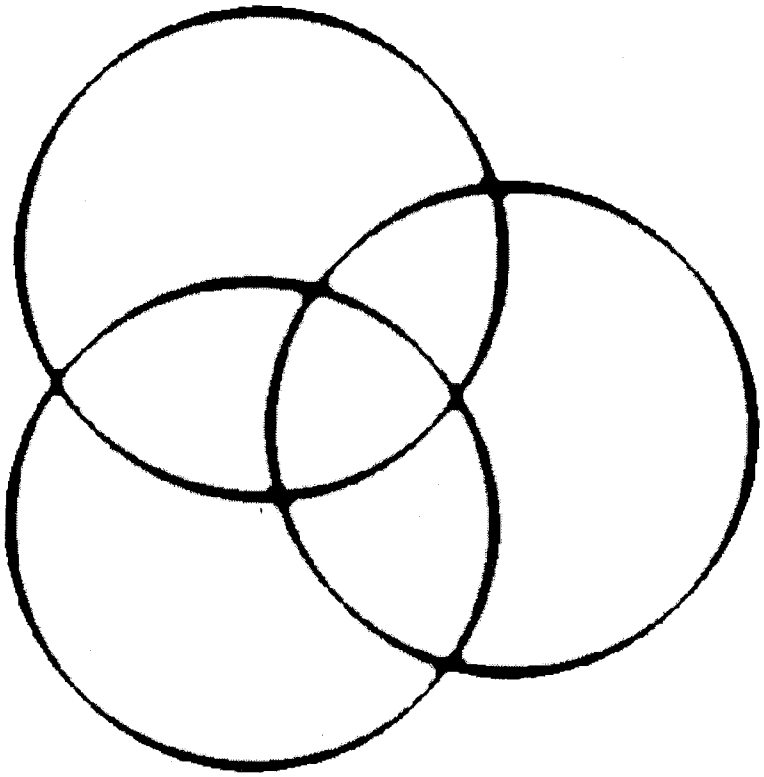
Circles



Billy Clarke
Methods: Secondary Mathematics
Fall 2008
Mr. Duncan

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good part 4's this helped

Billy,
You have a wonderful understanding of how long a topic might actually take. This will help you tremendously when you teach.

A

TITLE: GEOMETRY		SUBJECT/COURSE: MATHEMATICS
TOPIC: CIRCLES	GRADES: 9^{TH}-11^{TH}	DESIGNERS: WILLIAM CLARKE

Desired Results

Established Goals:

- Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving lines and circles
- Solves problems involving perimeter, circumference, or area of two dimensional figures

Understandings:	**Essential Questions:**
-That there are many parts of a circle -Circle properties can be used to solve real world problems -Pictures and sketch's are essential to solving problems involving circles	-What's the relationship between the radius of a circle and the tangent? -What's the relationship between an inscribed angle and a central angle that form the same arc?
Students will know…	***Students will be able to…***
-The different parts of a circle -The relationship between angles and arcs -	-Calculate the circumference and area of a circle -Use geometric properties to solve problems involving circles

Assessment Evidence

Performance Tasks:	**Other Evidence:**
-The students will be asked to investigate the circumference and areas of circles in real world problems	-There will be homework given out at appropriate times during the unit - Daily pre-class quizzes to assess knowledge and progression - There will be a review and an exam on the last day of the unit to assess understanding and knowledge

Learning Plan

Learning Activities:

-The students will use circles made out of construction paper to investigate the area of a circle.

-The students will work in groups and investigate the ratio that leads to Pi

Circumference and Area

Standards:

- Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving lines and circles.
- Solves problems involving perimeter, circumference, or area of two dimensional figures.
- Applies the concepts of congruency by solving problems.

Objectives:

By the end of the lesson:

- The students will be able to define a circle and terms that are related to them.
- The students will be able to calculate the circumference and area of different circles.
- The students will be able to identify congruent and concentric circles.

Materials:

Teacher:

- Globe
- Compasses and Straightedges
- Calculators

Student:

- Notebook
- Pencil
- Compasses and Straightedges
- Construction Paper
- Scissors
- Calculators

Procedure (Step-by-Step):

Day 1:

1. Begin the class by drawing a circle (without the center point) on the board and asking the students to give you some examples of circles that they see every day.
    a. List these example on the board.
    b. Expect some that are ovals or maybe even spheres.
        i. If you get these, just explain that it was a good effort but we will discuss those shapes later in the course.
2. Now that they gave you examples ask them to turn to their neighbor and try and come up with a definition of a circle.
    a. Give them 2 or 3 minutes to come up with some.

3. Write down all of their definitions or ideas that they came up with on the board.
4. Now put the center point in the circle and ask if any of them would like to change the definitions that they came up with.
    a. Give them 2 or 3 more minutes to discuss and see if they want to change their definitions.
5. Ask for their new definitions, if there are any.
6. If there are now new definitions then ask the students to look at what was put on the board and ask them what idea on the board was the best.
7. As a class, have them decide what one is the best.
8. Now write the actual definition of a circle on the board.
    a. Circle: The set of all point in a plane that are a given distance from a given point called the center. The given distance is denoted r.
9. State that we can use any letter to denote the center point.
10. Now draw a line from the center point to any point on the circle.
    a. Ask the students what this line is called?
        i. Should say r from the definition of a circle.
        ii. Some may say the radius if they have prior knowledge.
11. If no one says radius, then ask them what the certain segment might be called.
    a. If there are still are no answers then explain that it is called the radius.
    b. Radius: A segment extending from the center to any point on the circle.
12. Ask: Why must all the radii in a given circle be congruent?
    a. Assess the answers.
13. Now draw a line that goes directly through the center of the circle and touches the edge of the circle twice.
14. Ask the students what they think this segment is called.
    a. Expect no answers unless prior knowledge.
    b. You may even get 2r from a real experienced student.
15. If no one has an answer then state that it is the diameter of the circle.
    a. Diameter: Line segment containing the center, with its endpoints on the circle.
16. Pass out compasses and rulers to every student along with Guided Practice 1.
    a. The students are going to make 2 circles of the same size and their outer edges should be connected at one point.
    b. They will make radii from both circles to that point and measure them.
    c. Then they will make a circle of the same size whose center point is the point in which the two other circles touched.
    d. They should then investigate and see what the diameter of the 3rd circle is.
    e. They should follow the questions on the worksheet to find the relationship between the diameter and the radius.
17. This can also be shown on the computer with a projection using The Geometer's Sketchpad. (Use if extra time for class instruction is needed)
18. Now hand out Circle Assignment 1, which has the students identify different parts of circles and also use d=2r.

Day 2:

1.) Ask students what questions they had about the homework from the previous night. If they have no questions then collect the homework, if there are questions then address them with examples or review of the homework.

2.) Next put a 1 question homework quiz on the board taken from the homework assignment. The students will have 4 minutes to complete this.

3.) The topic of today's lesson is circumference and area of a circle.
4.) Ask the students if any of them know what the circumference of a circle is.
    a. Possible Answer: The perimeter of the circle.
    b. Possible Answer: The length around the circle.
    c. If they do not know what the circumference is then explain that it is the same as the perimeter of any other shape.
    d. Write the formal definition on the board.
        i. Circumference: The distance around a circle.
5.) Utilize the projector and project the table found below (Write on the board and in their notes):

Object	Circumference (C)	Diameter (d)	Ratio $\frac{C}{d}$
Can			
Mug			
Wheel			

6.) Put the students in groups of 3 or 4.
    a. Each group should have 1 of each object listed in the table above.
    b. The groups are to fill in the table completely.
    c. Next they are to find the average of the ratios that they found.
7.) Once each group has found their average, have one member come to the board and write the average on the board.
8.) Compare the averages.
    a. Are they close to one another?
9.) Ask the students what they think the ratio should be.
    a. They should be convinced it is slightly above three.
10.) The number that they should be getting should be close to 3.1415
    a. Explain that this is a special number that we use a lot for circles.
    b. The number is called Pi and the symbol is π.
11.) So we know that $\frac{C}{d} = \pi$.
    a. Ask the students to solve for what C is.
    b. Answer: $C = d\pi$.
12.) Given any circle and its diameter we can find the circumference.
13.) Ask: What if we are given just the radius, are we still able to find the circumference?
    a. Answer: Yes, since $d=2r$ then $C = 2r\pi$.
14.) No homework will be given tonight.

Day 3:

1.) Welcome the students to class. Since there was no homework then there will be no pre-class quiz.
2.) Last class we discovered the formula for circumference. Today we will be looking at the area of a circle.
3.) Hand out premade construction paper circles.
    a. Have the students fold the circle in half once.

b. Then have them fold it in half a 2^{nd}, 3^{rd}, and 4^{th} time.
c. Unfold the circle and have the students cut out the wedges they made by the folds. (there should be 16 wedges)
d. Arrange the wedges in a row, alternating the tips so it begins to form a shape resembling a parallelogram.
e. The radius of the original circle was r and the circumference is $2\pi r$.
    i. Teacher note: As the parallelogram forms the base is made of 8 wedges and since the circumference of the circle is 16 wedges and that is $2\pi r$ then 8 wedges is πr. The height is the radius of the circle so that is just r. The area of a parallelogram is b*h so $\pi r * r$ or πr^2.
f. Have the students find the area of the parallelogram that they made in terms of r. This should also be the area of the circle since the wedges also make up the circle.

4.) Ask the students what they found about the area of the circle.
    a. Answer should be πr^2.
    b. If some students did not get this or are having trouble understanding then walk through the entire problem on the board step by step.
5.) Put on the board a number of circles with either the radius or the diameter labeled and have the students find the area and circumference of all the circles.
6.) Hand out the premade homework assignment 2. Includes problems about area and circumference of circles. To be collected at the beginning of the next class.

Day 4:

1.) Ask students what questions they had about the homework from the previous night. If they have no questions then collect the homework, if there are questions then address them with examples or review of the homework.
2.) Next put a 1 question homework quiz on the board taken from the homework assignment. The students will have 4 minutes to complete this.
3.) The topic of today's lesson will be congruent vs. concentric circles.
4.) From their previous geometry classes, ask the students what the definition of congruency is? Or what does it mean for two things to be congruent?
    a. Answers may be link to triangles since this is where most of the congruency properties are learned.
    b. If students are having a difficult time then ask them what it means for two triangles to be congruent.
        i. Means that one triangle can be put onto the other one and they would be exactly the same.
        ii. The measures of all of their sides and angles are the same.
5.) So congruent triangles have all the same measures then what do congruent circles must have?
    a. Their measures must be the same.
        i. If this is said then ask what are the measures of a circle?
            1. Should be the radius and some may say the diameter.

6.) So congruent circles have the same measures and the measures that we know are the radius and diameter so thus the measures that must be the same for congruent circles are…(Pause for answers)
    a. Radius and Diameter must be the same.
7.) If the diameters are the same, doesn't that mean that the radii must be the same?
    a. The answer should be yes.
    b. If the students say no then ask why they shouldn't be the same.
    c. Then work through why the radii should be the same if the diameters are the same.
8.) Now give the formal definition of congruent circles.
    a. Congruent Circles: Two or more circles with the same radius measures.
9.) Give some examples of congruent circles.
    a. Hold up some CD's.
    b. Tires on a car.
10.) Now ask by a show of hands how many students know what concentric circles are?
    a. Don't expect many hands to go up.
    b. If one does go up then ask them to write down their definition and save it for after we work through it as a class.
11.) Ask: If congruent circles had radii that were the <u>same</u> measures, then what do you think concentric circles will have?
    a. Possible answer: Different radii.
    b. Possible answer: Different diameter.
12.) If these answer come up then put two circles on the board with two different radii and ask if they are concentric or of the students believe they are concentric.
13.) Ask if the students have any suggestions about what concentric circles might be?
    a. If there are some then have the students explain to you what they believe for you to draw it on the board or have them draw it on the board.
14.) Next tell the students that concentric circles have one thing in common. Ask them what they think that similarity is?
15.) Take one guess at a time and walk through it as a class.
16.) If they are no guess then ask the students what are some things that two circles could have in common?
    a. Same radii or diameter.
        i. Cannot be because we have already said that then they would be congruent triangles.
    b. Share a point on the outer part of the circle.
        i. Just mention to them that this is a good guess but the circles will not be concentric.
        ii. State that they are on the right track by thinking about points.
    c. They have the same center point.
        i. Ask if the circles would have the same radii?

1. May confuse them but it will allow them to think and problem solve.
2. They will not have the same radii.

17.) Now write the formal definition of concentric circles.
   a. Concentric circles: Two or more circles that share the same center and have different radii.
18.) Now ask any students who believed they knew what concentric circles were before the lessons if the definitions that they wrote down were the same as the definition that was just written on the board?
19.) If so then congratulate them.
20.) Hand out premade homework assignment 3. Assignment will be involving some radius and diameter questions as well as questions about concentric and congruent triangles. It will be due at the beginning of the next class.

Assessment:

Formative:

- Questions asked to the students to find out their prior knowledge and challenging them to problem solve and make conjectures.
- Pre-class quizzes
- Challenging them to come up with definitions of terms they may or may not know.

Summative:

- Homework Assignments

Technology Used:

- Overhead
- Computer with projection and The Geometers Sketch Pad may be utilized.
- Calculators

Arcs, Chords, and Tangents

Standards:

- Demonstrates conceptual understanding of spatial reasoning and visualization.
- Perform and justify constructions with a compass and straightedge.

Objectives:

By the end of the lesson:

- The students will be able to define an arc, chord and tangent of a circle.
- The students will understand the relationship between a chord and a tangent.
- The students will be able to solve word problems involving circles and their related figures.
- The students will be able to identify different parts of a circle.

Procedure (Step-by-Step):

Day 1:

1.) Welcome the class and ask them what questions they have about the previous nights' homework. If they have questions then address them with examples and review of the homework.
2.) If they have no questions then collect the homework and put up a 1 question pre-class quiz. The question will come from the previous nights' homework.
3.) Today's lesson will be about arcs, chords and tangents.
4.) Begin by asking the students what an arc looks like.
    a. Draw any of the suggestions on the board.
5.) Ask the class to decide what picture they feel represents an arc the best.
6.) If this is a proper picture for an arc then use it as the basis for what an arc is. If it is not a proper picture for an arc then give the students an example of what an arc is (be ready with a picture of the St. Louis Arc) and then ask if they have any other guesses.
7.) Once the class has decided on a proper picture of an arc then use it and erase all the other pictures on the board.
8.) Since you have drawn the arc (making sure that it was made to look like a piece of a circle), ask the students what they think the definition of an arc is?
    a. Expect some confusion and uncertainty about their definitions.
    b. Some definitions might be correct.
9.) To help the students with the correct definition then draw a dotted line that makes the outline of the rest of the circle.
10.) Ask if any of them think they know what the definition is now. Ask for their guesses.
11.) Write the formal definition of an arc on the board.

a. Arc: Two points of a circle and the continuous part of the circle between the two points.
b. Endpoints: The two points of the arc.

12.) Ask: When teachers in elementary school tell you to sit in a semicircle around them what do they mean? What is a semicircle?
a. Possible answer: A piece of a circle.
b. Possible answer: Half of a circle.

13.) Both answers are correct.

14.) So if a semicircle is half of a circle then where would the endpoints be located?
a. Possible answer: Exactly opposite from one another.

15.) Draw this on the board, with the two endpoints opposite each other on the circle.

16.) If the endpoints are opposite one another then part of a circle can we make using these endpoints?
a. Answer: Diameter.

17.) Now draw the diameter in.

18.) Ask the students if they can now write a definition of a semicircle.
a. Have them work with their neighbors to come up with a definition.
b. Give them 2 minutes to do so.

19.) Ask the groups to tell you the definitions that they came up with.

20.) If any of them are correct then write it on the board.

21.) Then write the formal definition on the board.
a. Semicircle: An arc of a circle whose endpoints are also the endpoints for the diameter.

22.) Ask: Now in relation to the semicircle, what do you think a minor arc would be? A major arc?
a. Have then work with their same groups to come up with some possible definitions.

23.) Ask each group for what they came up with.

24.) Write the formal definitions on the board to see if any of them were correct.
a. Minor Arc: An arc of a circle that is smaller than the semicircle.
b. Major Arc: An arc of a circle that is larger than the semicircle.

25.) Now begin with what a chord is.

26.) Tell the students that a chord is a segment and ask a couple of them to come to the board and draw what they think a chord might be.
a. Possible answer: A segment inside the circle but not touching the outside part of the circle.
b. Possible answer: A segment that touches the outer part of the circle once.
c. Possible answer: A segment whose endpoints are both on the circle.

27.) If no student draws a proper chord ask if there are any more guesses.

28.) On the board should be a number of segments within a circle.
a. One by one erase the incorrect guesses and tell the students that it was a good effort.
b. The one segment that should be left is a true chord.

29.) Have the students come up with a proper definition for the segment that is on the board. Should be done with their neighbors.
30.) Ask for each groups' definitions.
31.) Write the formal definition of a chord on the board.
    a. Chord: A line segment whose endpoints lie on the circle.
32.) Pass out the premade homework assignment 4. Contains questions on arcs and chords of circles. To be collected at the beginning of the next class.

Day 2:

1.) Welcome the class and ask what questions they have about the previous nights' homework. If they have any then address them with examples and review of the homework questions.
2.) If there are no questions then collect the homework and reveal the 1 question pre-class quiz. The questions will come from the previous nights' homework.
3.) The basis of todays will be a review of chords and we will be discussing tangents.
4.) Ask the students what a chord is from the previous class.
    a. Chord: A line segment whose endpoints lie exactly on the circle.
5.) What if the line only touches the circle once?
    a. Have the students make suggestions on what this might look like.
    b. Possible answer: segment within the circle with one endpoint on the circle.
    c. Possible answer: segment outside the circle with one endpoint on the circle but not tangent to the circle.
    d. Possible answer: line outside of the circle with one point touching the circle (tangent to the circle).
    e. Draw these on the board on the same triangle.
6.) Explain that the first two are not tangents but instead just segments within or outside of a circle.
7.) Erase the other two incorrect pieces and so just the tangent is left on the board.
8.) Ask the students to come up with a definition within their groups. They will have 2 or 3 minutes to come up with one.
9.) Once the time is up, ask for the groups to state their definitions. See if any of them are close to the formal definitions.
10.) Inform the students that these are all good guess and then write the formal definition on the board.
    a. Tangent: A line that intersects the circle only once.
11.) Ask: What do you think the point of intersection for a tangent and a circle is called?
    a. Expect a wide range of answers.
12.) The intersection is called the point of tangency.
13.) Hand out Guided Practice 2.
    a. Construct a circle and label the center O.
    b. Use a straightedge and construct a line that appears to be tangent to the circle. Label the point of tangency T.

c. Construct segment OT. This is clearly also the radius.
d. Fill in a guess about the relationship between the radius OT and the tangent line at T.
e. Using a protractor, measure the angles at point T.
f. Write down the measures and make a rule for the radius and the tangent line.

14.) The rules will vary but should be similar to the one below.
a. A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

15.) Hand out the premade homework assignment 5. It will include problems regarding arcs, chords, and tangents. To be collected at the beginning of the next class.

Assessment:

Formative:

- Pre-class quiz
- Questions asked throughout class
- Work with compasses and protractor to assess their accuracy using mathematical tools.
- Students writing down educated guesses for the terms we are trying to define.

Summative:

- Homework assignments

Technology Used:

Arc Measure and Arc Length

Standards:

- Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems.
- Uses units of measure appropriately and consistently when solving problems across content strands.

Objectives:

By the end of the lesson:

- The students will be able to identify parts of a circle.
- The students will be able to identify central angles of circles.
- The students will be able to determine arc measures.
- The students will be able to identify inscribed angles of circles.
- The students will be able to determine arc lengths.
- The students will be able to explain the different between arc measure and arc length.

Materials:

Teacher:

- Whiteboard
- Markers

Student:

- Protractor
- Notebook
- Pencil

Procedure (Step-by-Step):

19. Begin by welcoming the class and asking them what questions they have about their homework. If there are questions then address them with examples and problems from the homework.
20. If there are no questions about the homework then reveal the 1 question homework quiz which will be a problem from the previous night's homework. They will have 5 minutes to complete.
21. The topic of today's lesson will be arc measures and arc length.
22. To begin the lesson, the students need to know what a central angle.
23. Ask the class if anyone knows what a central angle is.
    a. If anyone knows then ask them to explain to the class what it is.
    b. Possibly draw a central angle in a circle.

c. Field any guesses at the same time.

24. If no one knows what as central angle is or has any guesses then tell them to break down the term.
    a. Central: What does this mean?
        i. Center; Center of the circle.
    b. Angle: What does this mean?
        i. What does an angle look like?
25. Now ask them since they have broken down the term, what do they think it is now?
    a. Answer: Angle that's vertex is at the center on the circle and has 2 points on the circle.
    b. Central Angle: An angle with its vertex at the center of the circle and its sides are radii.
26. Since they have found out what a central angle is, ask them what it looks like the central angle forms on the circle?
    a. Possible Answer: Two points on the circle.
    b. Possible Answer: An arc.
27. Explain that the central angle creates an arc on the circle and this arc has a **measure.**
    a. The measure of the arc that is made by a central angle is equivalent to the measure of the central angle.
28. Put some examples on the board:
    a. Circles with angles of:
        i. 90°
        ii. 45°
        iii. 180°
29. Ask: What is the arc that has a measure of 180°?
    a. Answer: Semicircle.
30. Now put a circle on the board and specify an arc measure.
    a. Ask the students what the central angle measure is?
31. Review what the following are:
    a. Central Angle.
    b. Arc Measure.
32. Now begin with the topic of inscribed angles.
33. Ask the students, if any of them know what an inscribed angle is?
    a. You will probably get no correct answers.
    b. If you get a correct answer then ask how the student came up with the answer or idea.
34. Inscribed angle: An angle that has its vertex on the circle and its sides are chords.
35. Now pass out protractors and compasses and have the students turn to a new sheet of paper.
36. Have them create a circle of any size and place two points on the circle.
37. Tell them to make the central angle with those two points and an inscribed angle with those same two points.
38. Have them measure both angles and tell them to write down the comparisons between the two angles.
39. Now have them construct their own circle and new central and inscribed angles. Measure these angles and write down what you noticed.
40. Have them state their conjecture about inscribed angles and arc measures.
    a. The inscribed angle is half the value of the intersected arc measure.

41. Write a circle on the board and create an inscribed and central angle that intersects the same arc.
    a. Put down an arc measure and have the students find the inscribed and central angle measures.
    b. Erase above; put down a central angle measure and have the students find the inscribed angle and arc measure.
    c. Erase above; put down an inscribed angle measure and have the students find the central angle and arc measure.
42. That concludes this lesson. Hand out the homework assignment 6 that includes central angle, inscribed angles, and arc measures. To be collected at the beginning of the next class.

Day 2

1. Begin by welcoming the class and asking them what questions they have about their homework. If there are questions then address them with examples and problems from the homework.
2. If there are no questions about the homework then reveal the 1 question homework quiz which will be a problem from the previous night's homework. They will have 5 minutes to complete.
3. The topic of today's lesson will be arc length.
4. Ask the students if there's a difference between arc measure and arc length.
    a. The answer should be yes but don't expect an answer.
5. Draw a circle on the board and a central angle.
    a. Show the distance between the radii in two positions: one close to the vertex and one further away from the vertex.
6. Ask the students if these are the same distance or length?
    a. They should say no.
    b. If they are still confused explain that it will take someone further to walk along the arc that's further from the center than it would for the person to walk on the one closer.
    c. Use a clock and the minute and second hand to show this.
7. Now ask the students if the arc length can be different even though the arc measure is the same?
    a. Answer: Yes they can be different.
8. Now put 3 circles on the board with central angles of 90°, 180°, and 120°.
    a. Ask what fraction each arc length is of the entire circle.
    b. Should be ¼, ½, and 1/3.
    c. Ask them how they found these numbers.
        i. Should know the first two because they have seen things like that in life.
        ii. If any of them found the answer to the third one then ask how they found it.
9. Return to the first circle and ask the students for the arc measure of the circle.
    a. 90°
10. Then ask the students how many degrees a circle is?
    a. 360°
11. Ask the students what $90^{\circ}/360^{\circ}$ is.
    a. ¼
12. So that angle produces an arc that is ¼ of the entire way around the circle.
13. What do we call the entire path around the circle?

a. Circumference.

14. So to find the total length of the arc what do we need to do to the circumference and the number that we just calculated?
    a. If a 90^0 angle produces an arc that is ¼ around the entire circle then what should we do to find the total length?
    b. Multiply ¼ and the circumference.
15. Arc Length equals the circumference times the measure of the central angle divided by 360^0.
16. Use that same circle with the 90^0 central angle and specify that the radius is 12 m.
    a. Have the students figure out the arc length.
        i. They must first find the circumference ($2\pi r$).
        ii. They multiply that by $90^0/360^0$.
17. Have the students do the same thing for radius 12 m for the other 2 circles with central angles of 180^0 and 120^0.
18. The lesson for today is over, Hand out the premade homework assignment 7 involving arc lengths. It is to be collected at the beginning on the next class.

Assessment:

Formative:

- Pre-class quiz
- Questioning the students to find knowledge level
-

Summative:

- Homework assignment

Technology Used:

Inscribed Polygons

Standards:

- Uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles.
-

Objectives:

By the end of the lesson:

- The students will understand the relationship between circles and inscribed polygons.
- The students will understand the relationship between circles and circumscribed polygons.

Materials:

Teacher:

- Protractor
- Whiteboard

Student:

- Protractor
- Notebook
- Pencil

Procedure (Step-by-Step):

43. Begin by welcoming the class and asking them what questions they have about their homework. If there are questions then address them with examples and problems from the homework.
44. If there are no questions about the homework then reveal the 1 question homework quiz which will be a problem from the previous night's homework. They will have 5 minutes to complete.
45. The topic of today's lesson is polygons and circles.
46. Ask the students to recall the definition of an inscribed angle.
47. Now ask them what they think you mean when you state that a polygon is inscribed within a circle.
    a. Possible answer: A polygon inside a circle.
    b. Possible answer: A polygon that is connected to the circle like an inscribed angle.
        i. Meaning the vertices of the polygon lie on the circle.
    c. Expect other answers.

48. Ask if any students want to come to the board and draw what they think might be an inscribed triangle.
49. If the correct picture is on the board then use that picture.
50. If not then try to walk the students through the construction to get the correct picture.
51. Begin by having one student come to the board and draw one inscribed angle.
52. Then ask how many angles a triangle has.
    a. Then proceed to state that if a triangle is *inscribed* then what should that tell you about the three angles of the triangle.
        i. They should all be inscribed angles.
53. Now finish making the triangle by connecting the two points (not the vertex) to one another.
    a. Ask the students if this is a triangle and if all the angles of the triangle are inscribed.
54. Since they are all inscribed then it must be that the triangle is inscribed within the circle.
55. Now ask them to create an inscribed square.
    a. Walk around the room and make sure they all completed.
    b. Then create one yourself on the board.
56. Now ask the students to work with their neighbor to come up with a definition for an inscribed polygon.
    a. Have each group state their definition to the class or any thoughts that they may have about the definition.
57. Write the formal definition on the board.
    a. Inscribed polygon: A polygon is inscribed about a circle if and only if all of its vertices lie on the circumference of the circle.
58. Ask the students what they notice about the sides of the inscribed polygon.
    a. Answer: All the sides are chords of the circle.
59. Ask the students to work with their neighbors again and come up with what an inscribed circle about a triangle would look like.
    a. Expect a bunch of answers but they should be similar.
    b. 3 points in which the circle is connected to the 3 sides of the triangle.
60. Now ask them to come up with some definitions.
    a. Have them read their definitions aloud.
61. Write the formal definition on the board.
    a. Inscribed circle: A circle is inscribed about a polygon if and only if it touches each side of the polygon at exactly one point.
    b. This is also called a circumscribed polygon.
62. Ask the students to construct an inscribed circle about a square and a pentagon.
    a. What do you notice about the points where the circle and the polygons connect?
        i. The circle connects with the sides at the midpoint of the side.
63. Pass out the premade homework assignment 8 with problems involving inscribed polygons and circles. It will be collected at the beginning of the next class.

Assessment:

Formative:

- Pre-class quiz
- Walk around room examining notes

- Questioning about previous knowledge
- Questions provoking thoughts about new concepts

Summative:

- Homework assignment

Technology Used:

-

Review

Standards:

- Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving lines and circles.
- Solves problems involving perimeter, circumference, or area of two dimensional figures.
- Applies the concepts of congruency by solving problems.
- Demonstrates conceptual understanding of spatial reasoning and visualization.
- Perform and justify constructions with a compass and straightedge.
- Uses units of measure appropriately and consistently when solving problems across content strands.
- Uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles.

Objectives:

By the end of the lesson:

- The students will understand and know the necessary concepts of linear functions including equations, graphing, domain, range, different forms, inequalities, parallel and perpendicular lines.
-

Materials:

Teacher:

- Whiteboard
- Calculator with projection capabilities
- Projector

Student:

- Notebook with notes
- Pencil
- Calculator

Procedure (Step-by-Step):

64. Welcome the class and ask what questions they have about their homework. If there are questions then address them. If there are no questions then reveal the one question homework quiz. They will have 5 minutes to complete it.
65. Have the students ask any questions that they have about the topics that we have covered in the class so far.

66. If they have any questions then address their questions and if not then take out the exam and put some example problems on the board that look similar to the questions that they will see on the exam.
67. This should bring up some questions that they have and some areas in which they are confused about.
68. This will help them study for their test.
69. At the end of the class, suggest that they should review their quizzes and homework's to review for the exam in the next class.

Assessment:

Formative:

- The example problems on the board will help you assess the students.
- Pre-class quiz

Summative:

- Homework assignment

Technology Used:

- Projector
- Calculator

Appendix A
Definitions

Circle:

-The set of all point in a plane that are a given distance from a given point called the center. The given distance is denoted r.

Radius:

-A segment extending from the center to any point on the circle.

Diameter:

-Line segment containing the center, with its endpoints on the circle.

Circumference:

-The distance around a circle.

-$2\pi r$

Area:

- The number of square units enclosed in the interior of a circle.

-πr^2

Congruent Circles:

-Two or more circles with the same radius measures.

Concentric circles:

-Two or more circles that share the same center and have different radii.

Arc:

-Two points of a circle and the continuous part of the circle between the two points.

Endpoints:

-The two points of the arc.

Semicircle:

-An arc of a circle whose endpoints are also the endpoints for the diameter.

Minor Arc:

-An arc of a circle that is smaller than the semicircle.

Major Arc:

-An arc of a circle that is larger than the semicircle.

Chord:

-A line segment whose endpoints lie on the circle.

Tangent:

-A line that intersects the circle only once.

-Intersection point is called the point of tangency.

Central Angle:

-An angle with its vertex at the center of the circle and its sides are radii.

Arc Measure:

- The measure of an arc that is made by a central angle is equivalent to the measure of the central angle.

Inscribed Angle:

-Inscribed Angle is defined as the angle formed by two chords that meet at the same point on a circle.

-It is equivalent to half of the central angle that shares the same arc.

Arc Length:

- The distance along the curved line making up the arc.

- Arc Length equals the circumference times the measure of the central angle divided by 360^{o}.

Inscribed polygon:

-A polygon is inscribed about a circle if and only if all of its vertices lie on the circumference of the circle.

Inscribed circle:

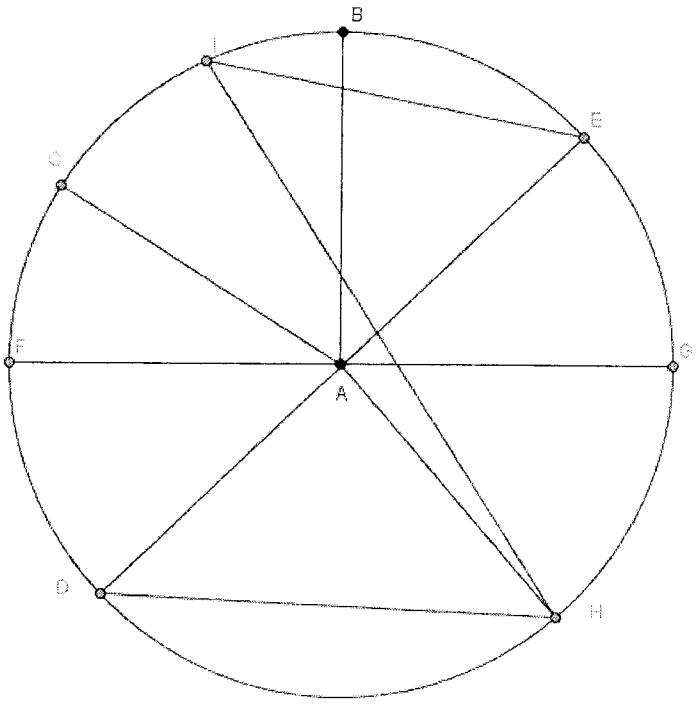
-A circle is inscribed about a polygon if and only if it touches each side of the polygon at exactly one point.

-Also known as a circumscribed polygon.

Appendix B
"Do Nows"

Definitions, Circumference and Area lesson:

Pre-class quiz (From homework 1)
1.)



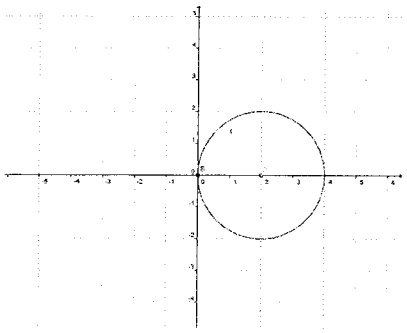
Identify the following:

a.) Radii: b.) Diameters: c.) Neither:

Pre-class quiz (From homework 2)
Use the definition of circumference to solve the following exercises.
1.) If $C = 5\pi$ cm, find d. 3.) If r = 5 cm, find C.

Pre-class quiz (From homework 3)
1.) Use the ordered pair rule to relocate the four points of the circle. Can the four new points be connected to make a circle? Is the new circle congruent or concentric to the original circle?

a.) $(x,y) \longrightarrow (x-1, y+2)$



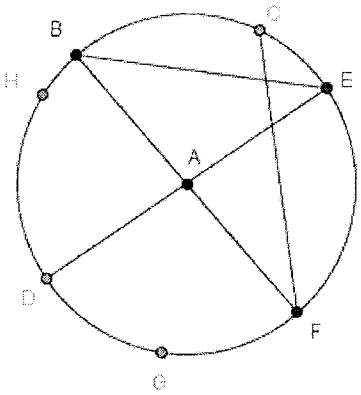
Arcs, Chords and Tangents lesson:
Pre-class quiz (From homework 4)

1.) Identify the different parts of the circle:

a. Arcs:

b. Minor Arcs:

c. Major Arcs:

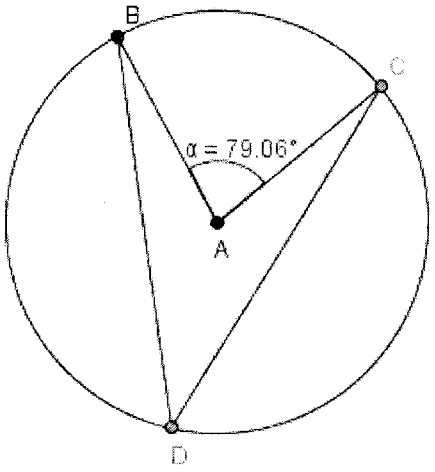


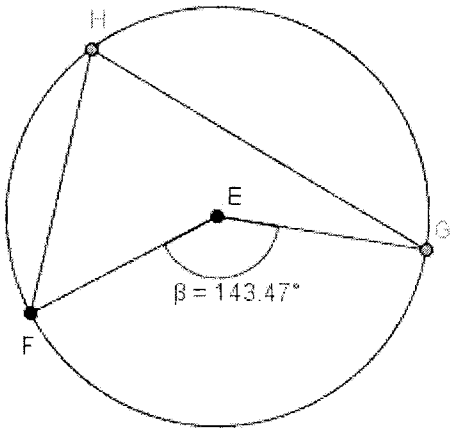
Pre-class quiz (From Homework 5)

1.) Can two circles be tangent to the same line at the same point? Different points? Explain using a sketch for both situations.

Arc Measure and Arc Length Lesson
Pre-class quiz (From Homework 6)

1.) State the measures of the central angle, arc, and inscribed angle of the following circles:

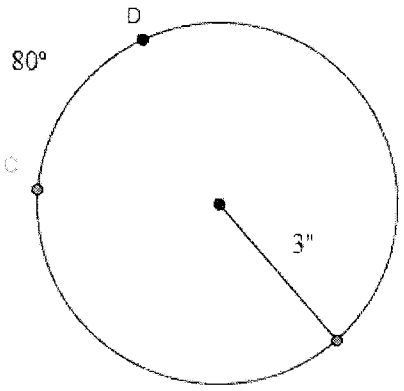




Pre-class quiz (From Homework 7)

State your answers in terms of π

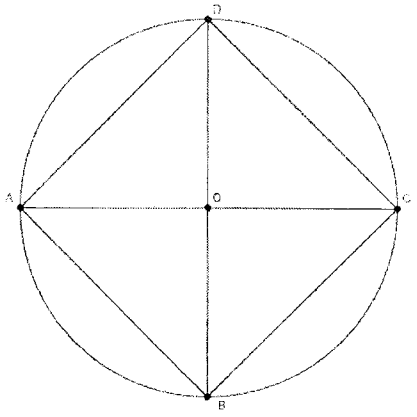
1.) Length of arc CD:



Inscribed Polygons lesson

Pre-class quiz (From homework 8)

1.) How many inscribed polygons are there in the figure? Name them.



Appendix C
In Class Handouts

Guided Practice 1

1.) Using your compasses, create two circles of equal size that connect at one point.

2.) Measure the radii of the two circles and record them below.

3.) Create a circle with same radii with its center at the point of intersection of the first two circles.

4.) What do you notice about the 3 circles?

5.) Write a statement about the relationship between the radii of the first two circles and the diameter of the third circle.

Guided Practice 2

1.) Construct a circle below with center point O.

2.) Use a straightedge and create a line that appears to be tangent to the circle. Label the point of tangency T.

3.) Construct segment OT. What is this segment called?

4.) What do you think is the relationship between the tangent line and the radius?

5.) Using a protractor, measure the angles at point T.

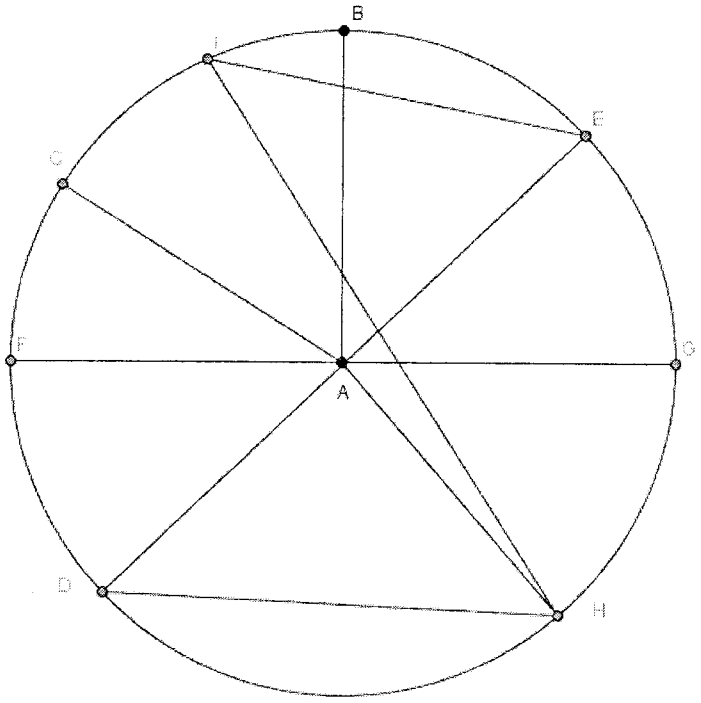
6.) Write a rule involving the tangent line and the radius of the circle.

Appendix D
Homework Assignments

Name_____ Date_____

Assignment 1

1.)



Identify the following:

b.) Radii: b.) Diameters: c.) Neither:

2.) In the circle above, assume that AH=3cm. Given the length of AH, what are the lengths of FG, EA and AC?

3.) Explain why FA+AB=FG.

4.) Tommy's mother asked him to go to the middle school to pick up his little sister Rachael. After picking up his little sister he must go to the store and pick up some milk for dinner. Once they get to the store Rachael realizes that she forgot her homework at the middle school so they must go back. Once at the middle school, Tommy gets a call from his mother and asks him to go to Mrs. Stevens' house to pick up something for her. Finally after visiting Mrs. Stevens, Tommy returns home with his sister and the milk. The school is 1.5 km from their house. Given the information below, how far did Tommy travel this afternoon?

Tommy's House is point F
The middle school is point A
The store is point H
Mrs. Stevens' house is point G

Name_____ Date_____

Assignment 2

Use the definition of circumference to solve the following exercises.

2.) If $C = 5\pi$ cm, find d.

3.) If r = 5 cm, find C.

3.) If d = 5.5 m, find C.

4.) If C = 24 m, find r.

5.) If a circle has a circumference of 46 π m, what is its diameter?

6.) If a circle has a diameter of 12 cm, what is its circumference?

7.) If the distance from the center of a Ferris wheel to one of the seats is approximately 90 feet, what is the distance that a rider travels, to the nearest foot, in one revolution?

Use the definition of the area of a circle to solve the following exercises.

1.) If r = 3 in, find A.

4.) If $A = 3\pi$ in^2, find r.

2.) If r = 7 cm, find A.

5.) If $C = 12\pi$ in, find A.

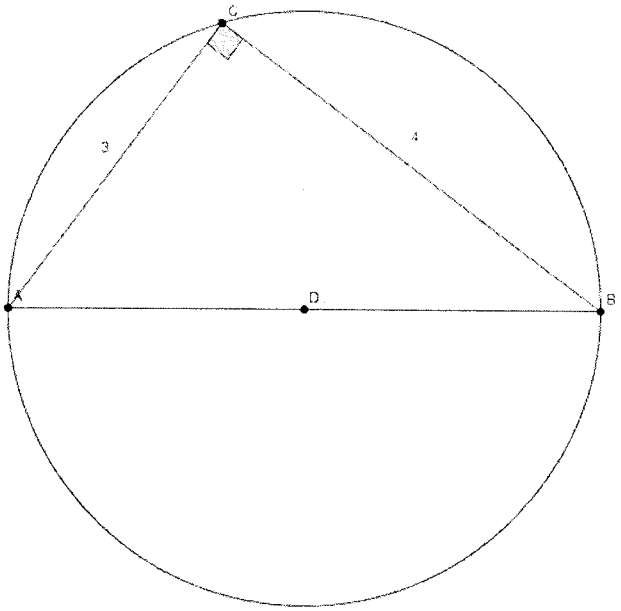
3.) If $A = 9\pi$ in^2, find r.

6.) If C= 314 m, find A.

7.) A small college TV station can broadcast its programming to household within a radius of 60 kilometers. How many square kilometers of viewing area does the station reach? Express your answer to the nearest square kilometer.

8.) Sampson's dog, Cecil, is tied to a post by a chain that is 7 meters long. How much play area does Cecil have? Express your answer to the nearest square kilometer.

9.)



Assume that the circle is the area that certain king of a country controls. The king gives his oldest son a piece of his land, the triangular portion. What is the area of land that the king did not give his oldest son?

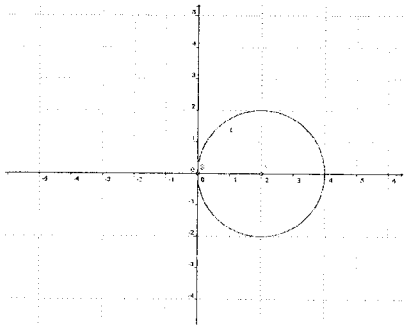
Name_____ Date_____

Assignment 3

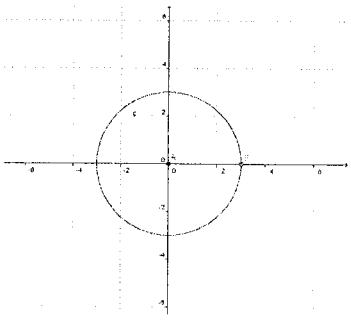
2.) Give some examples of concentric and congruent circles that you may see during a day of walking around town.

3.) Use the ordered pair rule to relocate the four points of the circle. Can the four new points be connected to make a circle? Is the new circle congruent or concentric to the original circle?

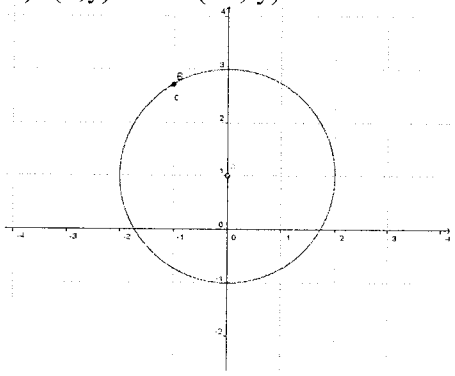
b.) $(x,y) \longrightarrow (x-1, y+2)$



b.) $(x,y) \longrightarrow (2x, 2y)$



c.) $(x,y) \longrightarrow (2x, y)$



Name_____ Date_____

Assignment 4

2.) Identify the different parts of the circle:

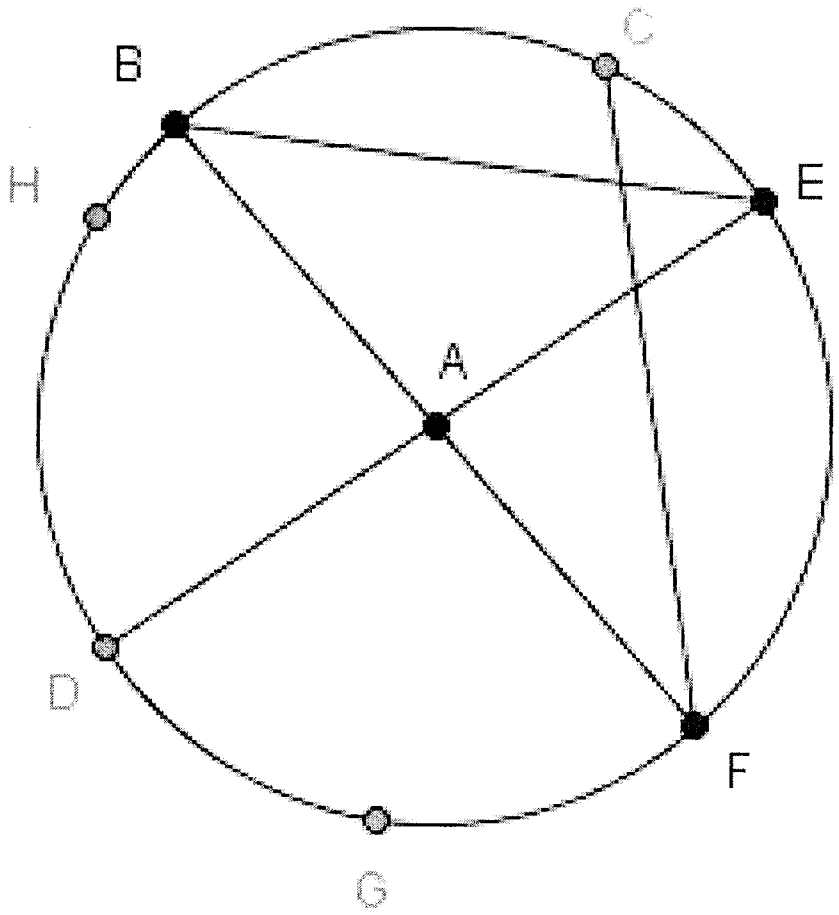
a. Arcs:

b. Minor Arcs:

c. Major Arcs:

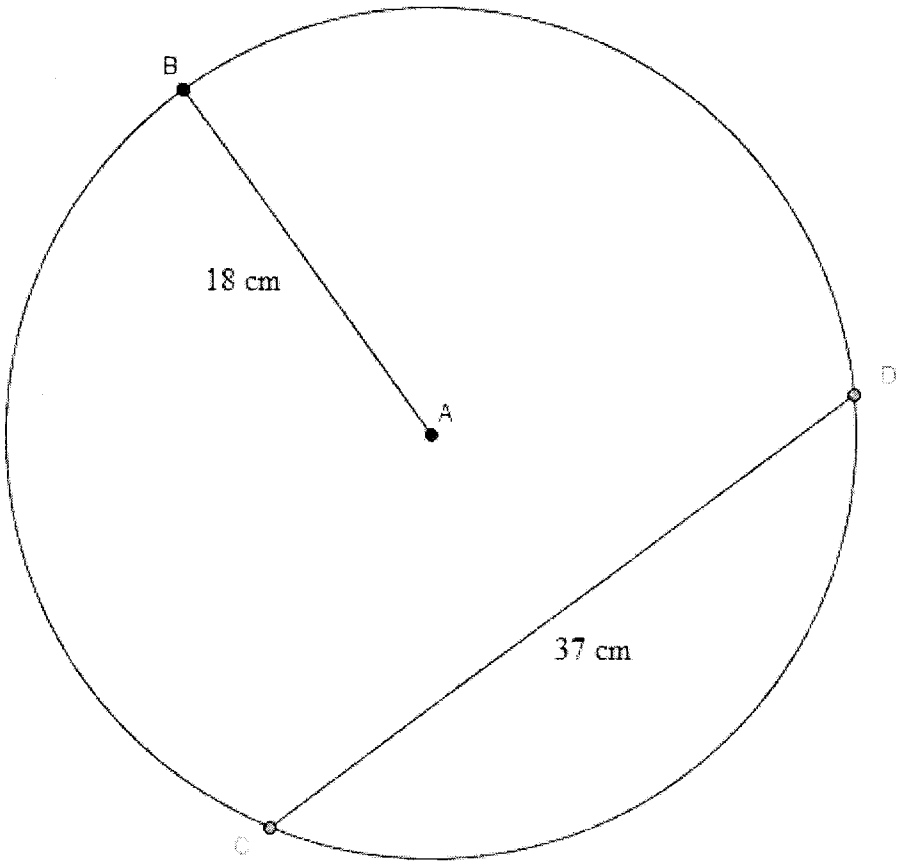
d. Semicircles:

e. Chords:



3.) Draw a circle and two chords of unequal lengths. Which chord is closer to the center of the circle, the longer chord or the shorter chord? Explain.

4.) What is wrong with the following picture?



Name_____ Date_____

Assignment 5

2.) Can the chord of a circle also be tangent to the circle? Explain.

3.) Can two circles be tangent to the same line at the same point? Different points? Explain using a sketch for both situations.

4.) Pam participates in the hammer-throw event. She throws a 16 lb ball at arm's length, about eye-level. The she releases the ball at the precise moment when the ball will travel in a straight line toward the target area. Draw an overhead view that shows the ball's circular path, her arms at the moment she releases it, and the ball's straight path toward the target area.

5.) Identify the following:

a.) Tangents:

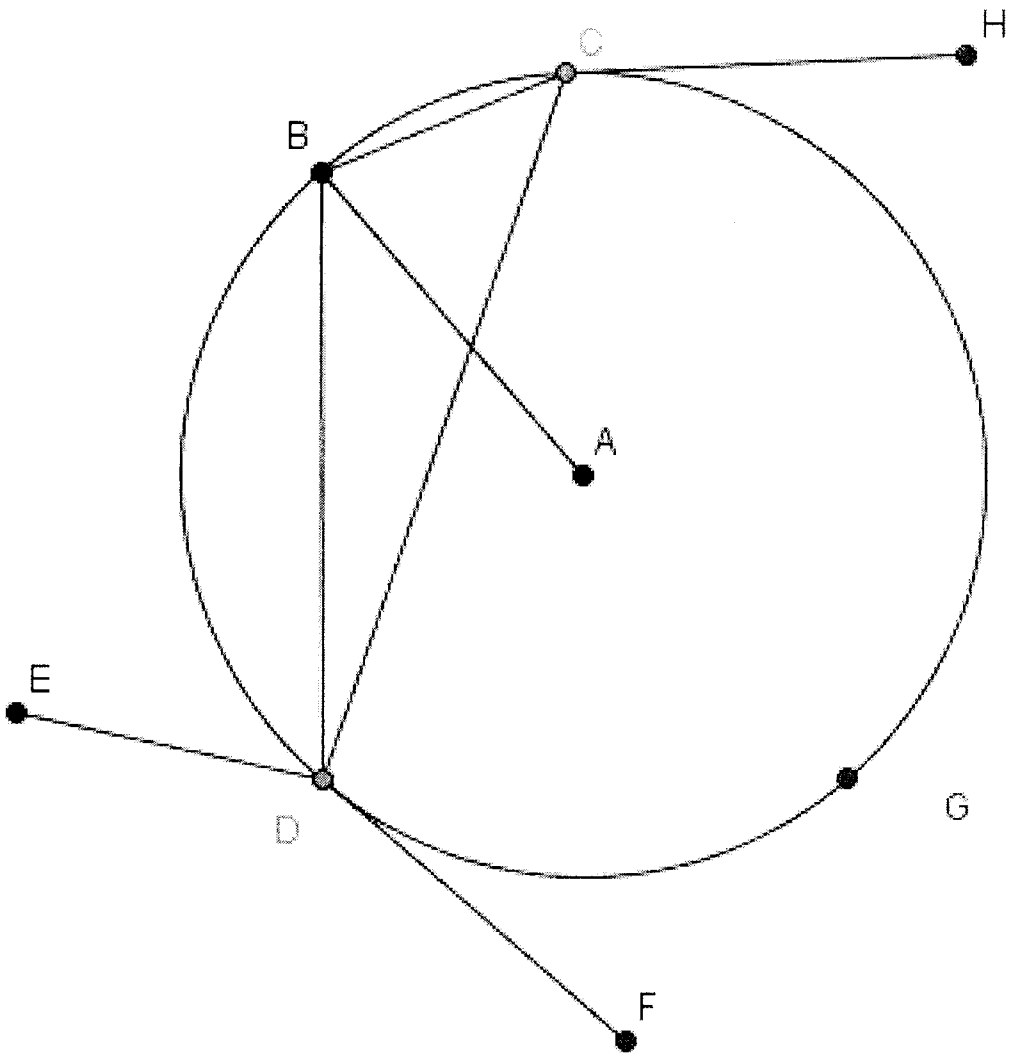
b.) Chords:

c.) Arcs:

d.) Minor Arcs:

e.) Major Arcs:

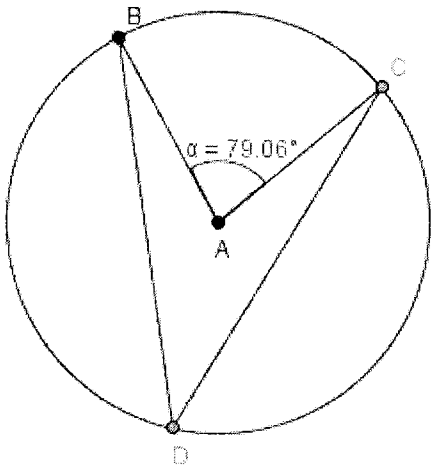
f.) Semicircles:

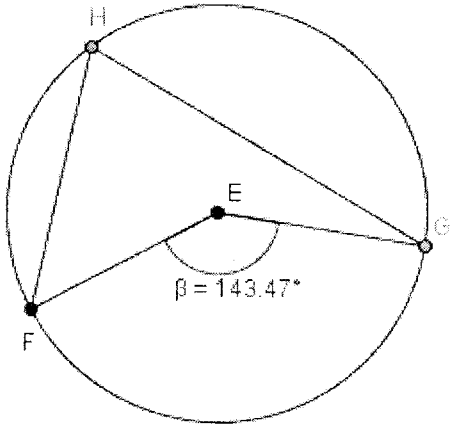

H
C
B
A
E
D
G
F

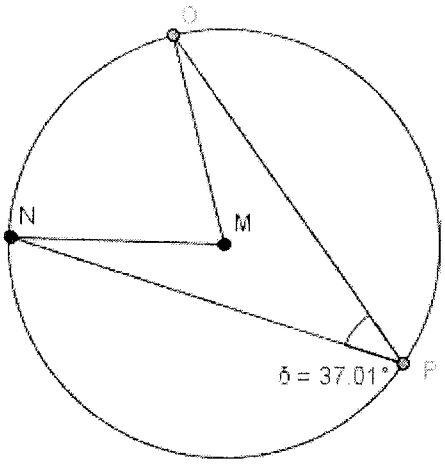
Name_____ Date_____

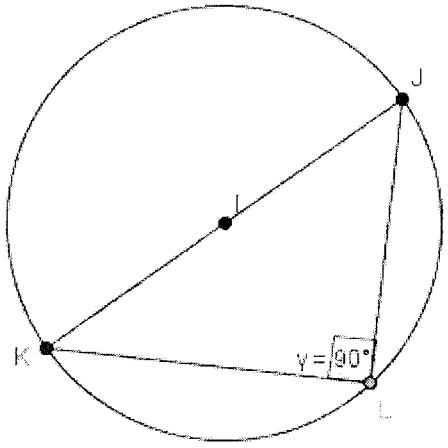
Assignment 6

2.) State the measures of the central angle, arc, and inscribed angle of the following circles



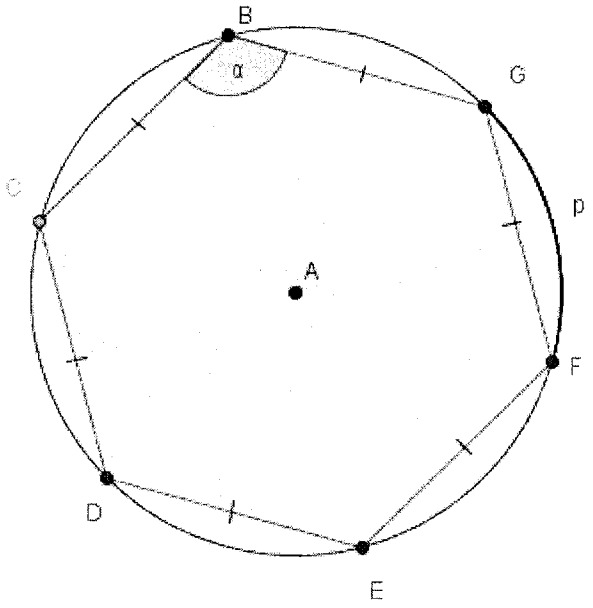






3.) What do you notice about the arc of a circle when the inscribed angle is 90°?

4.) What is the measure of α and arc p? Explain how you arrived at your answer.

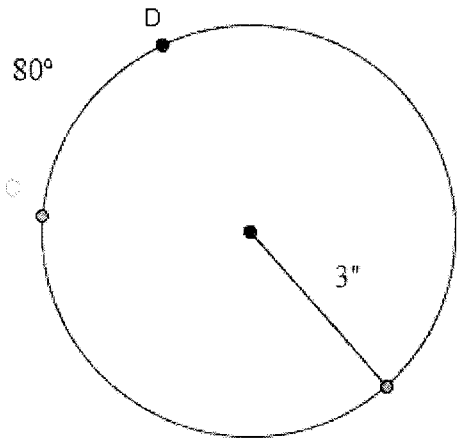


Name_____ Date_____

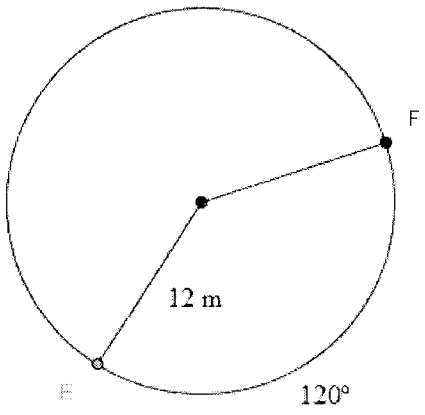
Assignment 7

State your answers in terms of π

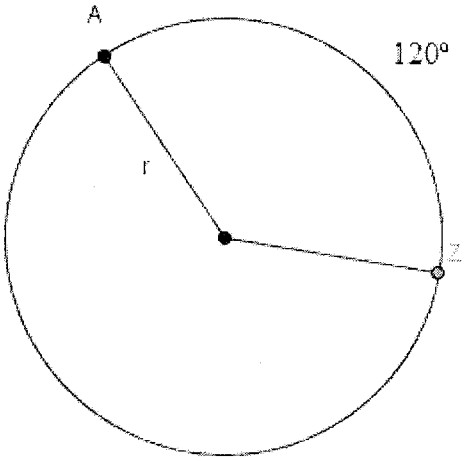
2.) Length of arc CD:



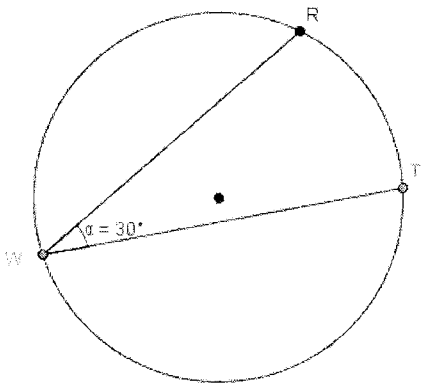
3.) Length of arc EF:



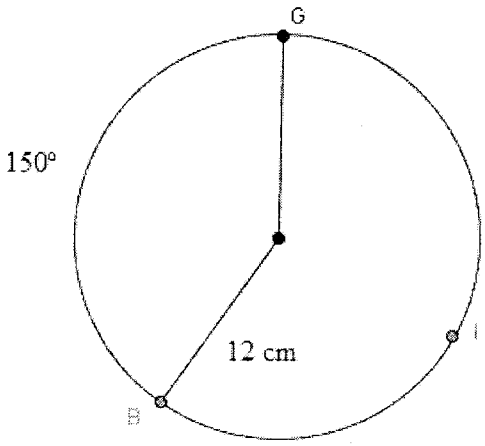
4.) Length of arc AZ is 6π m, what is the radius?



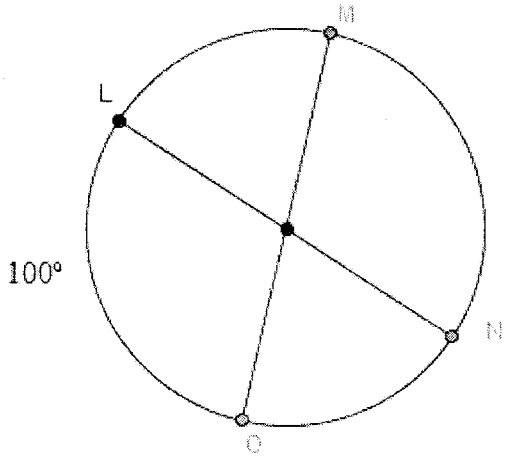
5.) The radius is 18 ft. What is the length of arc RT?



4.) Length of arc BIG:



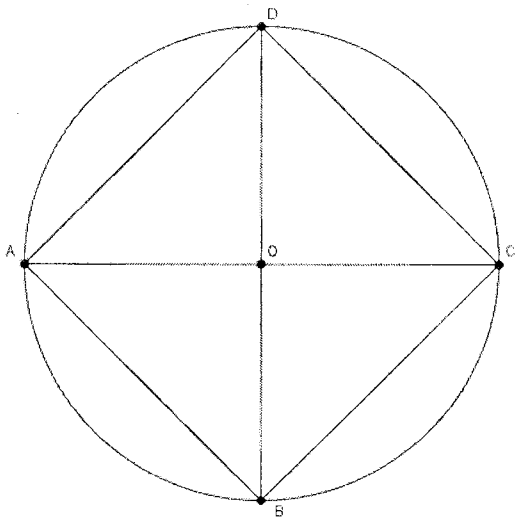
6.) The radius is 9m. What is the length of arc LM?



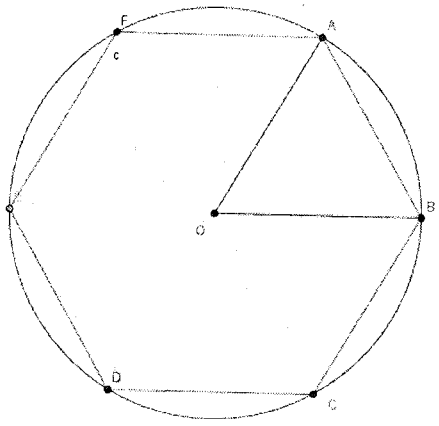
Name_____ Date_____

Assignment 8

2.) How many inscribed polygons are there in the figure? Name them.

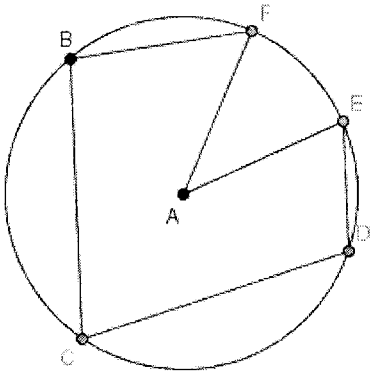


3.) A 6 sided regular polygon (hexagon) is inscribed in a circle of radius 10 cm; find the length of one side of the hexagon.

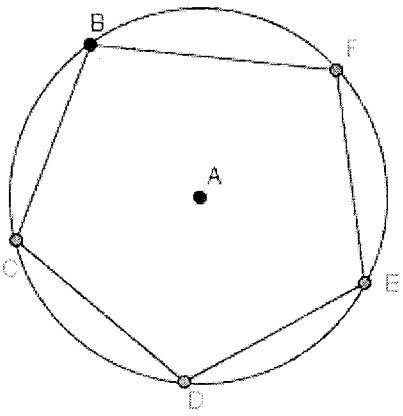


4.) In the following figures, decide whether a polygon is inscribed in a circle, a circle is inscribed in a polygon, or neither.

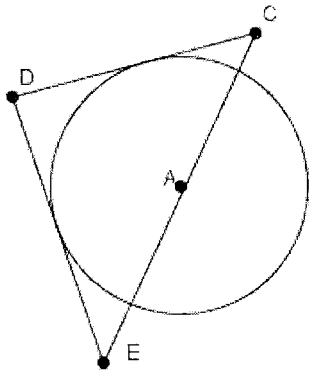
a.



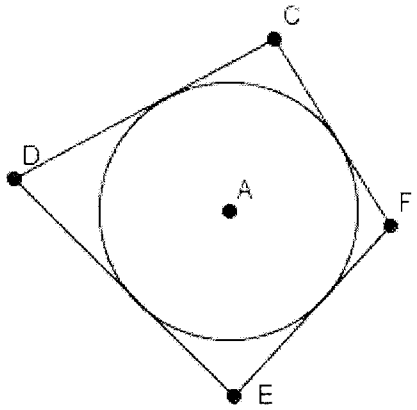
c.



b.



d.



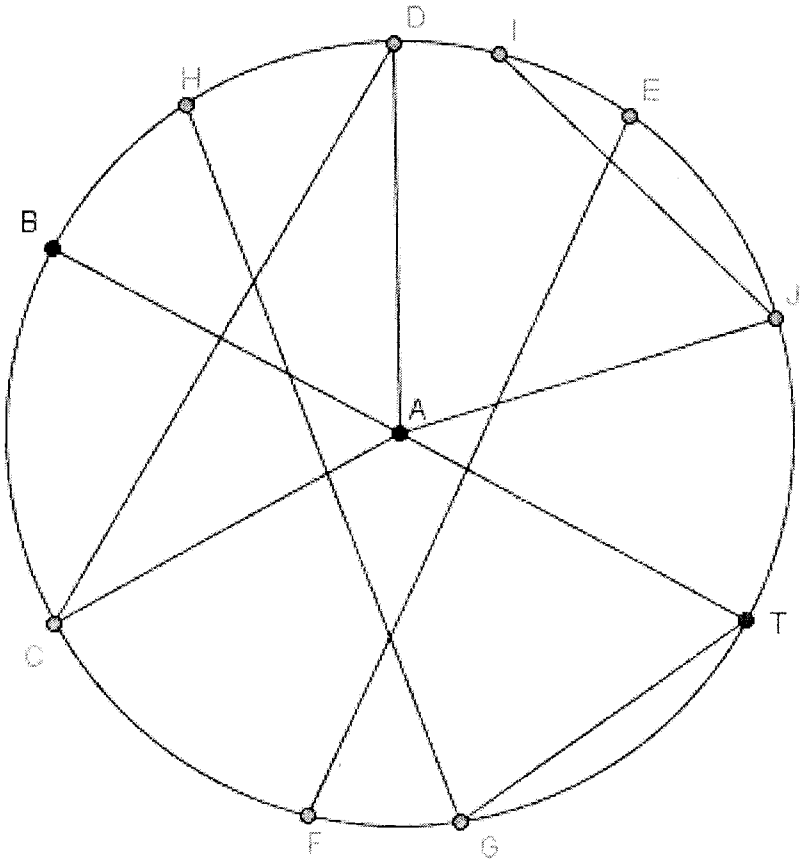
Appendix E

Assessment

Name:_____

Date:_____

1.) Identify the following parts of the circle:



a.) Radii:

b.) Diameter:

c.) Arcs:

d.) Major Arcs:

e.) Minor Arcs:

f.) Chords:

2.) Tommy's mother asked him to go to the middle school to pick up his little sister Rachael. After picking up his little sister he must go to the store and pick up some milk for dinner. Once they get to the store Rachael realizes that she forgot her homework at the middle school so they must go back. Once at the middle school, Tommy gets a call from his mother and asks him to go to Mrs. Stevens' house to pick up something for her. Finally after visiting Mrs. Stevens, Tommy returns home with his sister and the milk. The school is 1.5 km from their house. Given the information below, how far did Tommy travel this afternoon?

Tommy's House is point J
The middle school is point A
The store is point C
Mrs. Stevens' house is point B

3.) In the figure, explain why DA+AJ=BT.

4.) If the distance from the center of a Ferris wheel to one of the seats is approximately 75 feet, what is the distance that a rider travels, to the nearest foot, in three revolutions?

5.) Use the definition of circumference for the following.

a. If $C = 8\pi$ cm, find d.

b. If r = 10 cm, find C.

c. If C = 19 m, find r.

d. If d = 16 m, find C.

6.) Use the definition of the area of a circle to solve the following problems.

a. If r = 7 in, find A.

b. If r = 12 cm, find A.

c. If A= 5π in^2, find r.

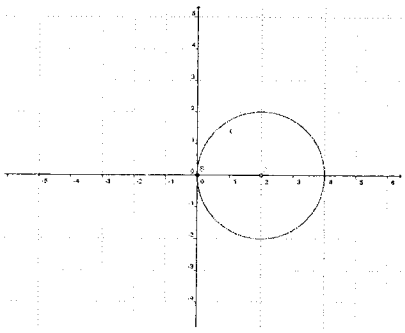
d. If C= 254 m, find A.

7.) Bobby and Tim got a bee and froze it. After a few minutes they took it out of the freezer and tied a string around it. The bee then warmed up and began to fly around. If Bobby was holding the string and the bee was 3 feet from him. How far would the bee fly if it flew around Bobby once? How much area does the bee have to fly in while attached to the string?

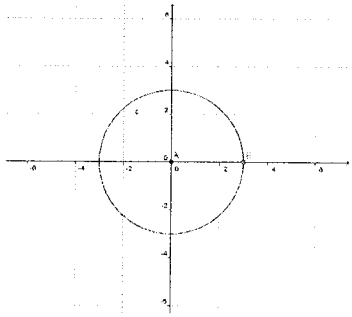
8.) The Smith family bought a piece of land but the problem was that the land was circular and they wanted a square house. They built the largest square house that they could. From the center of their house, the edge of their land is 15 ft away. If the house was one story and has a square footage of 144 ft^2, how much area do the children have to play in? Create a sketch to show what the question is asking then answer the question.

9.) Use the ordered pair rule to relocate the four points of the circle. Can the four new points be connected to make a circle? Is the new circle congruent or concentric to the original circle?

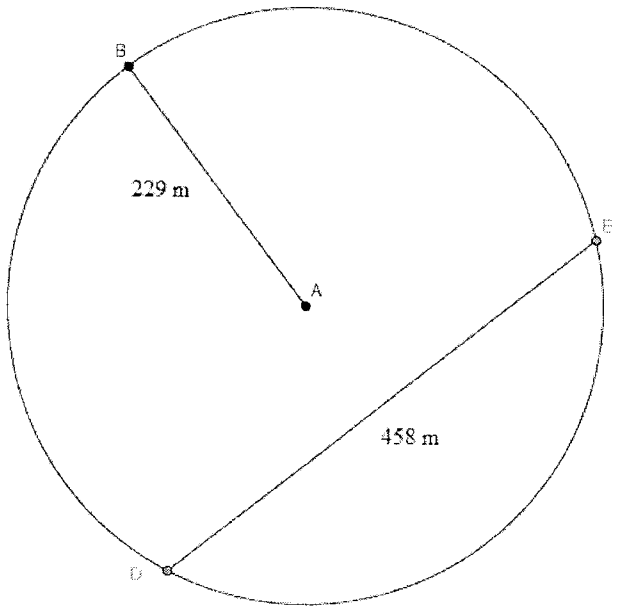
c.) $(x,y) \longrightarrow (x-3, y+3)$



b.) $(x,y) \longrightarrow (3x, y+1)$



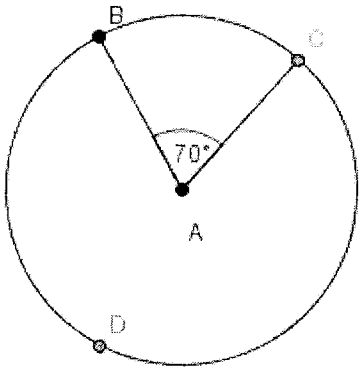
10.) What's wrong with the following picture?



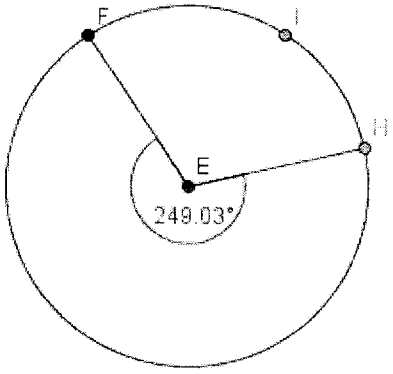
11.) Stacey was participating in an unusual event made up by her school. It was the hammer-throw but the hammer was attached to a rope 12 ft away from Stacey. Stacey spun around twice and then released the hammer. Once it landed she had to walk 15 ft to retrieve it. Including the 2 spins, how far did the hammer travel?

12.) What is the measure and lengths of the specified arcs?

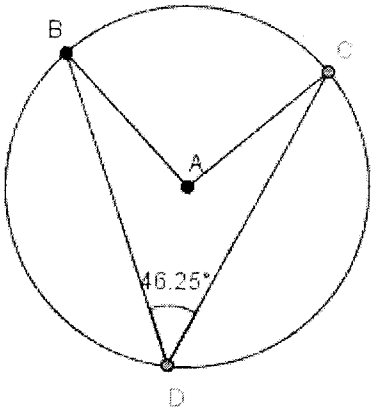
a. Arc BDC



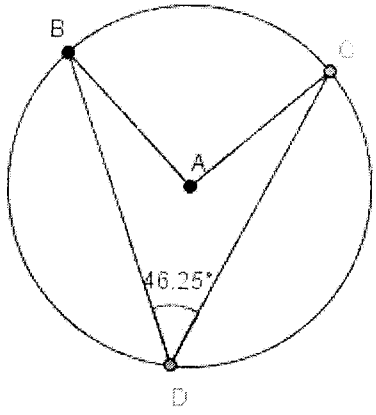
b. Arc FIH



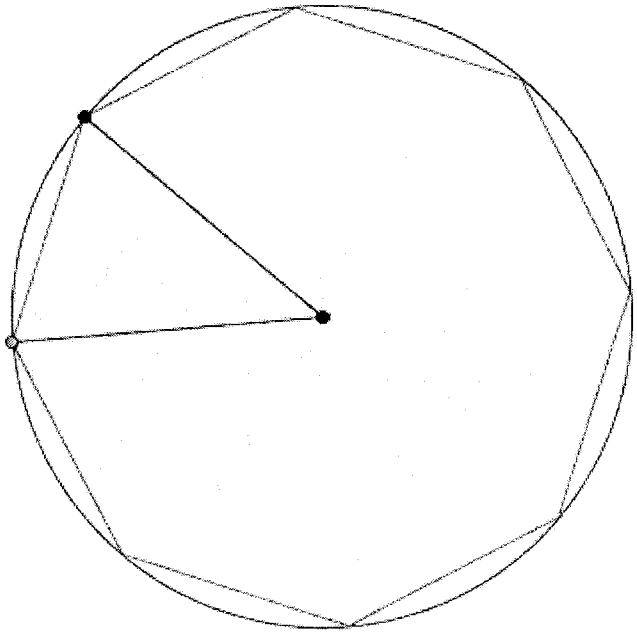
c. Minor Arc BC



d. Major Arc BC



13.) An 8 sided regular polygon (octagon) is inscribed in a circle of radius 10 cm; find the length of one side of the octagon.



Appendix F

Bibliography

Serra, Michael. Discovering Geometry : An Investigative Approach . 3rd. Emeryville, CA: Key Curriculum Press, 2003.