than the dividend, which is sometimes the case, make the last quotient figure one less, with which find a new subtrahend, and then subtract.

Proof.—When there is no remainder, the cube of the quotient or root, will be equal to the given number whose root is extracted.

* There are many different methods for extracting the cube root, but in none of them does the divisor appear to be found in so simple and easy a manner as in the rule here given. Nevertheless, as there is another rule which has formerly been much used, and which some may wish inserted, here it is.

Rule. Proceed as in the first rule, till you find the divisor; to find that, multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

Seek how often the divisor may be had in the resolvend, or dividend, and place the result in the quotient.

Multiply the triple square by the last quotient figure, and place the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and write this product under the last; under both set the cube of the last quotient figure, and the sum of all three numbers is the subtrahend, which subtract, and proceed as before, till the whole root is extracted.

Another Rule.—The following rule is

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Examples.

The two first may be solved mentally.

1. What is the cube root of 8? Ans. 2; because twice 2 are 4, and twice 4 are 8.

2. What is the cube root of 27? Ans. 3; because 3 times 3 are 9, and 3 times 9 are 27.

Method in Hutton's Mathematics, and with a little variation, in Webber's.
1. Find by trials, the nearest cube to the whole given number, whether it be greater or less, and call it the assumed cube.

2. Then say by the Rule of Three, as the sum of the given number and double the assumed cube, is to the sum of the assumed cube and double the given number, so is the root of the assumed cube to the root required, nearly. Then, by using the ^{cube of the} root so found, for the assumed cube, and repeating the operation, another root more exact than the first will be obtained, and so on, to any degree of accuracy.

Another Method. - The following rule is given in Webber's Mathematics, excepting that ^{it} is made rather plainer here than he has given it.

1. Proceed as in rule first, till you find the divisor. To find ^{proceed thus:} that, to three times the square of the root (as far as it is found) add three times the root itself, setting the root one place more to the right hand, and call their sum the divisor, by which divide the dividend or resolvend, except the last figure, and place the quotient figure for the next figure of the root.

2. Add together these three products, namely, thrice the square of all the figures of the root except the last, multiplied by the last; thrice all the figures of the root except the last, multiplied by the square of the last; and the cube of the last, setting each of them one figure further to the right than the former, and call their sum the subtrahend, which subtract, and proceed as before.

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3. What is the cube root of 1728?

The Operation.

Ans. 12.

$$\begin{array}{r}
 1728(12 \\
 300 \overline{) 728} \text{ resolvent.} \\
 \underline{600} \\
 120 \\
 \underline{8} \\
 728 \text{ subtrahend.}
 \end{array}$$

$$\begin{array}{r}
 2 \\
 \underline{2} \\
 4 \text{ square of 2.} \\
 \underline{30} \\
 120
 \end{array}$$

Explanation. In the first place the given number is pointed into periods, as the first rule directs. 2. As the first period is 1, the cube root of it is 1, because if 1 be multiplied by itself, the product will be but 1. Third, the root or quotient figure 1, is subtracted from its period, and the next period is brought down. 4. As the square of 1 is but 1, 300 is the divisor, for 1 multiplied by 300 is 300. Then 300 is contained in 728, two times, and twice 300 is 600. Next as all the root except the last figure, is but 1, multiplying any number by it will neither make it more or less, therefore square the last figure 2 and multiply the square by 30, and the number produced is 120, and the cube of 2 is 8, and the sum of 600, 120, and 8, is 728, which subtracted from the resolvent, leaves no remainder.

To prove the work, cube 12, and it will be 1728, which shows the work to be right.

[Reason of the Rule. — The reason why three figures make a period, is because the cube of any figure except 4, consists of three figures, therefore the period must consist of as many figures, as the cube of any single figure which may happen to be a root.

The reason of the operation of extracting the cube root may be explained by the third example, thus.

1. Take eight blocks of wood, or of any other substance, of the following dimensions, namely; the 1st, 10 inches long, 10 wide, and 10 high; the 2d, 3d, and 4th, 10 inches square and 2 inches thick, each; the 5th, 6th, and 7th, 10 inches long, and 2 inches square at each end; and the 8th, a little block 2 inches square on

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each side. These eight blocks together contain 1728 solid inches, and if put together in the following manner, will make a block 12 inches long, 12 wide, and 12 high, thus. First take the 2d, 3d, and 4th, and put one on the top, and one on each of two sides. Next take the 5th, 6th, and 7th, and place them so as to fill up the three long vacant corners made by putting on the 2d, 3d, and 4th pieces. Then there will be but one little space to fill up, viz., between the ends of the 5th, 6th, and 7th pieces, which is filled by the 8th block or little cube, and the whole is completed.[†]

Now extracting the root of the first period of 1728, reduces 1000 solid inches to the 1st block; multiplying the divisor by the second quotient figure (2), gives the content of the blocks or pieces 2d, 3d, and 4th; the square of the last quotient figure multiplied by 30, gives the content of the 5th, 6th, and 7th blocks or pieces; and the cube of the last figure is the content of the 8th block or little cube.

Having shown how to construct and put together the eight blocks which compose a cube, in such a proportion as the rule for extracting the cube root requires, I now proceed to show the reason for dividing by 300 times the square of the quotient, and why the square of the last quotient figure is multiplied by all the other figures of the root, and that product by 30, and that the reason may appear as plain as possible, the number 1728 is divided into its component parts as follows.

$$\begin{array}{r} 1000 \text{ (10) } 12. \\ 300 \overline{) 600} \text{ (2) } \\ 120 = 2 \times 2 \times 30, \text{ or } 2 \times 2 \times 10 \times 3. \\ 8 = 2 \times 2 \times 2. \end{array}$$

Here the cube root of 1000 is 10, and as the square of 10 is 100, and 3 pieces 10 inches square are wanted to put on 3 sides of the 10 inch block, 100×3 , or 300 must be the divisor for finding how many ^{whole} inches thick each piece must be, which in this question is 2, because 300 is contained but twice in 728: so twice 300 disposes of 600 solid inches more of the 1728. Now the reason for dividing by 300 times the square of 1, is because the left hand figure (1) of the root of 1000, is used

[†] As a cube whose side is 12 inches is larger than is necessary for use, any convenient parts of an inch, as quarters or tenths will do just as well.

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according to the rule, instead of the whole root 10, and 300 times the square of 1 is the same as 3 times the square of 10, and the number of times 300 times the square of 1 (or the first figure of the root) is contained in 728; viz. 2, added to the root of 1000, makes 12, the whole cube root of 1728.

But it is to be considered that, although the whole root is found in this example, after dividing by 300 times the square of the quotient or first figure of the root, yet the work is not done till the the divisor is multiplied by the last quotient figure, &c. Now the last quotient figure is squared, to find the superficial content of the end of the 5th, 6th, or 7th block, and the square is multiplied by the figure 1 (all the rest of the root) because each block, 5, 6, and 7, is the length of one side of the first or large block, viz. 10; and as the square of 2 is 4, this multiplied by the length 10, produces 40, and as there are 3 such pieces, $40 \times 3 = 120$ solid inches in all three of the long blocks, 5, 6, and 7. But, instead of 10, the cube root of 1000, the rule directs, for greater convenience, to multiply the square of the last figure of the root by the left hand figure only of the cube root of 1000 in this example, viz. 1, and as 1 is ten times less than 10, the product must be multiplied by ten times more than 3, viz. 30, to produce the same number. For, $4 \times 1 \times 30 = 120$, and $4 \times 10 \times 3 = 120$, which shows the reason of that part of the rule.

The reason why the cube of the last figure of the root completes the whole cube at the last step, is obvious from a view of the whole cube after the 8 blocks are put together as directed.

As a drawing or representation of the blocks on paper, convey to the mind rather a confused, than a clear idea of them, I have omitted it, and endeavoured to supply the omission by substituting plain directions in its stead.]

4. What is the cube root of 1000000?

Ans. 100

5. What is the cube root of 84604519? Ans. 439.
6. What is the cube root of 259694072? Ans. 638.
7. What is the cube root of 48228544? Ans. 364.
8. What is the cube root of 27054036008? Ans. 3002.

9. What is the cube root of 22069810125? Ans. 2805.
10. What is the cube root of 12261532732? Ans. 4968.
11. What is the cube root of 219365327791? Ans. 6031.
12. What is the cube root of 673373097125? Ans. 8765.

When the given number consists of a whole number, and decimal together, make the number of decimals consist of 3, 6, 9, &c. places, by adding cyphers thereto, so that there may be a point fall on the units place of the whole number.

13. What is the cube root of 12,977875? Ans. 2,35.
14. What is the cube root of 36155,027576? Ans. 33,06+.
15. What is the cube root of ,001906624? Ans. ,124.

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16. What is the cube root of 33,230,979,637? *Ans.* 3,215+.
 17. What is the cube root of 159,26,972,504? *Ans.* 25,16+.
 18. What is the cube root of 053,157,376? *Ans.* 376.

To extract the cube root of a Vulgar Fraction.

R U L E.

Reduce the fraction to its lowest terms, then extract the cube root of the numerator and denominator, for a new numerator and denominator; but if the fraction be a surd, reduce it to a decimal, and then extract the root from it.

E X A M P L E S.

19. What is the cube root of $\frac{250}{8}$? *Ans.* $\frac{5}{2}$.
 20. What is the cube root of $\frac{32}{1000}$? *Ans.* $\frac{2}{10}$.
 21. What is the cube root of $\frac{152}{3136}$? *Ans.* $\frac{2}{7}$.

SURDS.

22. What is the cube root of $\frac{4}{7}$? *Ans.* .829+.
 23. What is the cube root of $\frac{2}{3}$? *Ans.* .822+.
 24. What is the cube root of $\frac{2}{3}$? *Ans.* .873+.

To extract the cube root of a mixed number.

R U L E.

Reduce the fractional part to its lowest terms, and then the mixed number to an improper fraction, extract the cube roots of the numerator and denominator for a new numerator and denominator; but if the mixed number given be a surd, reduce the fractional part to decimals, annex it to the whole number, and extract the root therefrom.

E X A M P L E S.

25. What is the cube root of $12\frac{1}{2}$? *Ans.* 2 $\frac{1}{3}$.
 26. What is the cube root of $31\frac{1}{4}$? *Ans.* 3 $\frac{1}{4}$.
 27. What is the cube root of $405\frac{2}{3}$? *Ans.* 7 $\frac{2}{3}$.

SURDS.

28. What is the cube root of $7\frac{1}{2}$? *Ans.* 1,93+.
 29. What is the cube root of $9\frac{1}{4}$? *Ans.* 2,09+.
 30. What is the cube root of $8\frac{1}{2}$? *Ans.* 2,057+.

THE APPLICATION *and Use of the Cube Root.*

1. If a cubical piece of timber be of 47 inches long, 47 inches broad, and 47 inches deep—how many cubical inches doth it contain? *Ans.* 103823.

2. There is a cellar dug, that is 12 feet every way, in length, breadth, and depth—how many solid feet of earth were taken out of it? *Ans.* 1728.

3. There is a stone of a cubic form, which contains 389017 solid feet—what is the superficial content of one of its sides? *Ans.* 5329.

Between two numbers given, to find two mean proportionals.

R U L E.

Divide the greater extreme by the less, and the cube root of the quotient multiplied by the less extreme gives the less mean; multiply the said cube root by the less mean, and the product will be the greater mean proportional.

E X A M P L E S.

4. What are the two mean proportionals between 6 and 162? *Ans.* 18 and 54.

5. What are the two mean proportionals between 4 and 108? *Ans.* 12 and 36.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

R U L E.

The cube root of the solid content of any solid body given, is the side of the cube of equal solidity.

E X A M P L E.

6. If the solid content of a globe is 10648—what is the side of a cube of equal solidity. *Ans.* 22.

The side of the cube being given, to find the side of the cube that shall be double, treble, &c. in quantity to the cube given.

R U L E.

Cube the side given, and multiply it by 2, 3, &c. the cube root of the product is the side sought.

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EXAMPLE.

7. There is a cubical vessel, whose side is 12 inches, and it is required to find the side of another vessel, that is to contain 3 times as much? *Ans.* 17.306.

EXTRACTING of the BIQUADRATE ROOT. *

To extract the biquadrate root is to find out a number, which being involved four times into itself will produce the given number.

RULE.

First extract the square root of the given number, and then extract the square root of that square root, and it will give the biquadrate root required.

EXAMPLES.

1. What is the biquadrate of 27? *Ans.* 531441.
2. What is the biquadrate of 76? *Ans.* 33362176.
3. What is the biquadrate of 275? *Ans.* 5719140625.
4. What is the biquadrate root of 531441? *Ans.* 27.
5. What is the biquadrate root of 33362176? *Ans.* 76.
6. What is the biquadrate root of 5719140625? *Ans.* 275.

A general RULE for EXTRACTING the ROOTS of all POWERS.

1. Prepare the number given for extraction, by pointing off from the units place as the root required directs.
2. Find the first figure in the root by the table of powers, which subtract from the given number.
3. Bring down the first figure in the next point to the remainder, and call it the dividend.
4. Involve the root into the next inferior power to that which is given, multiply it by the given power, and call it the divisor.
5. Find a quotient figure by common division, and annex it to the root; then involve the whole root into the given power, and call that the subtrahend.

* The biquadrate root of a number is the square root of the square root.

The biquadrate of a number is the square of its square.

ROOTS of all POWERS.

6. Subtract that number from as many points of the given power as are brought down, beginning at the lowest place, and to the remainder bring down the figure of the next point for a new dividend.

7. Find a new divisor, and proceed in all respects as before.

A Table of Powers.

Roots, or - -	1st. pow.	1	2	3	4	5	6	7	8	9
Squares, - - -	2d. pow.	1	4	9	16	25	36	49	64	81
Cubes, - - - -	3d. pow.	1	8	27	64	125	216	343	512	729
Biquadrates, - -	4th. pow.	1	16	81	256	625	1296	2401	4096	6561
Sur solids, - - -	5th. pow.	1	32	243	1024	3125	7776	16807	32768	59049
Square cubes, - -	6th. pow.	1	64	729	4096	15625	46656	117649	262144	531441
2d. Sur solids, - -	7th. pow.	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates sqd.	8th. pow.	1	256	6561	65536	390625	1679616	5765801	16777216	33056721
Cubes cubed, - -	9th. pow.	1	512	19683	262144	1953125	10077696	40353607	134217728	387520589

27)261 dividend.

50653 subtrahend.

4107)25043 dividend.

53157376 subtrahend.

3 × 3 × 3 = 27 divisor.

37 × 37 × 37 = 50653 subtrahend.

37 × 37 × 37 = 4107 divisor.

376 × 376 × 376 = 53157376 subtrahend.

3. What is the biquadrate root of 19987173376?

19987173376(376

81

Ans. 376.

108)1188 dividend.

1874161 subtrahend.

202612)1245563 dividend.

19987173376 subtrahend.

3 × 3 × 3 × 4 = 108 divisor.

37 × 37 × 37 × 37 = 1874161 subtrahend.

37 × 37 × 37 × 4 = 202612 divisor.

376 × 376 × 376 × 376 = 19987173376 subtrahend.

Questions for Examination.

1. What is the cube root of a number?
2. What is the cube of a number?
3. What is the form of a solid body called a cube?

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4. How is the cube root extracted?
5. How is the extraction of the cube root proved?
6. What is the biquadrate root?
7. What is the biquadrate of any number?
8. What is the rule for extracting the biquadrate root?
9. How is the extraction of the biquadrate root proved?
10. What is the general rule for extracting the roots of all powers?

Additional Explanations, relating to Roots and Powers.

Besides the general rule for extracting the roots of all powers, the extraction of many of them may be more easily performed as follows.

1. The square root of the square root is the biquadrate root, or root of the 4th power.
2. The square root of the cube root is the square cubed root, or root of the 6th power.
3. The square root of the biquadrate root is the squared biquadrate root, or root of the 8th power.
4. The cube root of the cube root, is the cubed cube root, or root of the 9th power.
5. The square root of the sur-solid or fifth root, is the square sur-solid or 10th root, or root of the 10th power.
6. The cube root of the biquadrate root is the squared square cube root, or 12th root, being the root of the 12th power.

Reason. — The reason why roots may be extracted in this way, is because their powers are raised on the same principle by multiplication. Thus, the square of any number squared, or multiplied by itself, produces the biquadrate, or 4th power, and so of all the others. Now, the number of any required root, is called the index of the root; thus, the index of the square root is 2, of the cube root 3, of the biquadrate 4, of the sur-

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solid 5, of the square cube 6, of the second sursolid 7, of the squared biquadrata 8, of the cubed cube 9, of the squared sursolid 10, of the third sursolid 11, of the square cube squared 12, of the fourth sursolid 13, of the second sursolid squared 14, of the sursolid cubed, 15, &c.*

Therefore, when the product of any two or more indices is equal to any other index belonging to a number whose root is wanted, extract out of the given number a root answering to one of the indices, and from that root another, and so on to the last. Thus, to extract the 12th root, or squared square cube, whose index is 12, find two other indices whose product is 12, viz. ^{the} the cube and biquadrata roots; first extract the biquadrata root of the given number, and then the cube root of the biquadrata root, and that will be the 12th root required, and so of all the rest.

Remark. — Though in pure arithmetick and algebra where numbers are considered abstractly, powers whose roots may be wanted, often rise very high, yet, in geometry, where dimensions are expressed by numbers, the square and cube are the only forms or figures which convey clear ideas to the mind, and are all that are properly used in mensuration.

In geometry or mensuration, the square root is the length of one side of a square, without regard to thickness. Thus, an inch square of paper or of the thinnest gold leaf, is as much a square surface as a mile square of land, whose thickness may be considered equal to the earth's semidiameter. But as a cube, in geometry, is composed of as many squares placed evenly upon or against one another, as one square is inches, &c. in length or breadth, a square in this case, is supposed to be as thick as one of its equal parts in length and breadth is long.

* These indices, or exponents as they are sometimes called, are placed at the right hand near the top of the numbers whose powers they express, thus. q^2 , q^3 , q^4 &c. Here the small figures 2, 3, and 4, are the indices or exponents of q . The figure or index 2, shows that q is to be squared, or raised to the 2d power; 3 signifies the cube or 3d power; and 4 the biquadrata, or 4th power, or that q must be multiplied by itself, and the product by q again, and the 2d product again by q .

As the sum of the indices 2 and 3 is 5, if the square and cube of any number be multiplied together, the product will be the sursolid or 5th power of that number. The product of the 3d and 4th power is the 7th power, &c.

