Introduction

My career at Keene State College has been nothing short of a great learning experience. I complete many different mathematics courses that have helped me to develop my knowledge. I really enjoyed how small my classes were because it allowed me to feel comfortable so I was able to ask for help. I was also able to make many great friends to collaborate with.

I have learned about the mathematicians in my History of Mathematics course that have made math what it is today. I have learned that men are not the only people that have made great contributions in mathematics. I have also been able to conquer my fears in classes that I have struggled with in my past. I was able to really understand what was going on in geometry class because I put my mind to it. I learned about all the different types of geometry that I had never heard of before. In hyperbolic geometry, a straight line is not a straight line anymore. In linear algebra I learned about matrices and how to use them to find the determinate. In algebra and analysis I learned how to write proofs. I still struggle with proofs but I can say that I have more confidence in my abilities now. Lastly, my Issues and Trends class has helped me to understand why the education in the US is lacking compared to the rest of the world. I feel from all that I have read and done in this class it has helped to shape my philosophy of mathematics education. All the classes that I have taken have helped me to feel more confident in my mathematical abilities.

Issues and Trends

In Issues and Trends, I learned a lot about mathematics education. We read a lot of articles about approaches to teaching mathematics. All the reading that we did really helped to shape my view and philosophy of mathematics education. I feel strongly that student's need to know the basics before they are ever given a calculator. We read many articles that allowed me to understand that in elementary classrooms teacher's let their students use calculators because the teacher's themselves do not know how to do the math. I also learned that there are many different approaches to teaching. However, the US is lacking in their mathematics teaching. Some of the reasons are because teachers are not taking the time to understand the underlying concepts. The just teach in the way that they were taught. We were able to see that the Japanese have a really great system of teaching. Their teachers have a lot of time to collaborate and discuss mathematics. In the US the teacher's do not have the same amount of time and more time is spent going over homework instead of learning new material. That is the reason why the US is so far behind compared to where the Japanese are in their learning.

NCATE Standards
Math 475 - Issues and Trends in Mathematics Education

Assignment 1 addresses the following standard.

1.3 Build new mathematical knowledge through problem solving. *(secondary and middle)*

Juanita's Problem

Assignment 2 addresses the following standards.

1.4 Monitor and reflect on the process of mathematical problem solving *(secondary and middle)*

3.4 Analyze and evaluate the mathematical thinking and strategies of others *(secondary and middle)*

Write a synopsis review of "Using a Model Approach to Enhance Algebraic Thinking in the Elementary School Curriculum" from *Algebra and Algebraic Thinking in School Mathematics*, Seventieth Yearbook, National Council of Teachers of Mathematics. Analyze and comment on the mathematical problem solving and strategies used by the Singaporean students as they solved the problems within the article.

Assignment 3 addresses the following standard.

8.5 Participate in professional mathematics organizations and use their print and on-line resources. *(secondary and middle)*

Go to NCTM's website to your chosen teaching level (elementary school, middle school, high school). Select an NCTM Focus Points Related Resource (elementary and middle school) or an NCTM Resources (high school). Write a synopsis review of the resource and give your reason for making the particular selection. If you cannot access it electronically, go to Mason Library to retrieve it.

Assignment 4 addresses the following standard.

8.6 Demonstrate knowledge of research results in the teaching and learning of mathematics. *(secondary and middle)*

Write a synopsis review of "An Investigation into the use of graphics calculators with pupils in Key Stage 2" from the *International Journal of Mathematical Education in Science and Technology*, 35 (2), 227-237. Discuss the research results and its implications for the teaching and learning of mathematics.

Assignment 5 addresses the following standard.

9.10 Demonstrate knowledge of the historical development of number and number systems including contributions from diverse cultures. *(secondary)*

Describe the development of the number systems of the Mayans, Chinese, and Japanese. Compare and contrast their numeration systems with those of other modern day cultures such as those from Central America, India, and Papua New Guinea.

Homework #1 – *my philosophy and goals of mathematics education* *Abby Dutch*

The reason I want to be a mathematics teacher is because I have my own philosophy of how a classroom should be run. I have had mathematics teachers in the past who have shaped my views and goals either in a positive or negative way. These philosophies and goals will help me to create a warm environment in which learning is encouraged and results yield increased knowledge.

According to my philosophy of mathematics education, being able to teach in a way that the students have a more active role in class is very important. As long as students are physically involved they are increasing their ability to understand concepts. If the class only requires taking notes, many concepts are not understood because students tend to zone out and not pay attention, missing crucial information. For many students mathematics is their least favorite subject. These students do not see how they will be able to use the math they are taught ever again. That is why it is important to use real life problems and examples whenever possible in class because it encourages students they are learning to benefit themselves.

It will also benefit students if classes are not always structured the same way everyday. For students, the class becomes too predictable if all you do is take notes every class. To the students class becomes less interesting, and there is no excitement. Even if the slightest adjustment is made students will not know what to expect and incorporating new activities gets students animated. It is my philosophy that something new needs to be brought to the classroom to challenge student's minds at least once a week. The atmosphere of a classroom can also add to the excitement of a student and encourage learning. A classroom has to have aspects displayed to encourage students and show them what can be done with mathematics in everyday life.

Teachers are very important because they are the ones who shape our future. They are role models that students look up to, and sometimes want to be like. It is important for teachers to be passionate and show their students that they matter and are not just another number in the classroom. Students are more apt to overcome struggles if they know that their teacher is passionate and caring about their subject matter and students. Teachers also need to be approachable so students feel comfortable asking for help. Having this relationship with students allows those students who struggle to succeed.

Students should feel comfortable in the classroom but at the same time realize they are held accountable for their actions. Each student will be disciplined if behavior permits and consequences will be consistent for every student. That way they know who is in control. It also creates a better learning environment for the students.

In order to create this great learning environment and follow my philosophy, I have set goals for myself so that when I become a teacher I am able to stick by my views. My goals are as follows:

1. Gain confidence when teaching in front of a class.
2. Try to relate to the students and understand who they are.
3. Make the classroom a comfortable place.
4. Incorporate different learning styles.
5. Be confident in what I am teaching by being prepared and organized with lessons.
6. Be strict so students know that what I say goes.

7. To get at least one student to change their view of math being their least favorite subject.

If I am able to follow my goals and adhere to my philosophy, I will be able to accomplish the one thing I have wanted since I was little, and that is to be a great teacher. A teacher who is ready to continue on the path of learning so I can constantly be increasing my knowledge which I will pass on to all the students I teach. Helping to create more opportunities and improve lives.

Topic Paper- teaching using technology Abby Dutch

Teachers have a very important role in a student's education. Their pedagogical views can either make or break whether or not a student enjoys a subject. More specifically this is what happens in many mathematical education classes, students lose the desire to learn mathematics because of some teacher's pedagogical strategies. That is why I feel it is very important to involve technology as long as it is used at appropriate times because it increases student's excitement during class. Students enjoy those times when they are doing something other than taking notes. For most students, the ability to do hands-on activities increases their knowledge to retain the material being taught. That is why I would incorporate Geometer's Sketchpad into a geometry class, Excel into a statistics class and the overhead projector with calculator hook up into an algebra class.

In geometry classes students sometimes have difficulties drawing their shapes by hand, to scale, and grasping some of the harder concepts that deal with constructions and proofs. When incorporating the use of Geometer's Sketchpad into a class it allows students to be actively engaged in their learning so they can visualize concepts and try different scenarios that may be difficult to understand on the chalk board. The software comes with tutorials that students ~~can do~~ that will allow them to become more familiar with the program. It is set up with a tool bar at the top of the screen which allows students to write text, draw points, segments, shapes, and it has measuring capabilities. By incorporating the use of Geometer's Sketchpad in lesson, there will be increased excitement for the students when using the technology because it is not something that they would be using all the time. Worksheets can be made for students ~~to do~~ when using the program which can eliminate ~~a lot of~~ distractions for students. However, the only downfall of incorporating this software in the classroom is that it will take students a little while to get acclimated ~~with all of~~ its capabilities and some students might struggle more than others.

The use of Excel in a statistics classroom has the same downfall as Geometer's Sketchpad; it has many capabilities that students need to know. When you open Excel it is set up like a spreadsheet. Students can input data into the boxes and then use its graphing capability, the cart wizard, to produce a variety of graphs. Incorporating the use of Excel into the classroom allows for students to graph data, calculate the mean, median, mode, standard deviation, and even the correlation coefficient. Having students use Excel in the classroom allows them to work on more difficult multi-step problems that would take way too long to do out by hand. The software allows students to build on what they have learned to calculate certain aspects of a problem that they can use when computing a more challenging problem by hand. Excel can be used to reinforce student's skills when graphing and performing calculations and also allow them to explore new skills.

In an algebra class, the use of an overhead projector with a graphing calculator and graphing calculators for all the students will help reinforce material that have been taught while exploring new concepts. The overhead and calculator attachment will project the screen that the students should have on their calculators and the teacher is able to show where buttons are on the calculator by pointing and verbalizing. Students will be able to graph linear functions, make different graphs as well as perform basic computations all on their calculators. The projector will help the students to start discussions when interpreting graphs and recognize patterns. It allows the students to see the larger picture and understand another method for finding the same result by hand.

However, the downfall is that some students want to take the easy route all the time when performing computations, forgetting the basic algorithms.

All in technology is important to incorporate into lessons part of the time after all the basics are learned or the help to teach the basics. It can be a very helpful tool in the classroom. Technology like Excel, Geometer's Sketchpad, overhead projectors, and calculators can be used to reinforce the basics and expand on newer material. Although it may make aspects of the class run more efficiently, it is very unreliable. Teachers always need to be prepared in cases where the technology is not running properly.

Synopsis- NCTM Article Abby Dutch

The article I chose to read is titled "Iterated Function Systems in the Classroom." This article discussed Iterated Function Systems (IFS) and how they can be useful in the classroom. This software allows students to do great things using fractal images. The author also suggests that this software can be used in a variety of classrooms, ranging from algebra to real analysis. The program emphasizes geometric transformations and really allows students to grasp the concepts. It is also free for anyone to download at invisiblegolg.com/math. At the site the author has a gallery of some of his student's work as well as many different downloads. The author suggests what a great geometric tool this is because it allows you to produce art while having fun doing math. In order for this program to be used in a classroom it is suggested that students become very familiar with the program first. Then students can be asked to make 5 different kids of images and answer questions relating to the images. There are endless questions to ask students in order to assess what they are doing. The author also suggests having an image contest. That way student's are encouraged to spend a lot of time working on their assignments. This program's capabilities are endless when it comes to transformations and making images.

Reaction-

I thought that this was an interesting article. I have never heard of anything like this. I like that there is a free download so you can test it out. The reason I choose this article is because in methods class we are working on a geometry unit. I know that I am not that knowledgeable of technology that can be used when teaching geometry so I thought this would be interesting. I think this could be a fun program for students because it allows them to be creative while learning at the same time. The author even said that the smartest kids are not always the ones with the most creative images. Those students who might other wise struggle in class are suggested to do well with this software package.

Synopsis- "Using a Model Approach…" Abby Dutch

The article titled "Using a Model Approach to Enhance Algebraic Thinking in the Elementary Classroom" discussed different approaches to solving word problems using models as a way of enhancing algebraic thinking. In Singapore they use this model approach with their students in elementary school as a problem solving method. The article then goes through some problems in which they use rectangles to represent these word problems. These challenging word problems can be solved in different ways depending on student's knowledge. Singapore's students know three different modeling procedures, the part- whole model, the comparison model, and the change model.

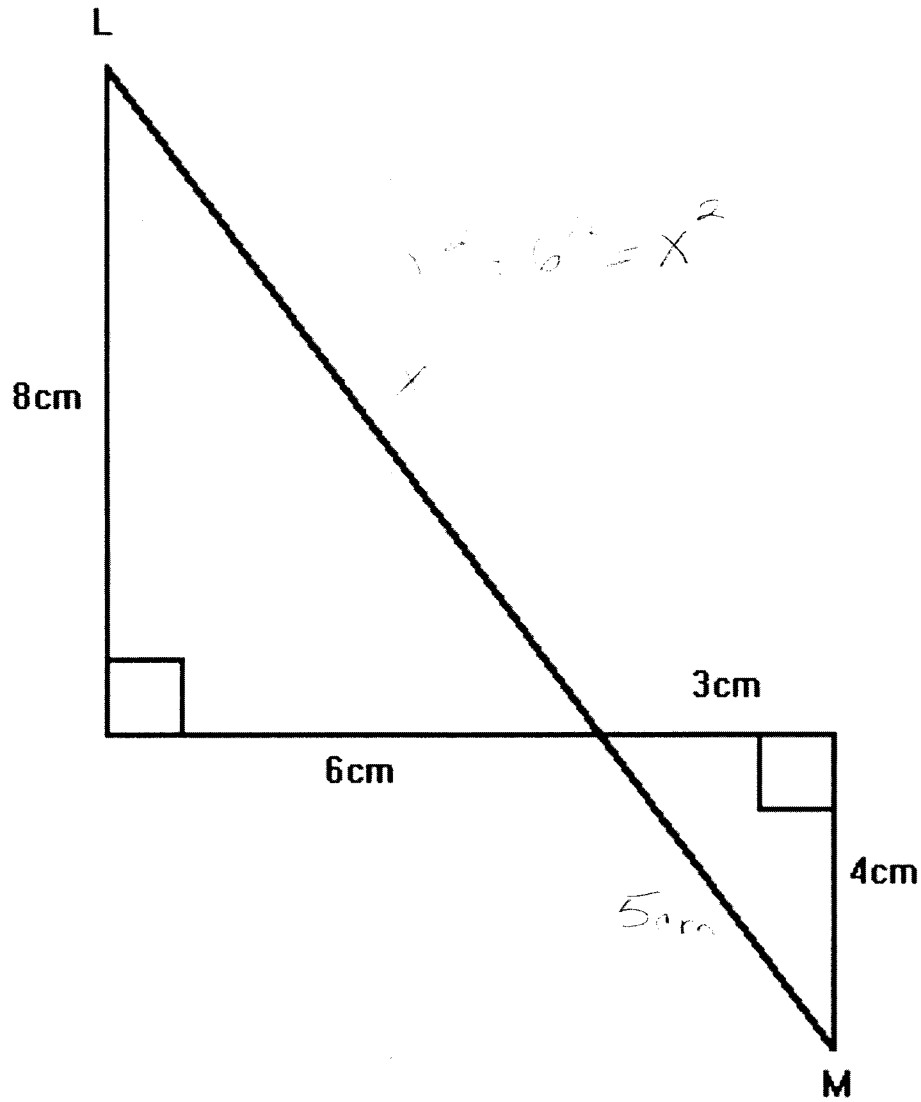
The part-whole model is used to illustrate the situation when a whole is composed of a number of parts. Students will be able to determine the whole once they are given the part. The comparison model demonstrates the relationship between two or more quantities when they are compared, contrasted, or described by differences. The last model is the change model. This model provides representations of the relationships between the new value of a quantity and its original value in a before and after situation. These different models that Singapore uses in elementary school are used to help foster algebraic thinking so that later on the students will be able to use a shortcut method to solve these same problems.

Reaction-

I thought that this was an interesting article. I wish that I had learned these methods for solving word problems. I know that today I still struggle with word problems because I never learned a good method. I think using models are a good way of teaching students word problems. I think it is important to build on students learning. This allows them to see that you can solve a word problem in many different ways.

Problem Solving Homework

Juanita Simpson was in her math class. She was told to make a figure with four thumbtacks, an elastic band and a piece of wood. She made the following figure. Her math teacher measured some of the sides of her figure and told Juanita to find the length of the side LM in mm. Can you help her?



Solve this problem two different ways. Thoroughly explain your solutions to this problem. What new mathematical knowledge could be obtained by working on this problem? Finally reflect on the problem solving processes used in the two problems.

Problem Solving Homework- *Juanita's Problem* Abby Dutch

Directions: Solve this problem two different ways. Thoroughly explain your solutions to this problem. What new mathematical knowledge could be obtained by working on this problem? Finally reflect on the problem solving processes used in the two problems.

Solution A: First we can consider the larger triangle with sides 8cm and 6cm. Because we are given that this is a right triangle we can use the Pythagorean Theorem.

$$8^2 + 6^2 = x^2$$
$$64 + 36 = x^2$$
$$100 = x^2$$
$$10\text{cm} = \text{x}$$

We can now consider the smaller triangle with sides 3cm and 4cm. Similarly we can use the Pythagorean Theorem.

$$3^2 + 4^2 = y^2$$
$$9 + 16 = y^2$$
$$25 = y^2$$
$$5cm = y$$

Since we want to find LM in mm we can add the values for x and the values of y.

$$x + y = LM$$
$$10 + 5 = LM$$
$$15cm = LM$$

We now want to convert 15cm to mm.

$$15cm * \frac{10mm}{1cm} = 150mm = LM$$

Method B: First we can convert each of the measurements to mm from cm.

$$8cm * \frac{10mm}{1cm} = 80mm$$
$$6cm * \frac{10mm}{1cm} = 60mm$$
$$4cm * \frac{10mm}{1cm} = 40mm$$
$$3cm * \frac{10mm}{1cm} = 30mm$$

We can now use what we know about triangles. Since we are given that both triangles are right we can use the Pythagorean Theorem. First we can consider the larger triangle which has sides 80mm and 60mm.

$$80^2 + 60^2 = x^2$$
$$6400 + 3600 = x^2$$
$$10000 = x^2$$
$$100mm = x$$

We can now use the triangle for the smaller triangle with sides 40mm and 30mm.

$$40^2 + 30^2 = y^2$$
$$1600 + 900 = y^2$$
$$2500 = y^2$$
$$50 = y$$

Since LM is made up of x and y we can now calculate LM in mm.

$$x + y = LM$$
$$100 + 50 = LM$$
$$150mm = LM$$

Thus, both methods lead us to the same result for LM.

What new mathematical knowledge could be obtained by working on this problem?

The mathematical knowledge that could be obtained would be how to use the Pythagorean Theorem by finding missing lengths. As well as learning to keep units and recognizing when you need to convert to different ones.

Reflect on the problem solving process used in the two problems.

In the first solution I used what I knew about right triangles. Then once LM was found in cm I used what I knew about conversions and converted LM to mm. In the second solution I used what I knew about conversions first and then after that what I knew about right triangles to get LM in mm.

Synopsis- Japanese Lesson Study Abby Dutch

I read the two articles, "Challenges to Importing Japanese Lesson Study: Concerns, Misconceptions, and Nuances," and "A Practical Guide to Translating Lesson Study for a US Setting." These two articles discussed how the US is trying to incorporate a Japanese method into US schools. This method is lesson study in which teachers work together to design lesson plans for specific classrooms. The process begins with the development of a lesson plan that is then tested by one teacher while the rest observe. Once the lesson has been completed all the teachers revised the lesson to see if there are any improvements that need to be made.

However, there are challenges associated with lesson study. One being that US teachers' do not have the time to plan each and every lesson for every teacher in the school. So they will have to pick and choose what lessons they will work with. Teachers will also have to teach in front of their peers. This could be nerve racking for many US teachers. Of course with these challenges there are many benefits as well. Lesson study will help teachers to focus on students' work ability, reflect and adapt their own teaching while focusing on students' needs. Teachers are able to work together combining all their knowledge into a single lesson. When teachers do this, they create many different ways to obtain the same goal. In general the authors of both articles suggest how the incorporation of lesson study is a good practice for US teachers to use.

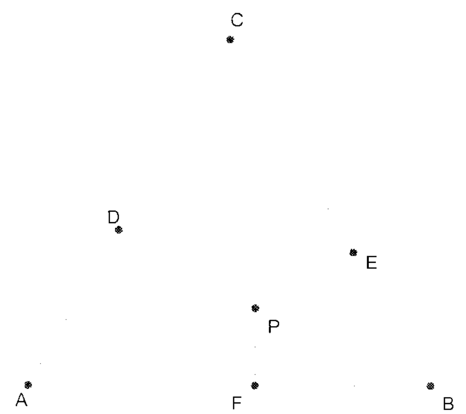
Reaction-

I thought that these two articles were very interesting. I never knew that Japanese teachers used lesson study. I think that if US teachers are able to incorporate lesson study into their schools then there will be better teaching. When teachers work together more ideas are generated and therefore better teaching happens. Also, when teachers work collaboratively they are able to increase their understandings about topics because they will work with other teachers who are stronger in an area they are not. In general I believe lesson study is a good idea. However, I do not think most US teachers have the time to incorporate it into schools. So I do not know if US schools will stick with it.

Geometry

I was really scared to take Geometry because I am not an abstract thinker. I worked that much harder and after completing the class it was very rewarding. I was able to accomplish something that I felt was a weakness of mine and now I feel more confident with my ability in geometry. In class I learned that there are many different types of geometry other than Euclidean Geometry. I was able to see what I always thought was a strait line isn't anymore. I learned how to use Geometer's Sketch Pad, and it really helped me to understand translations. I was able to work on improving my ability to write proofs.

3- 9. On your Geometer's sketchpad worksheet with the triangle, write a theorem (assume that the side of the quilateral triangle has length x) that expresses the situation above and then prove it. (Hint: The proof requires auxilliary lines and uses the area idea)



CA = 6.76 cm
AB = 6.76 cm
CB = 6.76 cm

CA = AB = CB = K; Where K is a constant

m∠ACB = 60.00°
m∠BCA = 60.00°
m∠BAC = 60.00°

DP = 2.65 cm
EP = 1.90 cm
FP = 1.30 cm

DP+EP+FP = 5.85 cm

As you move point P upward segments $\overline{DP}$ and $\overline{EP}$ get smaller and $\overline{FP}$ gets larger. The addition of all the segments says the same. When you move point P down, segments $\overline{DP}$ and $\overline{EP}$ get larger while $\overline{FP}$ gets smaller. The addition of the segments stays the same again.

Theorem: ΔABC is an equalateral triangle. Point P is contained in ΔABC. $\overline{PE} \perp \overline{CB}$, $\overline{PD} \perp \overline{CA}$, and $\overline{PF} \perp \overline{AB}$. Prove DP + EP + FP = K, for some constant K.

Proof: Since we know that ΔABC is an equaliteral triangle we know that CA = CB = AB. We also know that m∠CAB = m∠CBA = m∠ACB = 60°. We can draw auxillary line segment joing point P with B, point P with C, and point P with A. We can now consider the area of all the smaller triangles contained in ΔABC because we know that $\overline{PE} \perp \overline{CB}$, $\overline{PD} \perp \overline{CA}$, and $\overline{PF} \perp$ AB so all the triangles have 90° angles. By E26(a) we are able to use the formula area = $\frac{1}{2}$b*h, the areas of all the smaller triangles follows.

Area ΔCPD = $\frac{1}{2}$DP * DC   Area ΔCEP = $\frac{1}{2}$EP *EC

A rea ΔADP = $\frac{1}{2}$DP * DA   Area ΔAFP = $\frac{1}{2}$FP * AF

Area ΔBFP = $\frac{1}{2}$FP *FB   Area ΔBEP = $\frac{1}{2}$EP * BE

We know that the area of all these smaller triangles are equal to the area of ΔABC. By E26(b), the area of the equaliteral triangle equals $\frac{x^2\sqrt{3}}{4}$. So when we add all the little triangles together they are suppose to equal $\frac{x^2\sqrt{3}}{4}$.

$$\frac{x^2\sqrt{3}}{4} = \frac{1}{2}DP * DC + \frac{1}{2}DP * DA + \frac{1}{2}FP * FB + \frac{1}{2}EP * EC + \frac{1}{2}FP * AF + \frac{1}{2}EP * BE$$

$$\frac{x^2\sqrt{3}}{4} = \frac{1}{2}(DP * DC + DP * DA + FP * FB + EP * EC + FP * AF + EP * BE)$$

We can then simplify by taking out common terms.

$$\frac{x^2\sqrt{3}}{4} = \frac{1}{2}((DP * DC + DP * DA) + (FP * FB + FP * AF) + (EP * EC + EP * BE))$$

$$\frac{x^2\sqrt{3}}{4} = \frac{1}{2}(DP(DC + DA) + FP(FB + AF) + EP(EC + BE))$$

We can see that DC + DA = CA, FB + AF = AB, and EC + BE = CB. CA = AB = CB because they are the sides of ΔABC which is an equaliteral triangle. So we can call these sides x and replace x in the above equation.

$$\frac{x^2\sqrt{3}}{4} = \frac{1}{2}(DP(x) + FP(x) + EP(x))$$

Take out a common term.

$$\frac{x^2\sqrt{3}}{4} = \frac{1}{2}x (DP + FP + EP)$$

Multiply both sides by 2 and $\frac{1}{x}$ and simplify.

$$\frac{X\sqrt{3}}{2} = DP + FP + EP$$

Thus, DP + FP + EP is a constant because x equals the sides ΔABC and they will always remain the same no matter when you move point P.

9-4(a). Use the result from problem 9-3 to find the images of A(3,4) and B(5,0) [illegible] reflection [illegible]

solution: We can take the result from 9-3 for $R_{[\alpha,\beta,0]}$ which follows:

$$R_{[\alpha,\beta,0]} = \begin{bmatrix} \frac{\beta^2-\alpha^2}{\beta^2+\alpha^2} & \frac{2\beta\alpha}{\beta^2+\alpha^2} & 0 \\ \frac{2\beta\alpha}{\beta^2+\alpha^2} & \frac{\alpha^2-\beta^2}{\beta^2+\alpha^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We were also given $y = \frac{\alpha}{\beta}x$ and $y = 3x$ so

$$\frac{3}{1} = \frac{\alpha}{\beta}$$

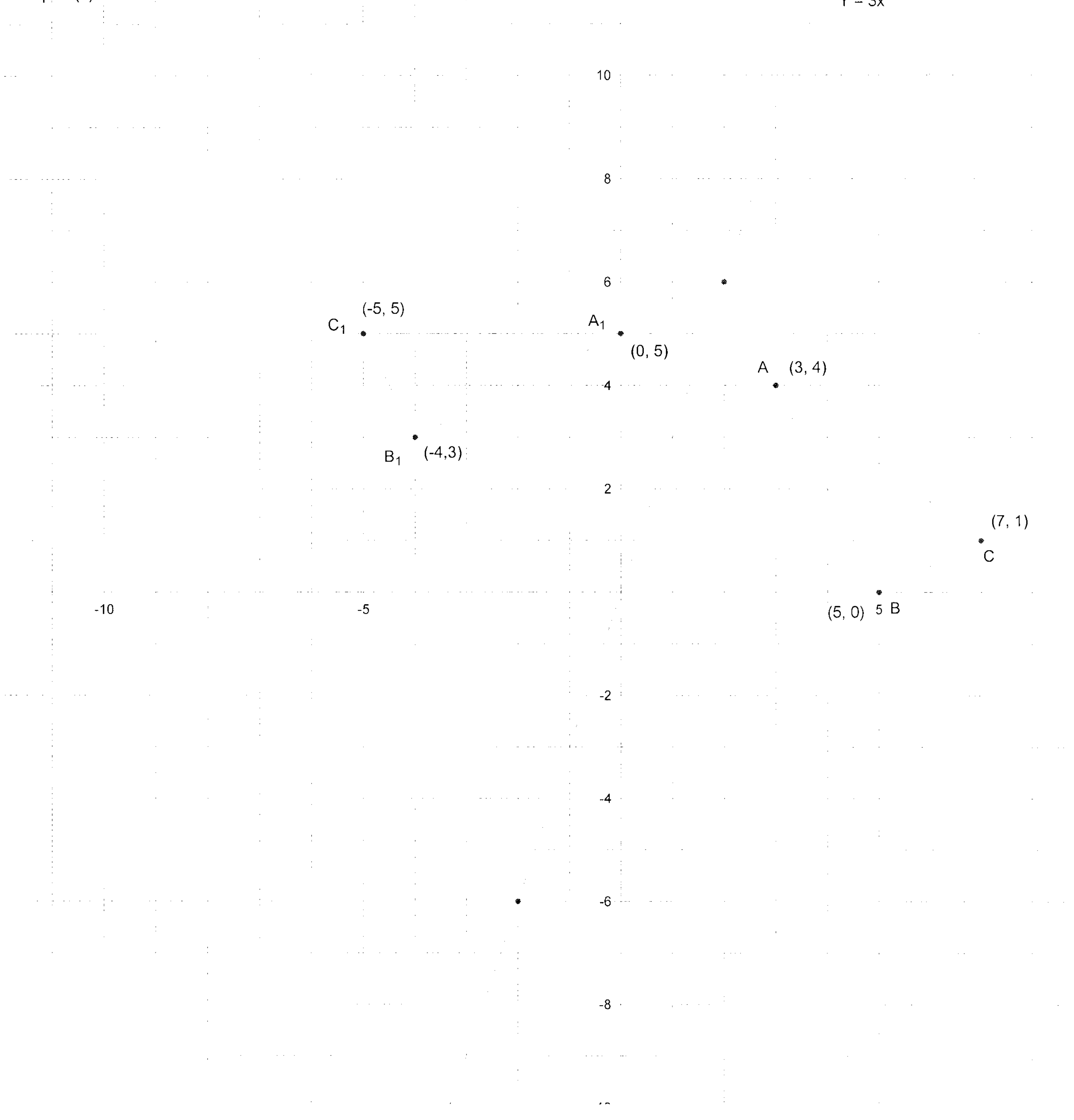
Thus, $\alpha = 3$ and $\beta = 1$. We can now substitute these values into $R_{[\alpha,\beta,0]}$

$$R_{[\alpha,\beta,0]} = \begin{bmatrix} -\frac{8}{10} & \frac{6}{10} & 0 \\ \frac{6}{10} & \frac{8}{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4/5 & 3/5 & 0 \\ 3/5 & 4/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = R_{[\alpha,\beta,0]}A = \begin{bmatrix} -4/5 & 3/5 & 0 \\ 3/5 & 4/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -12/5 + 12/5 \\ 9/5 + 16/5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}$$

$$B_1 = R_{[\alpha,\beta,0]}B = \begin{bmatrix} -4/5 & 3/5 & 0 \\ 3/5 & 4/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -20/5 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$ ✓

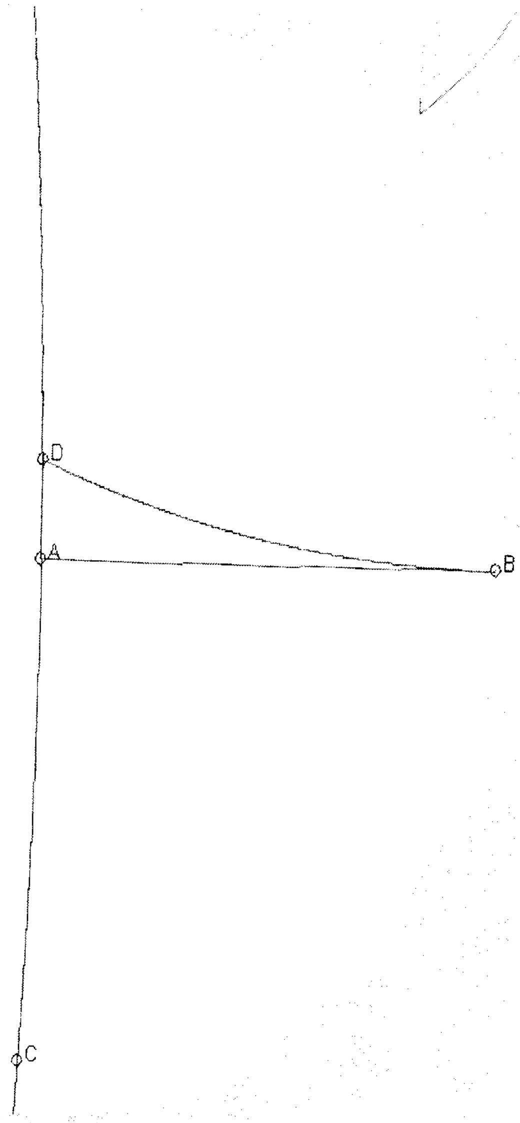
9-4 (b). Graph the line y = 3x and then use the reflection capability to reflect ΔABC about the lline to $\Delta A_1B_1C_1$. Label the vertices and type in the coordinates in your sketchpad worksheet. Check that the vertices A_1, B_1, and C_1 found in part (a) correspond with those found on the sketchpad worksheet. Print a copy of the worksheet and include it with part (a).



5-8.)

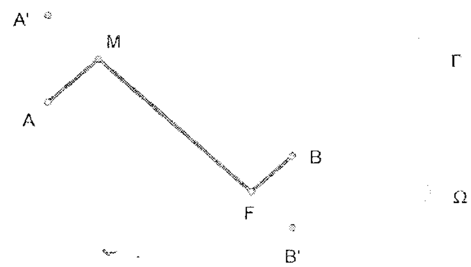
$AD = x$	$\angle ADB = \alpha$	e^{-x} (5 decimal)	$\tan(\frac{\alpha}{2})$ (5 deci
0.361	70.6°	0.69698	0.70804
0.797	48.8°	0.45068	0.45362
1.314	30.2°	0.26874	0.26982
1.918	16.8°	0.14690	0.14767
4.4	1.4°	0.01228	0.01222

✓



10 Formula given x as a function of α?

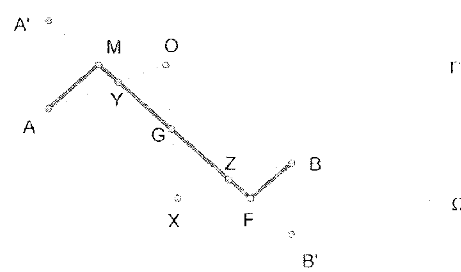
6-5.) For the lines ∏ and δ, assume ∏ ∥ δ. Determine the path from A to Γ to Ω and then to B that is the shortest. Show the construction using the appropriate transformation(s) on Geometer's Sketchpad. Use the thick line display for the path and the dashed line display for auxiliary lines. Then write a proof to justify that your construction yields the shortest path.



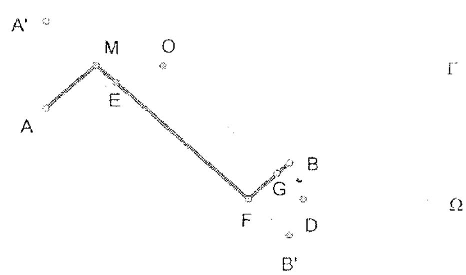
Solution: Reflect point A about line Γ to give point A'. Reflect point B about Ω to give B'. We can draw $\overline{A'B'}$ and where $\overline{A'B'}$ intersects Γ we can label that point M and where $\overline{A'B'}$ intersects W we can now label that point F. We can now draw $\overline{AM}$, $\overline{MF}$, and $\overline{FB}$. We can see that A'B' = the path from A to Γ to Ω to B since A and A' are rotations and B and B' are also rotations thus, A = A' and B = B'.

We now want to consider a couple of different cases.

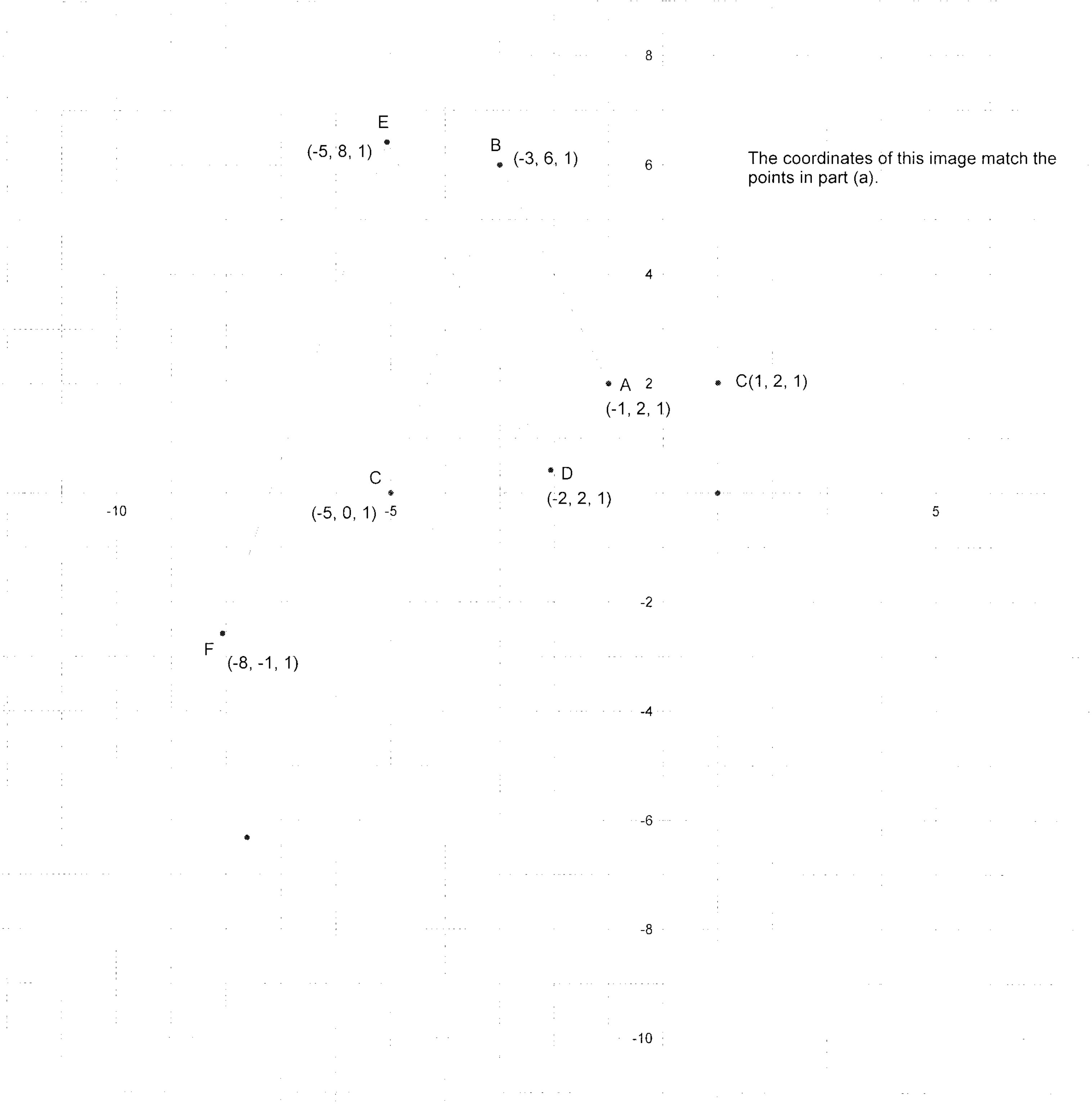
Case 1: Assume there is another point O on Γ such that point O is to the right of point M as seen in the figure to the right. We can also assume that there is another point X on Ω such that point X is to the left of point F. We can draw auxiliary lines $\overline{AO}$, $\overline{OX}$, and $\overline{XB}$. We can now label the intersection of AO with $\overline{A'B'}$ point Y, label the intersection of $\overline{A'B'}$ and $\overline{XY}$ point Z, and label the intersection of $\overline{OX}$ and $\overline{YZ}$ point G. We now want to show that A to M to F to B is the shortest path. We can now consider ΔYOG. By N9 (a) YO + GO > YG and similarly for ΔZXG, GX + ZX > GZ. From this we can see that the path of A to O to X to B is greater than the path from A to M to F to B. Thus, path A to M to F to B is the shortest path.



Case 2: Assume there is another point O on Γ such that point O is to the right of point M as seen in the figure to the right. We can draw auxiliary lines $\overline{AO}$, $\overline{OD}$, and $\overline{DB}$. We can now label the intersection of AO with $\overline{A'B'}$ point E, and label the intersection of $\overline{A'B'}$ and $\overline{XY}$ point G. We can now draw OF creating two triangles ΔEOF and ΔGOF. First we can consider ΔEOF. By N9(a) EO + FO > EF. We can now consider ΔGOF. By N9(a) GO + FG > FO but we already know EO + FO > EF so it must be that GO + FG > EF. From this we can see that the path from A to O to D to B is greater than the path from A to M to F to B. Thus, in both cases the path from A to M to F to B is the shortest then this must be the shortest path from A to Γ to Ω and then to B.



10-3 (b). Consult the Geometer's Sketchpad tutorial and determine how to perform a dilation. On a coordinate graph plot points A, B, and C and construct ΔABC. Use the dilation capability to dilate ΔABC with center and ratio computed in part (a). Print a copy of the worksheet.

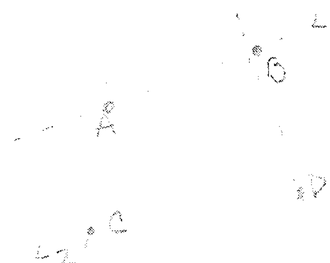


The coordinates of this image match the points in part (a).

Prove: There exists four points [illegible]

[illegible]

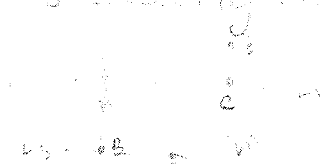
Proof: By Axiom [illegible] there exists a point, we will designate to be point A. By axiom 2, every point is contained on exactly two different lines so we will [illegible] point A [illegible] L_1 and L_2. By axiom 1, every line contains exactly two different points, so we will designate the other point on L_1 to be point B and the other point on L_2 to be point C. By axiom 2, every point is contained on exactly two different lines, we will designate [illegible] B to be L_3. By axiom 1, every line contains exactly two different points [illegible] designate the other point on L_3 to be Point D (see figure).



We are now going to consider two different cases.

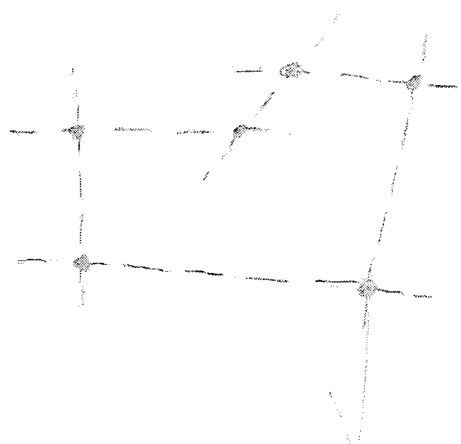
[illegible]

Case 1: By axiom 2, every point C is contained on exactly two different lines, we will choose the second line to be L_4. By axiom 1, every line L_4 contains exactly two distinct points so we will designate the other point on L_4 to be Point E. This contradicts Axiom 3, for each point A there is exactly one point E such that there is no line containing points A and E, point C also does not have a line containing A and C therefore contradicting that there is exactly one line. Thus, this case cannot happen.



[illegible] 2! By axiom 1, every [illegible] contains exactly 2 distinct points, we will designate the other point to be point C. By axiom 2, every point [illegible] contained in exactly [illegible] lines [illegible] designate [illegible] to be the line [illegible]. By axiom 3, for each point A, there is exactly one point D such that there is no line containing A and D. Axiom 3 holds for all the points. Thus, [illegible] geometry has [illegible] points.

Construct a model that satisfies axiom 1, 2, and 4 but not 3.



Linear Algebra

I really enjoyed linear algebra because algebra is my favorite. This class allowed me to work hard at something I enjoy doing. I worked with matrices, finding determinates, multiplying, and adding them. We had a project that we had to do during the semester in which we picked a section from our text and taught ourselves how to understand the concept. I was really excited about this assignment because it gave me the ability to use what I knew and teach myself how to understand a concept all on my own.

MATH 231 Stanish
May 3, 2007
Final Exam

Name: Abby Dutch

You may NOT use your calculator to complete the following problems. Once you have completed these problems, turn them in, and then you will receive the reminder of the exam on which you may use your calculator.

1. (8 pts) Consider the following linear system:

$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ 2x_1 - x_2 + 7x_3 &= 8 \\ -x_1 + x_2 - 5x_3 &= -5 \end{aligned}$$

Solve the system by reducing the augmented matrix to reduced row-echelon form.

2. (8 pts) Let $A = \begin{bmatrix} 1 & 0 & -3 \\ 4 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 & 0 & 1 \\ 1 & -1 & 4 & 1 \\ 0 & 3 & 1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & -1 & 4 \\ -3 & 0 & 0 \end{bmatrix}$

2 × 3   3 × 4

Calculate each of the following, if it is defined.

a) AB

= 5 -11 -3 1

19 -1 -2 3

0 - 4 + 2

4 - 1 + 0

b) BA

Can't be done because 3 × 4 ... 2 × 3

c) $A - C$

$= \begin{bmatrix} -2 & 1 & -7 \\ 7 & -1 & 2 \end{bmatrix}$

d) C^T

$= \begin{bmatrix} 2 & -3 \\ -1 & 0 \\ 4 & 0 \end{bmatrix}$

3. (4 pts) Find the determinant of each of the following matrices:

a) $A = \begin{bmatrix} 3 & 2 & -1 \\ 5 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

$-2 \begin{vmatrix} 5 & 2 \\ 0 & 3 \end{vmatrix} + 0 - 0$

15 - 2

13

det(A) = -26

b) $A = \begin{bmatrix} 8 & 4 & 25 & 0 \\ -50 & 12 & 1 & 0 \\ 0 & 13 & -4 & 0 \\ 7 & 68 & 100 & 0 \end{bmatrix}$

det(A) = 0

MATH 231 Stanish
May 3, 2007
Final Exam

Name: ____________________

You may use your calculator in any way on the following problems.

4. (10 pts) Consider the linear system

$$2x_1 + 3x_2 = 1$$
$$3x_1 + 4x_2 = 7$$

a) Write the linear system in the form $A\mathbf{x} = \mathbf{b}$.

$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

$A\mathbf{x} = \mathbf{b}$: $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$

b) Use an inverse matrix to solve the linear system.

$A^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

$A^{-1}\mathbf{b}$: $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 17 \\ -11 \end{bmatrix}$

c) Use Cramer's Rule to solve the linear system.

$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ $\det(A) = -1 \neq 0$ $A_1 = \begin{bmatrix} 1 & 3 \\ 7 & 4 \end{bmatrix}$ $\det A_1 = -17$

$A_2 = \begin{bmatrix} 2 & 1 \\ 3 & 7 \end{bmatrix}$ $\det(A_2) = 11$

$x_1 = \frac{|A_1|}{|A|} = \frac{-17}{-1} = 17 = x_1$

$x_2 = \frac{|A_2|}{|A|} = \frac{11}{-1} = -11 = x_2$

5. (8 pts) For the matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$,

a) find the characteristic equation

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \lambda - 2 & -1 \\ -3 & \lambda - 4 \end{bmatrix}$$

$$= (\lambda - 2)(\lambda - 4) - (-1)(-3)$$

$$\lambda^2 - 6\lambda + 8 - 3$$

$$\boxed{\lambda^2 - 6\lambda + 5 = 0}$$

b) find the eigenvalues

$$\lambda^2 - 6\lambda + 5 = 0$$

$$\lambda - 5 \qquad \lambda - 1$$

$$\boxed{\lambda = 5} \qquad \boxed{\lambda = 1}$$

c) find the corresponding eigenvector for **one** of the eigenvalues.

$\lambda = 1$

$$\begin{bmatrix} -1 & -1 & 0 \\ -3 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

$$x_2 = t$$

$$x_1 = -t$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -t \\ t \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ [illegible]

$\lambda = 1$

6. (8 pts) In each of the following, determine whether the set W is a subspace of the vector space V. Be sure to justify your answer.

a) $V = R^2$ $\quad W = \{(x,y) : x \text{ is an integer}\}$

b) $V = M_{2,2}$ $\quad W =$ the set of matrices of the form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$.

7. (4 pts) Determine whether the set $S = \{(1,3,0),(-1,4,2),(1,-1,5)\}$ is linearly independent or linearly dependent.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 4 & -1 & 0 \\ 0 & 2 & 5 & 0 \end{bmatrix} \underset{rref}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad x_3 = 0$$

S is linearly independent

8. (4 pts) Determine whether the set $S = \{(-2,5,0),(4,6,3),(1,5,0)\}$ spans R^3.

$$A = \begin{bmatrix} -2 & 4 & 1 \\ 5 & 6 & 5 \\ 0 & 3 & 0 \end{bmatrix} \qquad \det(A) = 45 \neq 0$$

yes; S spans R^3

9. (6 pts) Determine whether the set $S = \{1, x+5, x^2+x\}$ is a basis for P_2.

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 5 & 0 & 0 \end{bmatrix} \sim$$

independent

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 5 & 0 \end{bmatrix} \qquad \det = -1 \neq 0$$

yes S

10. (8 pts) Determine whether each of the following functions is a linear transformation. Be sure to justify your answer.

a) $T : R^2 \rightarrow R^2$ defined by $T(x,y) = (x + y, 4x)$

$u = (1, 3)$ $v = (2, 4)$

$u + v = (3, 7)$

$T(u+v) = T(3,7) = \ldots = (10, 12)$

$(10, 12)$

$c = -2$

Yes

b) $T : R^3 \rightarrow R^2$ defined by $T(x,y,z) = \left(x + y - z, \frac{1}{2}\right)$

$(0, \frac{1}{2}) + (3, \frac{1}{2}) = \ldots \notin W$

No

11. (8 pts) Use the standard matrix to determine whether the linear transformation $T(x_1,x_2,x_3) = (x_1 - 2x_2, x_3, 4x_1 + x_3)$ is invertible. If it is, find its inverse.

$T(1,0,0) = (1, 0, 4)$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 4 & 0 & 1 \end{bmatrix}$$

yes invertible

$$\begin{bmatrix} 0 & -1/4 & 1/4 \\ -1/2 & -1/8 & 1/8 \\ 0 & 1 & 0 \end{bmatrix}$$

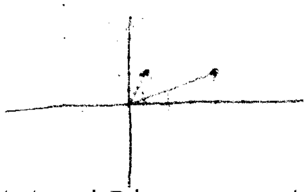
$T(x_1, x_2, x_3) = (0x_1 - 1/4(2x_2) + 1/4, -1/2$

12. (4 pts) Let $T(x,y) = (4x,y)$.

a) Identify the transformation geometrically.

horizontally

b) Graphically represent the transformation for an arbitrary vector in the plane.



13. (6 pts) Let A and B be $n \times n$ matrices such that AB is singular. Prove that either A is singular or B is singular.

AB is singular det (A) = 0 or det (B) = 0 Therefore, A is singular or B is singular.

14. (6 pts) Let $S = \{\mathbf{u}, \mathbf{v}\}$ be a linearly independent set in a vector space V. Let c be a nonzero scalar. Prove that the set $T = \{c\mathbf{u}, c\mathbf{v}\}$ is also linearly independent.

Proof: Let S = {u, v} be a linearly independent set in Vector space V. Let c be a nonzero scalar. Show T = {cu, cv} is linearly independent

Consider

$$c_1 u + c_2 v = 0$$

[illegible]

we have $c_1 = 0$ and $c_2 = 0$. [illegible], T = [illegible] independent.

15. (8 pts) Writing Problem: In a sentence or two, explain each of the following concepts.

a) A system of linear equations is consistent.

[illegible] system is always consistent because $c_1 = 0$, [illegible]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{matrix}$$

b) The dimension of a vector space V is n.

This means that the [illegible] of a [illegible] other the vector that is given.

ex. $M_{2,2}$ Dim = 4 S = { [illegible] }

The number of elements in the set is equal to the number [illegible]

History of Mathematics

In the History of Mathematics class I learned a lot about the early mathematicians and their contributions in mathematics. I did a project that helped me to understand that there are women that have made big contributions in mathematics. All we hear about is what men have done. I did not like that so I wanted to learn about what women have done. I was very impressed to find that women have made big contributions like men but, they are just not given the same recognition. Women earlier in history were not allowed to do anything other than work in the kitchen. This was an interesting class.

Women in Mathematics

Melissa Shinkwin
Abby Dutch

Overview of Presentation

- Men vs. Women during the late 19^{th} century
- The first six females to receive their PhDs in mathematics, in numerical order.
  - Winfred Edgerton Merrill (1886)
  - Ida Metcalf
  - Ruth Gentry
  - Charlotte Cynthia Barnum
  - Roxana Hayward Vivian
  - Charlotte Pengra (1901)
- Conclusion

Men vs. Women

Prior to the 19th century

- Women were perceived as the typical housewife.
- Men held all of the power.

Changes began to take place due to modernization and industrialization

- Women became more visible to society.
- An education became more acceptable for the women.

Winifred Edgerton Merrill

- Born- September 24, 1862 in Ripon, Wisconsin
- Her family was financially stable.
- Death- September 6, 1951.

Winifred

Academic Years

- Received bachelors degree from Wellesley College in 1883.
- A contribution that she made was the first ever computation of a comet.
- Initially denied a doctorate from Columbia University.
- She received her PhD in Mathematics from Columbia University in 1886.
  - Her dissertation was titled- “Multiple Integral”

Winifred

- What is a multiple integral?
  - is a type of definite integral extended to functions of more than one variable.
- She was the first American female to receive a PhD in Mathematics.

Winifred

Professional Years

- Founded Barnard College
  - Was New York's first secular school to award women liberal art's degrees.
- She taught mathematics at several institutions.
- She founded Oaksmere School for girls in 1906
- Following her retirement she wrote many published journals on educations.
- Also became a public speaker.

Roxanna Hayward Vivian

- Born- December 9, 1871
  - In Hyde Park, Boston, Massachusetts.
- Death is unknown

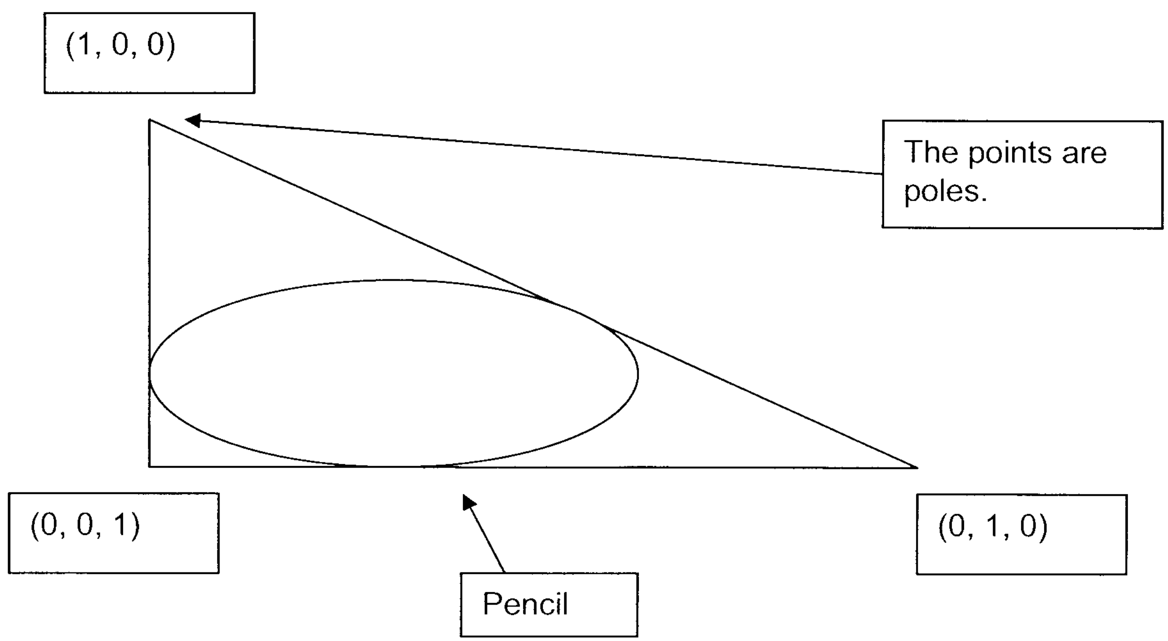
Roxanna

Academic Years

- BA degree from Wellesley College in 1894
  - In Greek and Mathematics
- She then taught for a year at a private preparatory school in Massachuesetts.
- Following teaching, she became a graduate student at the University of Pennsylvania
  - 1901- she received her PhD
    - Became the first woman from University of Pennsylvania to do so.

Roxanna

- Her dissertation was titled " Poles of a Right Line with Respect to a Curve of Order n."
  - Projective Geometry deals with curves in space and tangents which are described with right lines



Roxanna

Professional Years

- Became an instructor of mathematics in 1901 at Wellesley College
  - Associate professor 1908
  - Full professor in 1918
- Professor of Mathematics and Dean of Women at Hartwick College in New York (1929-1931)
- Instructor in Mathematics and Dean of Girls at the Rye Public High School (1931-1935)

Conclusion

- Overcame adversity by achieving their PhD
- PhDs came from a variety of different locations.
- Overcame misperceptions of the time.

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Algebra and Analysis

Algebra and Analysis was a very challenging class because it was abstract. The whole class involved writing lots of proofs. I struggled with this class but I worked hard to master writing proofs. I do not think that I have mastered proofs but I know my skills are better now. I worked on a project during the semester to understand the proofs of pi and Euler's number. This project helped me to be able to interpret other people's proofs. I worked with my group members to pick apart the meaning of the proofs. I feel that I learned a method for writing proofs and a way for interpreting them.

Standard #: 3.1, 3.2, 3.3

Pi (π) and Euler's number (e)

Abby Dutch
Ryan Farnsworth
Torey Cutting
Algebra and Analysis
O. Johnson
11/14/07

When dealing with mathematics there are many simple and complex topics which need to be explored and understood in order to use mathematics. For example, to fully understand mathematics one must first master the topic of numbers. Numbers can take many forms; they can either be positive, negative or even zero. In mathematics these types of numbers are called integers. To further build upon the idea of integer, there are numbers called rational numbers which fill in some of the gaps in-between the integers. These numbers take the form of fractions and "nice" decimals. To be a terminating decimal means have a place where the digits of the number come to an end. This now leads us to numbers that are irrational. These special kinds of numbers are numbers in which the decimal term does not terminate and can not be written as a fraction of integers. Two specific examples of irrational numbers are pi (π) and Euler's number (e). These two irrational numbers are able to provide a value for two different concept areas in mathematics and can be proved to be irrational.

When we look at the rational numbers versus the irrational numbers one can see a major difference. As stated above, rational numbers have a "nice" decimal. We can look at a rational number as a quotient. This means that a rational number can be written in the form $\frac{a}{b}$ where a,b are integers. While it is in this form, the b value cannot equal zero. A few examples of this can be seen in Table A1, which is found in the Appendix. If a number cannot be written in the form $\frac{a}{b}$ then it does not have a terminating decimal and the number is then called irrational.

Contrary to the rational numbers, irrational numbers can be viewed as "never ending". Even though the values of irrational numbers are never ending, they are still elements of the real numbers. However, an issue then arises when we look at the value $\frac{1}{3}$. We can calculate $\frac{1}{3}$ out in decimal form to be $.\overline{3}$. This decimal as we know is never ending; the three repeats. But, since we can write the value of $.\overline{3}$ in the form of $\frac{1}{3}$ it satisfies the requirement of being a rational number because it fits into the form $\frac{a}{b}$. This can be seen in Table A1, which is found in the Appendix. This leads us to two very special irrational numbers that have two different histories.

Our first irrational number is pi (π). Pi represents the circumference of any circle divided by its diameter [4]. Numerically written as 3.14159... This irrational number comes from as far back as the ancient Babylonian, Egyptians, Greek, and Indian geometers in which they calculated the area of a circle by measurement. Many mathematicians had various approximations for pi. However, it was not until Archimedes that the calculation of pi was formed. He used the Pythagorean Theorem to approximate the area of a circle. He did this by finding the area of two regular polygons, one inscribed and one circumscribed. Since the area of the circle is found between the two, this left for an upper and lower bound. Knowing this Archimedes knew that this was not the value for pi but rather an approximation of its limits. So Archimedes showed that pi was an irrational number found between $3\ \frac{1}{7}$ and $3\ \frac{10}{71}$ [1]. We can now look at the proof of pi and show that it is an irrational number.

Pi's irrationality can be proven with the technique of proof by contradiction. In Herstein's proof of the irrationality of pi, he does just that. With this technique we are going to assume something to be true, and later on prove the assumption is incorrect. For the proof of pi's irrationality we first make the assumption that pi is a rational and can take the form of $\frac{a}{b}$. Following this, a function is introduced. This function is $f(x) = \frac{x^n(a-bx)^n}{n!}$. This expanded out polynomial allows us to define integers within. Then the proof looks at the symmetry property of f(x). To be more specific, one can find that f(x) is the same as f (π -x). So when you substitute π -x into the equation for x, one would find that it is the same value as the original equation f(x). The reason for this is because of our assumption that $\pi = \frac{a}{b}$. The next part of Herstein's proof deals with the derivatives of f(x). He then uses the chain rule to differentiate the function and from this he is able to draw the conclusion $f^{(i)}(x) = (-1)^i f^{(i)}(\pi - x)$. From this the proof then looks at the value of $f^{(i)}(0)$ and $f^{(i)}(\pi)$. After doing out some work Herstein finds that they are both equal to an integer for all nonnegative integers i. The proof then introduces another function that is an alternating function of even derivatives. After doing more work the proof uses a corollary that if u is a real number, then $\lim_{n\to\infty} \frac{u^n}{n!} = 0$. Herstein comes to a point where x is trapped between 0 and 1. Since x is an integer and there are no integers between 0 and 1 we have a contradiction. Therefore, pi is an irrational number. To see this proof please refer to the appendix.

Our second irrational number is Euler's number (e). The number e is defined to be the following limit as n goes to infinity of $\left(1+\frac{1}{n}\right)^n$ [5]. Euler's number is equal to 2.7182... This number represents the base for natural logarithms. Euler's number was first introduction to mathematics by Napier in 1618. In his work on logarithms he used

e however, it was not really mentioned. As more mathematicians worked with logarithms, e was finally discovered through the study of compound interest by Jacob Bernoulli in 1683. He tried to find the limit of $(1 + {}^{1}/n)^n$ as n goes to infinity. He used the binomial theorem to show that the limit had to lie between 2 and 3 and this is how the approximation for e came about. It was not until 1748 when Euler published *Introductio in Analysin infinitorum* that he gave a full treatment of the ideas surrounding e. He showed that $e = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$ [3].

This leads us to the proof that e is irrational. To prove that e is irrational we can use a basic proof by contradiction. To do this we must assume that e is a rational number that can be expressed as $e = \frac{a}{b}$ in which a and b are both integers. We can then see that the proof defined e as $1 + \frac{1!}{1!} + \frac{1!}{2!} + \frac{1!}{3!} + \ldots$ Taking this equation and multiply it by b! found the following formula, $b!e = b! + \frac{b!}{1!} + \frac{b!}{2!} + \frac{b!}{3!} + \ldots + \frac{b!}{b!} + \frac{b!}{(b+1)!} + \frac{b!}{(b+2)!} + \ldots$

Because b! e is an integer and $b! + \frac{b!}{1!} + \frac{b!}{2!} + \frac{b!}{3!} + \ldots + \frac{b!}{b!}$ is an integer it must be that the sum of the rest of the right side of the equation is an integer as well. Taking the sum of the rest of the right side $\frac{1}{(b+1)} + \frac{1}{(b+1)(b+2)} + \frac{1}{(b+1)(b+2)(b+3)} + \ldots$ and replacing b+2, b+3… with b+1 would obtain $\frac{1}{(b+1)} + \frac{1}{(b+1)^2} + \frac{1}{(b+1)^3} + \ldots$ Thus, making the right terms larger and a geometric series on the right. Thus, its sum is $\frac{1}{(b+1)} \div (1 - \frac{1}{(b+1)}) = \frac{1}{b}$.

Therefore, b is larger than 1 because e is not an integer, so we have an integer between 0 and 1, this is the contradiction [6]. Therefore, e is an irrational number. To see this proof of the irrationality of Euler's e please refer to the appendix.

Since mathematics has many topics to explore, rational and irrational numbers are only a tiny piece of it. The history of pi (π) and Euler's number (e) is fascinating to follow. Many mathematicians have contributed to the history of these numbers. Without this history the basic understanding of pi (π) and Euler's number (e) would not have come about. One can take this history, analyze these numbers, and be able to prove them. The proofs of pi (π) and Euler's number (e) being irrational are just a few of many proofs that a mathematician should have an idea about, and understand the main principals involved. By understanding that pi and Euler's number are irrational, one is able to have a better understanding of the broader world of mathematics.

Appendix

Table A1

Number	Is it Irrational or Rational?	How it would look in the form $\frac{a}{b}$, if it can be written as such
7	Rational	$\frac{7}{1}$
$.\overline{3}$	Rational	$\frac{1}{3}$
$\sqrt{2}$	Irrational	Can't be done
e	Irrational	Can't be done
π	Irrational	Can't be done

rather long detour—but we will, at least, prove that π is irrational. The very nice proof that we give of this fact is due to I. Niven; it appeared in his paper "A Simple Proof That π Is Irrational," which was published in the *Bulletin of the American Mathematical Society*, vol. 53 (1947), p. 509. To follow Niven's proof only requires some material from a standard first-year calculus course.

We begin with

Lemma 6.7.1. If u is a real number, then $\lim_{n\to\infty} u^n/n! = 0$.

Proof. If u is any real number, then e^u is a well-defined real number and $e^u = 1 + u + u^2/2! + u^3/3! + \cdots + u^n/n! + \cdots$. The series $1 + u + u^2/2! + \cdots + u^n/n! + \cdots$ converges to e^u; since this series converges, its nth term must go to 0. Thus $\lim_{n\to\infty} u^n/n! = 0$. □

We now present Niven's proof of the irrationality of π.

Theorem 6.7.2. π is an irrational number.

Proof. Suppose that π is rational; then $\pi = a/b$, where a and b are positive integers.

For every integer $n > 0$, we introduce a polynomial, whose properties will lead us to the desired conclusion. The basic properties of this polynomial will hold for all positive n. The strategy of the proof is to make a judicious choice of n at the appropriate stage of the proof.

Let $f(x) = x^n(a - bx)^n/n!$, where $\pi = a/b$. This is a polynomial of degree $2n$ with rational coefficients. Expanding it out, we obtain

$$f(x) = \frac{a_0x^n + a_1x^{n+1} + \cdots + a_nx^{2n}}{n!},$$

where

$$a_0 = a^n, a_1 = -na^{n-1}b, \ldots, a_i = \frac{(-1)^i n!}{i!(n-i)!}a^{n-i}b^i, \ldots, a_n = (-1)^n b^n$$

are integers.

We denote the ith derivative of $f(x)$ with respect to x by the usual notation $f^{(i)}(x)$, understanding $f^{(0)}(x)$ to mean $f(x)$ itself.

We first note a symmetry property of $f(x)$, namely, that $f(x) = f(\pi - x)$. To see this, note that $f(x) = (b^n/n!)x^n(\pi - x)^n$, from whose form it is clear that $f(x) = f(\pi - x)$. Since this holds for $f(x)$, it is easy to see, from the chain rule for differentiation, that $f^{(i)}(x) = (-1)^i f^{(i)}(\pi - x)$.

This statement about $f(x)$ and all its derivatives allows us to conclude that for the statements that we make about the nature of all the $f^{(i)}(0)$, there are appropriate statements about all the $f^{(i)}(\pi)$.

We shall be interested in the value of $f^{(i)}(0)$, and $f^{(i)}(\pi)$, for all nonnegative i. Note that from the expanded form of $f(x)$ given above we easily obtain that $f^{(i)}(0)$ is merely $i!$ times the coefficient of x^i of the polynomial $f(x)$. This immediately implies, since the lowest power of x appearing in $f(x)$ is the nth, that $f^{(i)}(0) = 0$ if $i < n$. For $i \geq n$ we obtain that $f^{(i)}(0) = i!a_{i-n}/n!$; since $i \geq n$, $i!/n!$ is an integer, and as we pointed out above, a_{i-n} is also an integer; therefore $f^{(i)}(0)$ *is an integer for all nonnegative integers* i. Since $f^{(i)}(\pi) = (-1)^i f(0)$, we have that $f^{(i)}(\pi)$ is an integer for all nonnegative integers i.

We introduce an auxiliary function

$$F(x) = f(x) - f^{(2)}(x) + \cdots + (-1)^n f^{(2n)}(x).$$

Since $f^{(m)}(x) = 0$ if $m > 2n$, we see that

$$\frac{d^2F}{dx^2} = F''(x) = f^{(2)}(x) - f^{(4)}(x) + \cdots + (-1)^n f^{(2n)}(x)$$
$$= -F(x) + f(x).$$

Therefore,

$$\frac{d}{dx}(F'(x)\sin x - F(x)\cos x) = F''(x)\sin x + F'(x)\cos x$$
$$- F'(x)\cos x + F(x)\sin x$$
$$= (F''(x) + F(x))\sin x = f(x)\sin x.$$

From this we conclude that

$$\int_0^\pi f(x)\sin x\,dx = [F'(x)\sin x - F(x)\cos x]_0^\pi$$
$$= (F'(\pi)\sin\pi - F(\pi)\cos\pi) - (F'(0)\sin 0 - F(0)\cos 0)$$
$$= F(\pi) + F(0).$$

But from the form of $F(x)$ above and the fact that all $f^{(i)}(0)$ and $f^{(i)}(\pi)$ are integers, we conclude that $F(\pi) + F(0)$ is an integer. *Thus $\int_0^\pi f(x)\sin x\,dx$ is an integer. This statement about $\int_0^\pi f(x)\sin x\,dx$ is true for any integer $n > 0$ whatsoever.* We now want to choose n cleverly enough to make sure that the statement "$\int_0^\pi f(x)\sin x\,dx$ is an integer" cannot possibly be true

We carry out an estimate on $\int_0^\pi f(x) \sin x \, dx$. For $0 < x < \pi$ the function $f(x) = x^n(a - bx)^n/n! \leq \pi^n a^n/n!$ (since $a > 0$), and also $0 < \sin x \leq 1$. Thus $0 < \int_0^\pi f(x) \sin x \, dx < \int_0^\pi \pi^n a^n/n! \, dx = \pi^{n+1}a^n/n!$.

Let $u = \pi a$; then, by Lemma 6.7.1, $\lim_{n \to \infty} u^n/n! = 0$, so *if we pick n large enough, we can make sure that* $u^n/n! < 1/\pi$, hence $\pi^{n+1}a^n/n! = \pi u^n/n! < 1$. But then $\int_0^\pi f(x) \sin x \, dx$ is trapped strictly between 0 and 1. But, by what we have shown, $\int_0^\pi f(x) \sin x \, dx$ is an integer. Since there is no integer strictly between 0 and 1, we have reached a contradiction. Thus the premise that π is rational was false. Therefore, π is irrational. This completes the proof of the theorem. □

INDEX

Appendix

Proof of *e*

The basic idea of this proof is to assume that the number in question is algebraic, and then you arrive at a contradiction. The contradiction always takes the form of showing that the assumption implies the existence of an integer between 0 and 1.

e is defined as $1+\frac{1!}{1!}+\frac{1!}{2!}+\frac{1!}{3!}+\ldots$

Suppose e is rational, say $e=\frac{a}{b}$, where a and b are integers. Then b! e is certainly an integer. So multiply the equation for e by b!.

This gives us:

$$b!\,e = b! + \frac{b!}{1!}+\ldots+\frac{b!}{b!}+\frac{b!}{(b+1)!}+\frac{b!}{(b+2)!}+\ldots$$

Now all the terms up to b!/b! on the right are integers, and the left side is an integer, so the sum of the rest of the terms on the right must be an integer. This is:

$$\frac{1}{(b+1)}+\frac{1}{(b+1)(b+2)}+\frac{1}{(b+1)(b+2)(b+3)}+\ldots$$

Now if we replace b+2, b+3, ... by b+1 we make the terms on the right bigger, so the right side is less than:

$$\frac{1}{(b+1)}+\frac{1}{(b+1)^2}+\frac{1}{(b+1)^3}+\ldots$$

which is a geometric series. Its sum is:

$$\frac{1}{(b+1}\div(1-\frac{1}{(b+1)})=\frac{1}{b}$$

Note that b is certainly bigger than 1, since e is not an integer, and we have arrived at a contradiction: an integer between 0 and 1.

This proof is taken from Ask Doctor Math - http://mathforum.org/library/drmath/view/53910.html

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Pi (π) and Euler's number (e)

By. Ryan Farnsworth
Abby Dutch
Torey Cutting

Overview

- History of pi
- Proof of pi's irrationality
- History of *e*
- Proof of *e*'s irrationality

History of pi

π
3.141
5926535
8979323846
2643383279502
8841971693993751

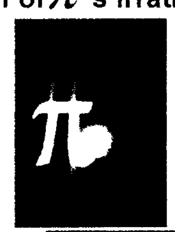
History of Pi (π)

- Pi represents the circumference of any circle divided by its diameter
- Numerically written as 3.14159...
- This irrational number comes from as far back as the ancient Babylonian, Egyptians, Greek, and Indian geometers in which they calculated the area of a circle.

History Continued

- Many mathematicians had their tries with pi however, it was not until Archimedes that the calculation of pi was formed. He used the Pythagorean Theorem to approximate the area of a circle. In which he used upper and lower bounds.
- So Archimedes showed that pi was an irrational number found between $3\frac{1}{7}$ and $3\frac{10}{71}$

Proof of π's Irrationality



3.141592654...

Herstein's Proof of π

- Uses proof by contradiction.
- Introduces a polynomial to aid in getting the desired outcome.
- Uses derivatives and integration.
- Uses inequalities to trap a variable between 0 and 1.

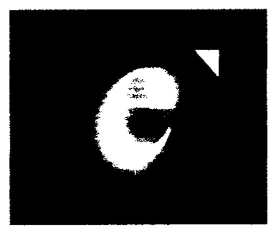
Pi's Irrationality

- Assume pi is rational → pi = a/b
- The polynomial introduced is as follows.
- $f(x) = \frac{x^n(a-bx)^n}{n!}$ → $f(x) = \frac{a_0x^n + a_1x^{n+1} + \ldots + a_nx^{2n}}{n!}$
- In the equation all the a_n are integers.

Pi's Irrationality

- The proof then looks at the symmetry property, namely f(x)=f(π -x)
- Then the proof uses differentiation.
- Then an auxiliary function is introduced
  - With that the proof concludes that by using integration a variable x, which is an integer, is between 0 and 1.
- We of course know this is not possible thus arriving at the desired outcome.
- Thus, pi is irrational.

History of e



History of Euler's Number e

- Euler's number (e) is defined to be the following limit as n goes to infinity of

$$\left(1+\frac{1}{n}\right)^n$$

- Euler's number is equal to 2.7182… This number represents the base for natural logarithms.
- Euler's number was first introduction to mathematics by Napier in 1618.

History Continued

- As more mathematicians worked with logarithms, was finally discovered through the study of compound interest by Jacob Bernoulli in 1683.
- He tried to find the limit of $(1+\frac{1}{n})^n$ as n goes to infinity. He used the binomial theorem to show that the limit had to lie between 2 and 3 and this is how the approximation for e came about. He used e in his work however it wasn't really mentioned.
- In 1748 Euler published work that gave full treatment to e. He showed that $e = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$

Euler's number (e)

Proof of the irrationality of e

- To prove e is irrational, a basic proof by contradiction is used.
- e is defined as $1 + \frac{1!}{1!} + \frac{1!}{2!} + \frac{1!}{3!} + \dots$
- Assume e is rational; thus $e = \frac{a}{b}$ where a and b are both integers.

- Multiplying the definition of e by $b!$ would get the following;

$$b!\,e = b! + \frac{b!}{1!} + \frac{b!}{2!} + \dots + \frac{b!}{b!} + \frac{b!}{(b+1)!} + \frac{b!}{(b+2)!} + \dots$$

- We know that $b!e$ is an integer and $b! + \frac{b!}{1!} + \frac{b!}{2!} + \dots + \frac{b!}{b!}$ is an integer, thus it must be that the sum of the rest of the right side is an integer as well.

- Taking the sum of the rest of the right side

$$\frac{1}{(b+1)} + \frac{1}{(b+1)(b+2)} + \frac{1}{(b+1)(b+2)(b+3)} + \dots$$

and replacing b+2, b+3... with b+1 would obtain

$$\frac{1}{(b+1)} + \frac{1}{(b+1)^2} + \frac{1}{(b+1)^3} + \dots$$

Thus, making the right side larger and giving us a geometric series.

- So, the sum is

$$\frac{1}{(b+1)} \div \left(1 - \frac{1}{(b+1)}\right) = \frac{1}{b}$$

- Therefore, b is larger than 1 because e is not an integer, giving us an integer between 0 and 1; a contradiction.
- Thus, we conclude that e is irrational.

In closing...

- We have looked upon how pi has come about.
- We have seen that pi is irrational.
- We have looked upon the development of e.
- We have seen that pi is irrational.

1.) *Write an essay describing the parallels between F[x] and* ***Z*** *(up to and including ideals and quotient rings)*

When looking at F[x] and **Z** we are able to see many parallels considering their differences. F[x] is a ring of polynomials with coefficients in F. Where as **Z** is the set of integers, subject to the eight algebraic laws: commutative law of addition and multiplication, associative law of addition and multiplication, additive inverse, additive identity, and the distributive law. They parallel each other with it comes to being commutative rings, fields, the g.c.d. and Euclidean Algorithm, Unique Factorization, Ideals, Irreducible and $f|g * h \longrightarrow f|g$ or $f|h$, as well as quotient rings.

The basic structure of F[x] and **Z** is the same. They are both fields. To be a field they both have to be a commutative ring R if every non- zero element, a ε R is a unit. So in other words the commutative law for multiplication has to hold. For all integers a,b element of R, $ab = ba$; and for polynomials F[x], $f(x), g(x)$ elements of R, $f(x)g(x) = g(x)f(x)$. As a result that the two are fields they are also an integral domain because they do not have any zero divisors. A zero polynomial has no degree.

When it comes to the division algorithm, **Z** uses the absolute value and F[x] uses the degree of polynomials. Since both **Z** and F[x] are rings they have a division algorithm in which for integers, a,bε **N**, there are integers q (quotient), and r (remainder) such that $a = qb + r$, with $0 \le r < b$. An example follows:

Let 5, 3 ε **Z**

By proposition 1.3 for polynomials which is very similar to the algorithm for integers. It states let $f, g \varepsilon$ F[x] be non polynomials. Then there exists unique polynomials, $q(x)$ and $r(x)$ such that $f(x) = g(x) * q(x) + r(x)$ where the $\deg(r(x)) \pi \deg(g(x))$ or $r(x) = 0$. An example follows:

Let $4x^2 + 1$, $2x + 4$ ε F[x]

The two are very similar besides F[x] uses polynomials and **Z** is just integers.

This leads us to the Greatest Common Divisor or g.c.d. The g.c.d. of polynomials may be constructed as in the case of integers, by applying the division algorithm repeatedly. This process is called the Euclidean Algorithm for polynomials. The division algorithm has elements in F[x]. The procedure of computing the g.c.d. for two integers and the expression $d = m_0 a + n_0$ is the Euclidean Algorithm for integers. An example for an integer and polynomial follows to show the similarities of the two:

As you can see the two are very similar. They use the same form and the expression is similar because one is a polynomial and the other is an integer. The existence of the g.c.d and the Unique Factorization property of the integers form the Euclidean Algorithm.

The Unique Factorization Property in F[x] states that every non-constant polynomial in F[x] can be written as a product of irreducible factors; the resulting expression is unique, except for rearrangement and nonzero constant factors. A non constant polynomial $f(x)\varepsilon F[x]$ is called irreducible if it cannot be expressed as a product of non constant polynomials. So in other words the Unique Factorization in F[x] parallels a polynomial for integers primed. A product of irreducible factors for a given polynomial is the same as a product of prime integers for a given integer. An example follows to demonstrate this:

Here we can see how the two parallel each other. Also if f is irreducible and f/ g* h it yields f/g or g/h. The polynomials parallel the integers primed.

Another parallel come from the idea of ideals. Both **Z** and F[x] are ideals, if $I \subseteq R$ non empty of R if for all $a,b\varepsilon I$ and $r\varepsilon R, a+b\varepsilon I$ and $ra\varepsilon I$. An example of both follows:

They are very similar yet a little different because F[x] are all the polynomials with a factor of f(x). Since F[x] and **Z** are division algorithms they have to be principal ideal domain. By proposition 1.2 **Z** and F[x] are principal Ideal Domains. We are actually able to prove this fact. They are similar yet F[x] uses the degree of polynomials f(x) which would be polynomials of the smallest degree. Corollary 1.3 also shows the relationship of **Z** and F[x]. It states if $I \subsetneq \mathbf{Z}$ is an ideal and $p \varepsilon I$, $I = \langle p \rangle$. If $I \subsetneq F[x]$ and $f(x) \varepsilon I$, f(x) is irreducible, then $I = \langle f(x) \rangle$. The ideals like **Z** are in F[x].

This brings us to the last parallel between F[x] and **Z**, which is quotient rings. R/ I is called a quotient ring. By a preposition if R is a commutative ring and I is and Ideal, R/I (R mod I) is a commutative ring. The only difference between the two is that F[x] is congruent to a linear polynomial or constant. An example of both follows:

In conclusion F[x] and **Z** have many similarities. They have the same basic structure; so when it comes to some ideas their only difference is in the process. The fact is that one is a polynomial, and one is an integer. These are two very different ideas because one has a variable when the other is just a number. Many of the ideas build upon each other as well allowing for more parallels between F[x] and **Z**.

Standard 4.3

#3.) The proof of the Extreme Value Theorem is based off of the idea of extreme values which are the absolute minimum and maximum values of a function. This idea was built on and is included in the proof. The idea of bounded above meaning there is nothing in the set bigger than it. If a nonempty set of real numbers is bounded above, then there exists a least upper bound. So that any upper bound is greater than or equal to it. Similarly if a nonempty set of real numbers is bounded below, there exists a greatest lower bound. Bounded means for every m, M such that $m \le a_n \le M$ for every n. This is saying that m is the greatest lower bound and M is the least upper bound for the function $F(X_n)$ because if the function is bounded the function is found between the greatest lower bound and the least upper bound. This builds on the idea of monotone, meaning terms are increasing or terms are decreasing. This also brings up the idea of a sequence or a list of terms or more generally f: **N** $\rightarrow$ **R**. Knowing that a sequence is an ordered list there can be other sequences formed from the original, a subsequence. Knowing these ideas we can then expand on all of them to form bigger ideas or theories. Like the bounded monotone sequence theorem which says that if $\{a_n\}$ is a bounded, monotone sequence, then $\{a_n\}$ converges. This theorem means that a function converges or has some limit using the least upper bound and greatest lower bound. We can build upon this theorem with the Monotone subsequence theorem, saying that every sequence in the real numbers has a monotone subsequence. Knowing these facts the Bolzano- Weierstress Theorem for sequences helps us in the proof. It states that a bounded sequence in the real numbers has a convergent subsequence. This statement was used along with the proof of the lemma stating if f: [a, b] $\rightarrow$ **R** is continuous, then f(x) is bounded. In order to use this lemma you need to understand what it means to be continuous. Which means that for every $\varepsilon \succ 0$, there exists a $\delta \succ 0$ such that if $|x-c| \prec \delta$, $|f(x)-f(c)| \succ \varepsilon$. More generally, **f: R→R.** When you compile all these ideas together, the ideas in their proofs or when used separately you are able to prove the Extreme Value Theorem.

Take home
Abby Dutch

#1.) You might ask what a complex number ℂ is. ℂ is the set of all ordered pairs (a, b), a, b R or if easier the set of all vectors in $\mathbf{R}^2$. The algebraic structure of the complex numbers is a field. A field is every nonzero element a R is a unit, and R holds all the properties of a ring. The way a complex number is denoted is by (0, 1) by the symbol i and so (a, b) = a + bi. Where "a" is the real part and "b" is the imaginary part. The idea of complex numbers came about when trying to solve equations like $X^2 + 1 = 0 \Rightarrow X^2 = -1$. We know here that when trying to solve you cannot have a negative number under a radical. So we are given the rule $i^2 = -1$ thus, we can now solve $X^2 = -1 \Rightarrow X = \sqrt{i}$.

Complex numbers can also use the operations for addition and multiplication much like the real numbers. Some examples follow:

We define addition as: (a, b) + (c, d) = (a+c, b+d)

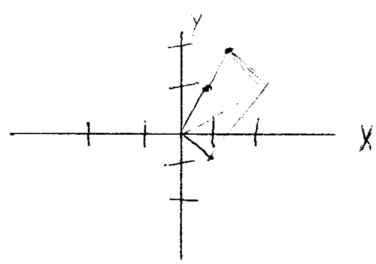
Ex.

(1, 2i) + (4, 3i) or more formally written (1 + 2i) + (4 + 3i)

Thus,

(1 + 4, 2i + 3i) = (5, 5i) formally (5 + 5i)

Picture: Ex. $(1 - i) + (1 + \sqrt{3}\,i) = 2 + (\sqrt{3} - 1)\,i$



(you use the tip to tail method)

We define multiplication as: (a, b) * (c, d) = (ac- bd, ad + bc)

Ex.

(1, 2i) * (4, 3i) or more formally (1 + 2i) * (4 + 3i)

Thus,

((1)(2i) – (2i)(3i), (1)(3i) + (2i)(4))

$(2i - (6i^2), 3i + 8i)$

Using the rule $i^2 = -1$ (2i + 6, 11i) or more formally 2i + 6 + 11i = 6 + 13i

Among the operations you can perform with complex numbers, you can represent complex numbers in different forms. One form is polar form. Where θ is only defined to multiples of 2π (360 degrees). In this form we have $a + bi = r(\cos\theta + i\sin\theta)$. Where $a = \cos\theta$ and $b = \sin\theta$. To find r in this equation we use the formula $r = \sqrt{a^2 + b^2}$. Then to find θ we use the equation $\tan\theta = b/a$. A picture and example follow:

Ex:

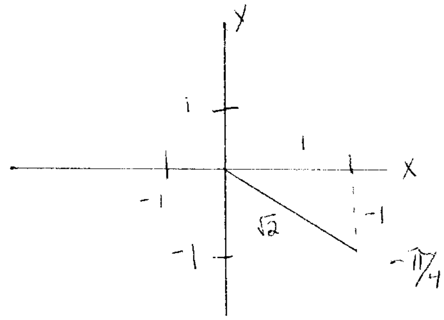
General pict:

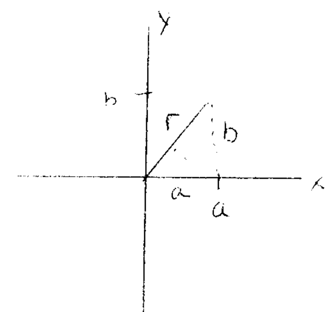
$a + bi = 1 - i$

$r = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$\tan\theta = -1/1 = -\pi/4$

$a + bi = r(\cos\theta + i\sin\theta)$
$= \sqrt{2}\,(\cos(-\pi/4) + i\sin(-\pi/4))$





You are also able to multiply in polar form. When we have: $r(\cos\theta + i\sin\theta) * \rho(\cos\phi + i\sin\phi)$ it yields $r * \rho(\cos(\theta+\phi) + i\sin(\theta+\phi))$. An example follows and a picture:

$(1-i)(1+\sqrt{3}i)$

where $1-i = \sqrt{2}(\cos^{-\pi}/_4 + i\sin^{-\pi}/_4)$

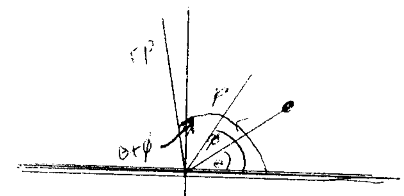
and, $1+\sqrt{3}i = 2(\cos{}^{\pi}/_3 + i\sin{}^{\pi}/_3)$

when multiplying the two

$(1-i)(1+\sqrt{3}i) = 2\cdot\sqrt{2}(\cos(^{-\pi}/_4 + {}^{\pi}/_3) + i\sin(^{-\pi}/_4 + {}^{\pi}/_3))$

$= 2\sqrt{2}(\cos{}^{\pi}/_{12}) + i\sin(^{\pi}/_{12}))$

General Pict for multiplicat



r: dilation by a factor of r. (shrinks or stretches)

θ. rotating a number

Another form using corollary 3.4 says if $z = r(\cos\theta + i\sin\theta)$ then, $1/z = 1/r(\cos\theta - i\sin\theta) = \frac{\bar{z}}{|z|^2}$. We are able to use this corollary to prove problems like $|z|^2 = z\bar{z}$. Also, corollary 3.4 (deMoivre's Theorem) says if $z = r(\cos\theta + i\sin\theta)$ then, $z^2 = r^2(\cos n\theta + i\sin n\theta)$. We can use this corollary to resurrect the double-angle formulas of trigonometry. An example follows:

If $z = \cos\theta + i\sin\theta$,

then, $\cos 2\theta + i\sin 2\theta = z^2 = (\cos\theta + i\sin\theta)^2$

$= (\cos^2\theta - \sin^2\theta) + i(2\sin\theta\cos\theta)$

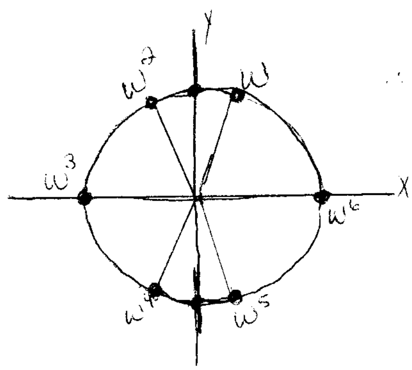
↑

(Getting this result by foiling $(\cos\theta + i\sin\theta)^2$)

One important application of deMoivre's Theorem is to solve $z^n = a$. This leads us in to roots of unity or the nth roots of 1. We want to find complex numbers so that $z = r(\cos\theta + i\sin\theta)$ so that $z^n = r^n(\cos n\theta + i\sin n\theta) = 1$. To do this we have to us proposition 3.5 which states, let $w = \cos 2\pi/n + i\sin 2\pi/n$ then $1, w, w^2, \ldots, w^{n-1}$ are solutions of $z^n = 1$ (nth roots). Using proposition 3.5 we are able to see that the six roots of unity are:

$$w = \cos \pi / 3 + i \sin \pi / 3 = ½ + \sqrt{3} / 2\, i$$
$$w^2 = \cos 2\pi / 3 + i \sin 2\pi / 3 = -½ + \sqrt{3} / 2\, i$$
$$w^3 = \cos \pi + i \sin \pi = - 1$$
$$w^4 = \cos 4\pi / 3 + i \sin 4\pi / 3 = -½ - \sqrt{3} / 2\, i$$
$$w^5 = \cos 5\pi / 3 + i \sin 5\pi / 3 = ½ - \sqrt{3} / 2\, i$$
$$w^6 = 1$$

A picture:



We can also use complex numbers in solving cube roots. To do so we use the roots of unity but also combine that with corollary 3.6 which says to let $a = \rho (\cos\phi + i \sin\phi)$, and set $b = n\sqrt{\rho} (\cos\phi /n + i \sin\phi /n)$. Then the n solutions of $z^n = a$, are $b, bw, bw^2, \ldots, bw^{n-1}$. An example follows:

$$\sqrt{3} - i = 2 (\cos (-\pi / 6) + i \sin (-\pi / 6))$$

$$b = 3\sqrt{2} (\cos(-\pi / 6/ 2) + i \sin (-\pi / 6/ 2))$$

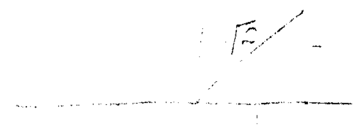
$$b = 3\sqrt{2} (\cos(-\pi / 12) + i \sin (-\pi / 12))$$

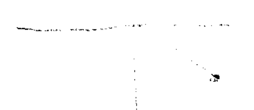
In conclusion we are able to see that complex numbers have many forms and yet some are very similar to those for real numbers. We are also able to see that many of the forms and idea of a root is built upon and expanded. These expanded forms help us solve many complex problems that we may have not been able to do otherwise.

#1

80.)

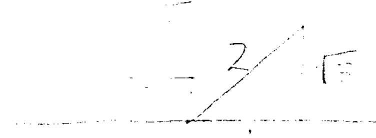
$r = \sqrt{2}$

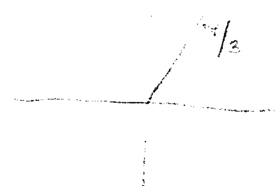


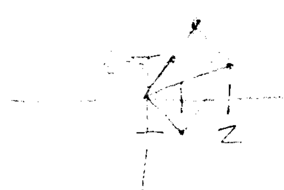


8?)

$= 2$

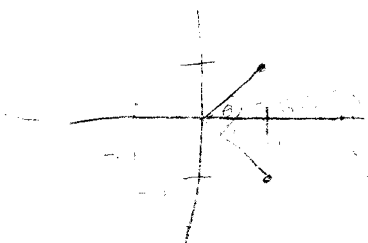






#2 cont.

$-.73 + .73i$



$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

[illegible]

Proof: Let $a, b, c, d, e, f \in \mathbb{R}$ and $a+bi$, $c+di$, $e+fi \in \mathbb{C}$. We know $\mathbb{R}$ is a field [illegible] [illegible] have property [illegible]

$a(b+c) = ab + ac$ [illegible]

Now [illegible] for $\mathbb{C}$.

[illegible]

$= [ac + \ldots$ [illegible] $]i$

[illegible]

We can [illegible]

$= [(ac + ae$ [illegible]

$= [(ac - bd) + (ae$ [illegible] $]$

[illegible]

Therefore the [illegible]

[illegible]

2) and in what time period

3) (a) solve

$z = \frac{-3 + \dots - 32 - 8i}{\dots}$

$= \frac{\dots 3 + 4i}{2}$

$z = z' - a/3$

$(z')^3 - \ldots = 0$

$- (-28) \ldots$

$13 + 13$

$z_1 = \ldots \boxed{4}$

$\ldots = \boxed{-2+\sqrt{3}\ldots}$

$-\frac{3}{2} - \frac{3\sqrt{3}}{2}i + \frac{3(-3/2 + \ldots)}{9/4 + \ldots}$

$4^2 - 9(4) - 28 = 0$

$64 - 36 - 28 = 0$

$64 - 64 = 0$

$0 \checkmark 0$

$z^3 - 9z - 28 = 0$

$(-2+\sqrt{3}i)^3 - 9(-2+\sqrt{3}i) - 28$

$(10 + ~~9\sqrt{3}i~~) + 18 ~~- 9\sqrt{3}i~~ - 28$

$28 - 28$

[illegible]

$(-2+\sqrt{3}i)(-2+\sqrt{3}i)$

$4 - 2\sqrt{3}i - 2\sqrt{3}i - 3$

$(1 - 4\sqrt{3}i)(-2+\sqrt{3}i)$

$-2 + \sqrt{3}i - 8\sqrt{3}i(\ldots 3i)$

$-2 + \sqrt{3}i - 8\sqrt{3}i + 12$

$10 + 9\sqrt{3}i$

HW 3

orne 5000 years ago

the use of

Each

Paired