Portfolio Assessment

Name: Geoff Benson

Expected Date of Graduation: 2010

Mathematics
Middle/Junior High Option

	Semester Completed
Math 141 – Introductory Statistics	Spring 2006
Math 151 – Calculus I	Spring 2008
Math 152 – Calculus II	
Math 225 – Introduction to Abstract and Discrete Mathematics	
Math 231 – Linear Algebra and Differential Equations	
Math 275 – Geometry K-8 Teachers	
Math 375 – Algebraic Concepts for K-8 Teachers	

Reflection Paper

As far back as I can remember I have been able to use mathematics to pursue my dreams of a higher education. For example, I took *Introduction to Statistics* because I knew I was good in mathematics. When I was assigned to write a history paper on Karl Pearson I was very nervous. I don't do well writing essays in my English class, so I figured the same results would show in my Statistics class. However once I started researching Standard Deviation my interest in probability motivated me to understand the information, so it would outweigh my technical flaws.

In order to learn new materials and ideas it is critical that I go to every class. In my past, I had not been as serious about my education and I would miss classes for no good reason. Also reading the material for the class before has helped me because when I get to class and something isn't clear I have the opportunity to ask the instructor. Helping other students outside of class is helpful too. This builds my understanding of the material and also gives me confidence in the course. Confidence has proven to be a tool for learning, and also is crucial in my motivation towards mathematics.

By doing my homework and preparing for exams, I have an understanding of the processes needed. If I am stuck on a problem I can do a number of things to help myself get through it. First I like to problem solve. I ask myself if the issue is like something that I have done before, or if I can find an explanation of it in one of my texts. I also can seek advice from another student or PCA at the math center. I had earned a poor grade in my final exam in *Calculus I*, without making excuses I know that I could have done more to prepare for the exam. What I like most about being in college is that the instructors are readily available to give me advice, which is what I knew should have utilized.

Now that I am in college I have become more independent. As a student I have planned my time into a work schedule, which helps me keep on track and motivated. My class is like when I can get advice on processes or ideas I do not understand. Out of class, depending on the day, I have my lunch then I have a two hour block of time in the library. This block is when I do my *Introduction to Abstract and Discrete Math* homework because I need to be in a quiet place so I don't get distracted. I can make conjecture and prove them using a direct method, contradiction or induction.

Continuous improvements are needed every year. For next year I will review materials over the summer and also remind myself that I will need to schedule more time for school work during the semester. I have found that since my freshmen year I have progressively increased my school work schedule. The higher level classes require more time of out of class work, which directly translates in my understanding or the subject matter.

In conclusion, my *Introduction to Statistics* class taught me how to use Microsoft Excel to make charts and make statistical conclusions. This is directly helps me in other courses because I can correctly make and interpret a chart on my graphing calculator or in Excel. My grade in *Calculus I* was low, but this has motivated me to do better in my *Calculus II* class. I can analyze change in various contexts, such as derivatives and integrals My effort in the class and effort on the homework has shown in my Friday quizzes. My most interesting class is *Introduction to Abstract and Discrete Math*. With our practice of writing proofs and fundamental ideas of number sense I am confident that I can explain or proof other math processes to peers or students. Also typing my assignments has increased my abilities in Microsoft Word.

Math 141 (Introductory Statistics) Artifacts

Applicable NCATE Middle School Standards

6.1 Use knowledge of mathematics to select and use appropriate technological tools.

14.1 Design investigations, collect data, and use a variety of ways to display data and interpret data representations.

14.2 Draw conclusions involving uncertainty by using hands-on and computer-based simulation for estimating probabilities and gathering data to make inferences and decisions.

14.3 Identify misuses of statistics and invalid conclusions from probability.

14.4 Use appropriate statistical methods and technological tools to describe shape and analyze spread and center.

14.5 Investigate, interpret, and construct representations for conditional probability, geometric probability, and for bivariate data.

14.6 Demonstrate knowledge of the historical development of statistics and probability including contributions from diverse cultures.

Karl Pearson

Standard Deviation

Geoffrey Benson
Intro. To Stats
History Paper

Geoffrey Benson

Karl Pearson

Karl Pearson was honored to be one of the three leaders of statistical revolution, along with Galton and Edgeworth. Karl was a courageous child and was sent off at the age of nine to the University College School in London. Here is where he studied physics, metaphysics and Darwinism. During his study of Darwinism originally named Carl, decided to change his name to Karl to coincide with his studies. Karl developed many ideas that are studied in statistic courses to this day, making him one of the more powerful members of the three leaders. Karl went back to University College working as a professor and lecturer until the last couple months of his life.

Karl wrote journals and essays that were important foundations in the history of statistical information. In these pieces he discussed the terms like Chi-Square Test, Histograms and Standard Deviation. These three topics discovered by Pearson are important bricks in the walls of statistics. Standard deviation is a statistic that tells you how tightly all the various examples of a population are compiled in relation to the mean. When the examples are real tight together the bell-shape curve is steep, and the standard deviation is small. When the examples are spread apart and the bell-curve is relatively flat, meaning there is a large standard deviation. One standard deviation from the mean in both directions on the horizontal axis accounts for 60 percent of the example population. Two standard deviations from the mean are 95 percent of the population. Three standard deviations are 99 percent of the population. The common area in a bell curve is in the second percentile, which are two standard deviations away from the mean.

In a statistics course, it is important to be able to identify and utilize the descriptive statistics. Standard deviation is a descriptive statistic that is used to measure the spread. The standard deviation measures spread by looking at how far the observations are from their mean. Standard deviation also is used in this course for hypothesis testing. In the third project in this course we were asked to use the standard deviation to conduct a hypothesis test for the claimed average class size at KSC. This data has helped develop my overall awareness and prevalence how math is important to my everyday life.

? overstated?

Work Cited

Moore, David S. *The Basic Practice of Statistics.* W.H. Freeman and Company, New York, NY. 2004.

O'Connor, J J and Robertson, E F. *Karl Pearson.* JOC/EFR. October 2003. http://www-history.mcs.st-andrews.ac.uk/Biographies/Pearson.html

Turner, Frank M. Karl Pearson: *The Scientific Life in a Statistical Age.* Acedemic Search Premier. June 2005, Vol. 110 Issue 3. http://0search.epnet.com.ksclib.keene.edu:80/login.aspx?direct=true&db=aph&an=17407119&scope=site

History Essay Checklist

Geoffrey Benson

A 20/20

1. Does this essay have the correct content?:

Definition of statistical topic	0	2	4
Brief biography of the statistician	0	2	4
Connection of the statistician to the topic	0	1	2
Connection of the topic to our course	0	1	2

2. Is the essay well-written?:

Clearly identifiable introductory and concluding paragraphs	0	1	2
Coherent sentence and paragraph structure	0	1	2
Minimal spelling, capitalization, and punctuation errors	0	1	2
Citation of all references used, to include at least one non-WWW source and one WWW source	0	1	2

Math 151 (Calculus I) Artifacts

Applicable NCATE Secondary Standards

10.1 Analyze patterns, relations, and functions of one and two variables.

12.1 Demonstrate a conceptual understanding of and procedural facility with basic calculus concepts.

12.2 Apply concepts of function, geometry, and trigonometry in solving problems involving calculus.

12.3 Use the concepts of calculus and mathematical modeling to represent and solve problems taken from real-world contexts.

12.5 Demonstrate knowledge of the historical development of calculus including contributions from diverse cultures.

Applicable NCATE Middle School Standards

10.5 Analyze change in various contexts.

12.1 Demonstrate a conceptual understanding of and procedural facility with basic calculus concepts.

12.2 Demonstrate knowledge of the historical development of calculus including contributions from diverse cultures.

MATH 151 Stanish
May 7, 2008
Final Exam

47

Name: Geoffrey Benson 151-01

For problems 1)-3) translate the sentences into mathematical notation.

1. (4 pts) a) My net worth (call it W) right now ($t = 0$) is \$55,000.

$W(\cancel{t}) = \$55,000$

b) Thanks to some good investments, my net worth right now is growing at the rate of \$6,000 per year.

$w'(t) = 6,000\,t$

2. (2 pts) From 1998 to 1999, the number of serious crimes that were reported in New York City (call it S) fell by 23,627 reports.

1998 = 0 1999 = 1

2

$\int_0^1 S' = -23,627$

3. (2 pts) The number of people with O-negative blood (call it O) in this country is directly proportional to the total population (call it P).

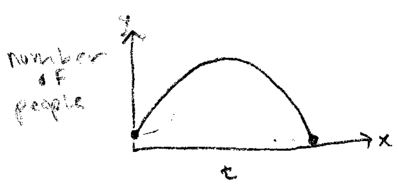
2

$O = kP$

4. (2 pts) Translate the following passage into a curve on a plane with appropriately labeled axes. In brackets you are given the quantity that should appear on the vertical axis.

An illness hit a small town. At first it spread rapidly, but as medicine became available it began to spread more slowly. It finally stopped spreading. Then people began to get better and the number of infected people started dropping more and more quickly until everyone had recovered. [number of ill people]

2



5. (4 pts) Let $f(x) = x^3 - 1$. Write an equation to the line tangent to the graph of f at $x = -2$

$f'(x) = 3x^2$

2

$y = f'(a)\, f(a-x) + f(a)$

$\cancel{y = 3x^2 (3x^2 - 2) + x^3 - 1}$

$y = 9x^4 - 6x^2 + x^3 - 1$

$y = 9x^4 + x^3 - 6x^2 - 1$

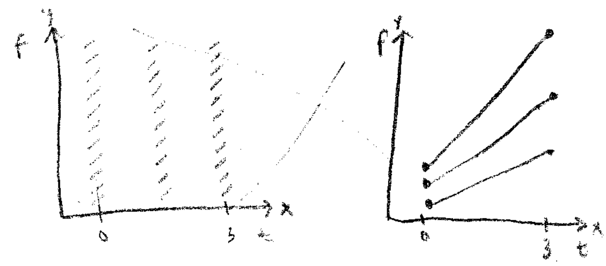
6. (8 pts) From 2002 to 2005, my retirement account grew at a rate of 2% per year. Let $t = 0$ represent 2002, and let $r(t)$ be the value of my retirement account in year t.

a) Translate the above statement into a differential equation.

$r(t) = .02t$ $\quad r(0) = .02(0)$ $\quad r(0) = 0$

$r'(t) = .02$ $\quad f(3) = .02(3)$ $\quad f(3) = .06$

b) Draw slope field for f given by this equation and sketch three possible graphs that could represent the value of my retirement.



c) Given that the value of my retirement account was \$50,000 in 2002, find the solution to the differential equation you found in part a) that gives the value of my retirement account in year t.

$r(0) = 50,000$

d) Find the value of my retirement account in 2005.

7. (6 pts) Let $f(x) = x^2 + 3x - 1$. Find the **difference quotient** for this function. Then show that the derivative is $f'(x) = 2x + 3$ by taking the **limit of the difference quotient** as Δx goes to 0.

$f(x) = x^2 + 3x - 1$ $\qquad f'(x) = \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

$f'(x) = \dfrac{x^2 + 3x - 1 + \Delta x - x^2 - 3x + 1}{\Delta x}$

$f'(x) = 2x + 3\ \Delta x$

$f'(x) = \lim_{\Delta x \to 0} 2x + 3$

8. **(6 pts)** Find each of the following limits (if it exits):

a) $\lim_{x \to -1}(x^2 + 3x - 4)$ $1+3+4=0$

$\lim_{x \to -1} = 0$

-6

b) $\lim_{x \to 2} \frac{x-2}{x^2-4}$ $\frac{0}{0}$

-2

2

c) $\lim_{x \to 0} \frac{x+1}{x}$ $\frac{1}{0}$ DNE

9. **(4 pts)** Determine the intervals on which the function $f(x) = \frac{3}{x+1}$ is continuous.

0

10. **(10 pts)** Differentiate each of the following functions.

a) $f(x) = x^5 + 3x^2 - 2x + 1$

$f'(x) = 5x^4 + 6x - 2$

b) $f(x) = x^3\sqrt{x-2}$

$f'(x) = 3x^2 (x-2)^{\frac{1}{2}} + x^3 (\frac{1}{2}(x-2))^{-\frac{1}{2}}$

8

c) $f(x) = \frac{\sin(x^2)}{x+1}$ $\frac{2\sin x}{x+1}$

$f'(x)\ \frac{(2 \cos(x))(x+1) - (2\sin(x))(1)}{(x+1)^2}$

-1

d) $f(x) = \ln(x^2 + 3) + e^{2x}$

$f'(x)\ \frac{1}{2}\ \cancel{\ln |x^2+3|} + e^{2x}(2)$

-1

e) $f(x) = -3\tan(x)$

$f'(x) = -3\sec^2(x)$

11. (10 pts) Find the following indefinite integrals.

a) $\int (x^4 + 2x - 1)\, dx$

$\frac{1}{5}x^5 + \frac{1}{2}(2)x^2 - x = \frac{1}{5}x^5 + x^2 - x + C$

(-1) for no C

b) $\int \left(\frac{1}{x^3} + \sqrt{x}\right) dx$ $\int \left(\frac{1}{x^3} + x^{\frac{1}{2}}\right)$ $\int \cancel{\frac{1}{3}\ln|x^3|} + \frac{2}{3}x^{\frac{3}{2}}$

(-2)

3

c) $\int (\sin(3x) + \sec(x)\tan(x))\, dx$ $= \int (\cos(3x) + \sec(x))$

(-1)

d) $\int 0.2e^{2x}\, dx$ $\int \frac{0.2}{2} e^{2x}$

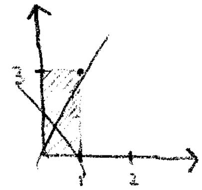
(-1)

e) $\int \frac{3x^2 - 2}{x}\, dx$ $\int \frac{3x^2}{x} - \frac{2}{x} = \int \cancel{\frac{1}{2}\ln|3x|} - \left(\frac{1}{2}\left(\frac{2}{x}\right)\right)$

(-2)

12. (6 pts) Consider the definite integral $\int_0^3 x\, dx$

a) Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.



$\int_0^3 x\,dx = \frac{1}{2}x^2 \Big|_0^3 = \frac{1}{2}(3)^2 + \frac{1}{2}(0)^2$

$= 9/2$

$= 4.5$

3

b) Use the Fundamental Theorem of Calculus to evaluate the definite integral.

13. (6 pts) Consider the definite integral $\int_0^3 e^{2x}\, dx$

a) Use the Method of Rectangles to approximate the value of this integral.

$\int_0^3 e^{2x}$ $\quad \frac{1}{2}e^{2(3)} \cdot \frac{1}{2}e^{2(0)}$

$\frac{403.4 \cdot 1}{2} = \frac{405.4}{2}$

2

b) Use the Fundamental Theorem of Calculus to evaluate the definite integral exactly.

14. (6 pts) For a dosage of x cubic centimeters of a certain drug, the resulting blood pressure B is approximated by

$$B(x) = 0.05x^2 - 0.3x^3, \qquad 0 \le x \le 0.16$$

Find the maximum blood pressure and the dosage at which it occurs.

4

$B(x) = 2(.05)x - 3(.3)x^2$

$B'(x) = .1x - .9x^2 \quad x(0.1x - 0.9x^2) = 0 \quad 0.1 - 0.9x = 0 \quad -0.9x = -0.1$

$x = .11$ critical point

$B(.11) = .05(.11)^2 - 0.3(.11)^3$

$= 6.05 - 3.993$

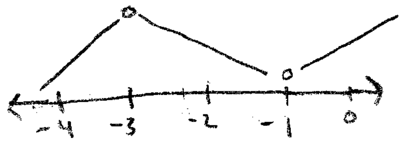
-2 $\quad B(.11) = 2.057 \times 10^{-4}$ $\quad$ endpoints?

$x+1 \quad x+3 \quad x^2+3x+x$

15. (10 pts) Let $f(x) = x^3 + 6x^2 + 9x$

a) On what intervals is $f(x)$ increasing/decreasing?

$f'(x) = 3x^2 + 12x + 9 \quad 3(3x^2+12x+9) = 0 \quad x^2 + 4x + 3 = 0 \quad (x+1)(x+3) = 0 \quad x = -1,\ x = -3$ critical points



$f(-4) = 9$
$f(-3) = 0$
$f(-2) = -3$
$f(-1) = 0$
$f(0) = 9$

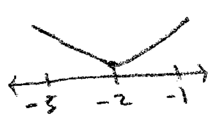
$f(x)$ is increasing on $(-\infty, -3) \cup (-1, \infty)$
$f(x)$ is decreasing on $(-3, -1)$

b) Find the relative extrema of $f(x)$.

There is a relative max at $x = -3$
There is a relative min at $x = -1$

c) On what intervals is $f(x)$ concave up/concave down?

$f''(x) = 6x + 12 \quad 6(x+2) \quad x + 2 = 0 \quad x = -2$



$f(-3) = -6$
$f(-2) = 0$
$f(-1) = 6$

7

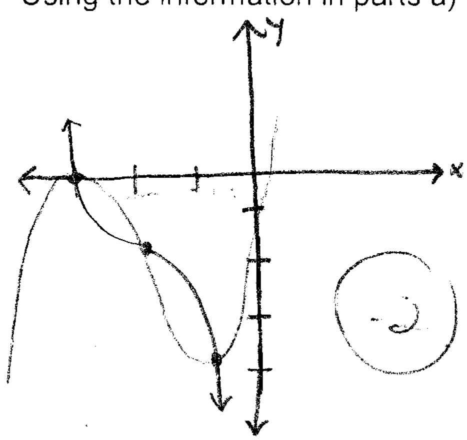
$f(x)$ is concave down on $(-\infty, -2)$
$f(x)$ is concave up on $(-2, \infty)$

d) Find the inflection points of $f(x)$.

~~$x=-3$~~
~~$x=-1$~~
~~$x=-2$~~

-1

e) Using the information in parts a)-d), sketch the graph of $f(x)$.



-2

X	Y
-3	0
-2	-2
-1	-4

$= x^3+6x^2+9x$
$-27+54-27=0$
$-8+24-18=-2$
$-1+6-9=-4$

16. (8 pts) Solve the following differential equations for y as a function of t.

a) $y' = ty^2$

$\frac{dy}{dt} = ty^2 \quad \frac{dy}{y^2} = ty^2\,dt$

$\frac{1}{y^2}\,dy = t\,dt \quad \int \frac{1}{y^2}\,dy = \int t\,dt + C$

$\ln|\cancel{y^2}| = \frac{1}{2}t^2 + C$

-2

$y^2 = e^{\frac{1}{2}t^2} + C$

$y = \sqrt{e^{\frac{1}{2}t^2}+C}$

5

b) $y' = t^2 y$

$\frac{dy}{dt} = t^2 y$

$\frac{dy}{y} = t^2\,dt \quad \frac{1}{y}\,dy = t^2\,dt \quad \int \frac{1}{y}\,dy = \int t^2\,dt \quad \ln|y| = \frac{1}{3}t^3 + C$

$y = \frac{1}{3e^3} t^3 + C$

-1

17. (6 pts) Calculate the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ for each of the following functions.

a) $f(x,y) = 4x^5 + 3x^2y - 2xy + y^3$

$f_x(x,y) = 20x^4 + 6xy - 2y$

$f_y(x,y) = 0 + 3x^2 - 2x + 3y^2$

3

b) $f(x,y) = \cos(xy)$

$\cancel{f_x(x,y) = -\sin(y)}$

$\cancel{f_y(x,y) = -\sin(x)}$

Extra Credit (3 pts each):

a) Differentiate $f(x) = \ln(\ln(x\cos(5x-1)))$

b) Evaluate $\int 2xe^{x^2}\,dx$

Geoffrey Benson
Math-151-01

Archimedes

The "father of Calculus" is a nickname crediting Archimedes for his contribution to mathematics. From as early as the Egyptian pharos, mathematics has been studied to benefit many different needs. Over the years mathematicians have invented ideas to bring us our modern day mathematics. Thanks to many of these early thinkers we can study the disciplines of Algebra, Trigonometry and Calculus.

In the year 287 BC Archimedes was born in Syracuse, Sicily under Greek rule. Archimedes went to Alexandria, where he studied with the successors of Euclid, before returning back to Syracuse where he focused on mathematics.[1] During the second Punic War, the Roman army seized the city of Syracuse. Despite his attributions to the warfare weapons, Archimedes was killed in 212 BC. His work was buried with him in his tomb and was not discovered until many years after.

Archimedes was sharp in astronomy, physics and engineering. This all helps him in the completion of Pychometry, which is the measurement of volume and density of an object. He made himself famous by a variety of mechanical inventions, which were considered diversions of geometry.[2] Development of area and volume is what Archimedes is best known for. Similar to the modern integral calculus Archimedes used the method of exhaustion in which he approximated the pi value. [3]

Heath describes on the sphere and cylinder that there are three main points to consider; first, that the surface area of any sphere is four times its greatest circle.

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[1] Heath xvi-xvii
[2] Heath xix
[3] Archimedes on spheres and cylinders

The "Father of Calculus" was well earned by Archimedes. He proved the measurement of a circle, the area of a circle and the volume of spheres and cylinders. It's because of Archimedes and few other early mathematicians that we have the knowledge and the technologies that make these complex writings seem as simple as a few calculator button presses.

Bibliography

Heath, L. T. *The Works of Archimedes*; New York: Dover Publications Inc.1953.

Archimedes on spheres and cylinders. http://www.mathpages.com.

Math 225 (Intro to Abstract/Discrete Math) Artifacts

Applicable NCATE Secondary Standards

2.1 Recognize reasoning and proof as fundamental aspects of mathematics.

2.2 Make and investigate mathematical conjectures.

2.3 Develop and evaluate mathematical arguments and proofs.

2.4 Select and use various types of reasoning and methods of proof.

9.5 Apply the fundamental ideas of number theory.

13.1 Demonstrate knowledge of basic elements of discrete mathematics.

13.2 Apply the fundamental ideas of discrete mathematics in the formulation and solution of problems arising form real-world situations.

13.3 Use technological tools to solve problems involving the use of discrete structures and the application of algorithms.

13.4 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

Applicable NCATE Middle School Standards

2.1 Recognize reasoning and proof as fundamental aspects of mathematics.

2.2 Make and investigate mathematical conjectures.

2.3 Develop and evaluate mathematical arguments and proofs.

2.4 Select and use various types of reasoning and methods of proof.

9.5 Apply the fundamental ideas of number theory.

13.1 Demonstrate a conceptual understanding of the fundamental ideas of discrete mathematics such as finite graphs, trees, and combinatorics.

13.2 Use technological tools to apply the fundamental ideas of discrete mathematics.

13.3 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

4R. A Mathematical Research Situation: Investigating the Relationship Between Two Sets

For any sets A, B, and C, we can form the sets $(A \cup B) - C$ and $A \cup (B - C)$. As budding mathematicians, we might then become interested in the relationship between these two sets. For instance, we might ask

- are the two sets $(A \cup B) - C$ and $A \cup (B - C)$ always equal?
- if they are not equal, is one always a subset of the other?
- if one of them is not a subset of the other, is there a condition on the sets A, B, and C which *would* guarantee a subset relationship?

By carrying out an investigation of the two sets $(A \cup B) - C$ and $A \cup (B - C)$ we might hope to formulate conjectures that propose answers to these questions, and perhaps others as well. We could then use what we have learned about proof writing to attempt to develop and write proofs for our conjectures.

> *Remark:* What we have just described illustrates the mathematical research process. We start with a mathematical issue of interest; pose questions related to the issue that we would like to answer; carry out an investigation leading to the formulation of conjectures that, if proved, will provide answers to our questions; and, finally, work toward producing proofs for these conjectures.

1. In attempting to answer the questions posed above, we might use Venn diagrams to illustrate the sets $(A \cup B) - C$ and $A \cup (B - C)$. Draw two Venn diagrams, one depicting $(A \cup B) - C$ and the other depicting $A \cup (B - C)$. Then try using the pictures you have drawn to answer the following questions:

   a. Must the sets $(A \cup B) - C$ and $A \cup (B - C)$ be equal?

   b. Does it appear that one of the sets $(A \cup B) - C$ or $A \cup (B - C)$ is always a subset of the other?

   c. If you believe that one of the sets $(A \cup B) - C$ or $A \cup (B - C)$ is not always a subset of the other, can you find a condition on the sets A, B, and C which you think *would* guarantee a subset relationship?

Use your answers to these questions to formulate several conjectures concerning the sets $(A \cup B) - C$ and $A \cup (B - C)$ that we could then attempt to prove.

2. For each of the conjectures you formulated in the investigation, either prove it or give evidence showing why it is not true.

3. Write up your work from (1) and (2) as a mathematical paper. Your paper should include:

- an introduction in which you describe the focus of your investigation;

Geoff Benson
10-20-2008

4R. A Mathematical Research Situation: Investigating the Relationship Between Two Sets

The focus of my investigation is for any sets A, B, and C that form the sets $(A \cup B) - C$ and $A \cup (B - C)$. Also my investigation will propose conjectures for the sets that I will prove of given evidence of. Furthermore I will answer the following three questions:

(a) Must the sets $(A \cup B) - C$ and $A \cup (B - C)$ be equal?

(b) Does it appear that one of the sets $(A \cup B) - C$ or $A \cup (B - C)$ is always a subset of the other?

(c) If you believe that one of the sets $(A \cup B) - C$ or $A \cup (B - C)$ is not always a subset of the other, can you find a condition on the sets A, B and C which you think would guarantee a subset relationship?

attempting

In attempt to answer the questions for the two sets $(A \cup B) - C$ and $A \cup (B - C)$, I needed to illustrate a Venn Diagram for each set. Once the diagrams were finished I was able to visually see the members of each set. With the two Illustrations I can answer my three focus questions. My answers to these questions are in my write up for *A Mathematical Research Situation.*

After answering the three questions, I was able to propose three conjectures that I would focus on proving. From my first answer I proposed that $(A \cup B) - C$ is not equal to

for some sets A, B, and C always

$A \cup (B - C)$. Secondly, I proposed that $(A \cup B) - C$ is a subset of $A \cup (B - C)$. Lastly, I proposed that if $A \cap C = \varnothing$, then $(A \cup B) - C$ is a subset of $A \cup (B - C)$.

Now I needed to make conjectures out of what I have proposed. I made four conjectures that I will prove:

Conjecture 1: $(A\cup B) - C \neq A\cup(B - C)$.
Conjecture 2: $A\cup(B-C) \subseteq (A\cup B) - C$.

indicate appropriate quantification

Conjecture 3: $A\cap C=\varnothing \Rightarrow A\cup(B - C) = (A\cup B) - C$.

Proofing Conjecture 1

Conjecture 1: There exists sets A, B, and C, for which $(A\cup B) - C \neq A\cup(B - C)$.

Proof: Let $A = \{1,2,3\}$, $B = \{2,4,6,8\}$, and $C = \{2,3,4,5\}$. Then $A\cup B =\{1,2,3,4,6,8,\}$, so that $(A\cup B)- C = \{1,6,8\}$. Also $(B - C) = \{6,8\}$, so that $A\cup(B - C) = \{1,2,3,6,8\}$. Thus since for instance $2 \notin (A\cup B) - C$ and $2\in A\cup(B - C)$ we can conclude that $(A\cup B) - C \neq A\cup(B - C)$. ♦

Proofing Conjecture 2

Conjecture 2: For any sets A, B, and C, $(A\cup B)-C \subseteq A\cup(B - C)$.

Proof: Consider any set A, B, and C. Let $x\in (A\cup B)-C$. We will show that $x\in A\cup(B-C)$. The definition of set difference tells us that $x\in (A\cup B)$ and $x\notin C$. So, the definition of union gives us $x\in A$ or $x\in B$ ~~and $x\notin C$.~~

Case 1: $x\in A$, so it follows that $\underline{x\in (A\cup B) \text{ and } x\notin C}$. ← not what you need, why does it follow that $x\in A\cup(B-C)$ in this case?

Case 2: $x\in B$, so it follows that $x\in B$ and $x\notin C$.
← Now explain why $x\in A\cup(B-C)$ in this case.

So, as $x\in B$ and $x\notin C$, it follows that $x\in (B-C)$. Since $x\in A$ or $x\in (B-C)$, we can conclude that $x\in A\cup(B-C)$.♦

Proofing Conjecture 3

Conjecture 3: For any sets A, B, and C, If $A\cap C=\varnothing$, then $A\cup(B-C)=(A\cup B)-C$.

Proof: Suppose $A\cap C=\varnothing$, we will show $A\cup(B - C)=(A\cup B) - C$. Since in Conjecture 2 we proved that $(A\cup B)-C \subseteq A\cup(B - C)$, we only need to prove $A\cup(B - C) \subseteq (A\cup B) - C$. So consider any $x \in A \cup (B - C)$, we will show $x \in (A \cup B) - C$. Using the definition of ~~set~~ difference $x \in A$ or $x \in (B - C)$.

Case 1: $x \in A$, using our hypothesis $A \cap C = \varnothing$, it follows that $x \notin C$. → Go further. How can we conclude that $x \in (A \cup B) - C$?

Case 2: $x \in (B - C)$, using our hypothesis $A \cap C = \varnothing$, it follows that $x \notin A$

Since $x \in (B - C)$, the definition of set difference tells us that $x \in B$ and $x \notin C$. Since our hypothesis is $A \cap C = \varnothing$, we deduce form our first case that $x \in A$ or $x \in B$ and $x \notin C$. Since $x \in A$ or $x \in B$ and $x \notin C$, it follows that $x \in (A \cup B)$. So as $x \in (A \cup B)$ and $x \notin C$, we can conclude that $x \in (A \cup B) - C$. ♦

Since the three conjectures above have been proven true, the conjectures are now theorems. These theorems explain the relationship between any some sets A, B, and C, $(A \cup B) - C$ is not equal to $A \cup (B - C)$. Secondly, that $(A \cup B) - C$ is always a subset of $A \cup (B - C)$. Lastly, that if $A \cap C \neq \varnothing$, then $(A \cup B) - C$ is a subset of $A \cup (B - C)$. These theorems conclude my investigation of the three sets A, B and C.

Reflective Summary

In my investigation of the relationships of the three sets A, B, and C, I used Venn Diagrams to see where members of these sets would be. Once I grasped membership of the sets, I was able to answer the questions. While answering the questions I could form conjectures in my answers to the questions. These processes were not difficult for me because of my practice with logic in my Math-225 course.

Proposing conjectures from the questions was not difficult because I knew where a member could be in a set, and it seemed that the sets did not look equal, so I thought that they could never be equal. While forming my conjectures to the questions I knew that $(A \cup B) - C$ is a subset of $A \cup (B - C)$. From this I stated thinking about if $A \cap C = \varnothing$, then $(A \cup B) - C$ is a subset of $A \cup (B - C)$. Once I figured my way onto this track I started to try to prove my conjectures into theorems.

Working through my proofs went well at the beginning. I got stuck when trying to show that something was a member of $A \cap C = \varnothing$. But I realized that this was simple, I=to prove an 'and' statement they both have to be true, so if something wasn't a member of one, then that something was not equal to the empty set. Also in arranging two cases to prove an 'or' statement is hard for me. Once I proved the conjecture I concluded that these are theorems.

Confused by what you have written here . . . never [illegible] trying to show something was a member of $A \cap C$ [illegible] in fact assumed $A \cap C = \varnothing$ [illegible] your first conjecture

Assessment of Math 225 Graded Assignment

Cullinane

The assessment of this graded assignment has taken into account all of the following:

- the correctness of your work;
- the validity of your reasoning;
- your ability to communicate your conclusions and your reasoning;
- the quality of your writing;
- the organization of your work;
- the degree to which you have completed the assignment.

Specific Areas of Concern

General

_____ incomplete assignment

_____ directions not followed

_____ handwriting not legible or difficult to read

_____ not following requirement of sentences organized into paragraphs

_✓_ writing incoherent or difficult to follow — at times … I have pointed out a few places where you needed to proofread better

_____ spelling, grammar, punctuation, or capitalization errors

_____ overall presentation of work not at an appropriately professional level

Mathematical (proofs or other types of problems)

_____ incorrect mathematics

_____ incorrect or inappropriate use of mathematical terminology or notation

_✓_ invalid reasoning — good job setting up cases … but in most of them you either did not complete the needed argument, or seemed to go in a direction not relevant to what you needed to show

_____ evidence provided not specific enough, conclusions not always justified, or incorrect justification

_____ interpretations not always accurate

Mathematical (proof-specific)

_____ appropriate labels (*Claim*, *Proof*, etc.) not always provided

_____ informal explanation given instead of proof

_____ not following standard proof formats or strategies

_____ incorrect or inappropriate application of a proof strategy

_____ confusing what is known and what needs to be shown

_____ sequencing of deductions incorrect

Grade _B–_

6C. Alternative Methods for Proving $n^2 + n$ Is Even

1. In Example 6.8 in §6.2 we defined what it means for an integer to be even. Create a similar definition for the notion of a natural number being *odd.*

2. Formulate conjectures concerning the sum of two even integers, the sum of two odd integers, the square of an even integer, and the square of an odd integer. Try to prove your conjectures.

3. Use your conclusions in (2) to prove, without using induction, that $n^2 + n$ is even for every positive integer n.

4. Formulate a conjecture concerning the product of an even integer and an odd integer. Try to prove your conjecture.

5. Use factoring and your conclusion in (4) to construct another proof, one that does not use induction and which is different from the one you developed in (3), that $n^2 + n$ is even for every positive integer n.

B+

Geoffrey Benson
Math 225
Graded Assignment

6C. Alternative Methods for Proving n^2+n is Even

1. Create a similar definition for the notion of an integer being odd.

An integer is odd provided that it is not divisible by 2, meaning that for every *n* that is even, there is an odd number *n*-1. Also, there is a natural number *k* such that *n-1*=2*k*, which means *n*=2*k*+1. ✓
integer

2. Formulate conjectures concerning the sum of two even integers, the sum of two odd integers, the square of two even integers, and the square of an odd integer. Try to prove your conjectures.

Conjecture 6C-2.1: For any $n,m \in \mathbf{Z}$. If *n* and *m* are even, then *n*+*m* is even. ✓
Conjecture 6C-2.2: For any $n,m \in \mathbf{Z}$. If *n* and *m* are odd, then *n*+*m* is even ✓
Conjecture 6C-2.3: For any $n \in \mathbf{Z}$. If *n* is even, then n^2 is even. ✓
Conjecture 6C-2.4: For any $n \in \mathbf{Z}$. If *n* is odd, then n^2 is odd. ✓

Now, I must directly prove the four preceding conjectures.

not yet introduced

Conjecture 6C-2.1: For any $n,m \in \mathbf{Z}$. If *n* and *m* are even, then *n*+*m* is even.
Proof of Conjecture 6C2.1: Assume *n* and *m* are even integers. Our definition tells us that *n*=2*h* and *m*=2*l*, for some $h,k,l \in \mathbf{N}$. Since integers are closed under addition, $n+m = 2k+2l = 2(h+l) = 2k$, so we can conclude that *n*+*m* is even.♠ ✓
where $K = h+l$

Conjecture 6C-2.2: For any $n,m \in \mathbf{Z}$. If *n* and *m* are odd, then *n*+*m* is even.
Proof of Conjecture 6C2.2: Assume that *n* and *m* are odd intergers. Our definition tells us that *n*=2*i*+1 and *m*=2*j*+1, for some $i,j,k \in \mathbf{N}$. Since integers are closed under addition, $n+m= (2i+1)+(2j+1) = 2(i+j)+2 = 2k$, since adding 2 to any even number gives you an even number. So we can conclude that *n*+*m*=2*k*, which follows that *n*+*m* is even.♠
where $K = i+j+1$

Conjecture 6C-2.3: For any $n \in \mathbf{Z}$. If *n* is even, then n^2 is even.
Proof of Conjecture 6C2.3: Assume that *n* is an even integer. Our definition tells us that *n*=2*k*, for some $k \in \mathbf{N}$. Now, $n \cdot n$ is another form of n^2, so it follows that $n \cdot n = (2k \cdot 2k) = 4k^2 = 2(2k^2)$, which is 2 times a number, we can conclude that since $n^2=2k$, n^2 is even. ♠
an integer (since $2k^2$ is the result of multiplying together integers)

Conjecture 6C-2.4: For any $n \in \mathbf{Z}$. If *n* is odd, then n^2 is odd.
Proof of Conjecture 6C2.3: Assume that *n* is an odd integer. Our definition tells us that *n*=2k+1, for some $k \in \mathbf{N}$. Now, $n \cdot n$ is another form of n^2, so it follows that $n \cdot n = (2k+1)\cdot(2k+1) = (4k^2 + 4k + 1) = 2(2k^2 + 2k)+1$, which is 2 times a number plus 1, we can conclude that $n^2=2(k)+1$, which we can conclude n^2 odd. ♠
an integer
(how do we know $2k^2+2k \in \mathbb{Z}$?)

3. Use your conclusions in (2) to prove, without ~~inductions~~ induction that n^2+n is even for integer n.

Conjecture 6C-3.1: For any $n \in \mathbf{Z}$, n^2+n is even.

Conjecture 6C-3.1: For any $n \in \mathbf{Z}$, n^2+n is even.

Proof of Conjecture 6C3.1: I will prove this conjecture using two cases.

Case 1: Consider n is an even integer. ✓

Our definition tells us that $n=2k$, for some $k \in \mathbf{N}$. Now, $(n \cdot n)+n$ is another form of n^2+n so it follows that $(n \cdot n)+n=(2k \cdot 2k)+2k=(4k^2+2k)=2(2k^2+k)$, which is 2 times a number, we can conclude that $n^2+n=2(k)$. Thus, $n^2=2k$ is even.

Case 2: Consider n is an odd integer. ✓

Our definition tells us that $n=2k+1$, for some $k \in \mathbf{N}$. Now, $(n \cdot n)+n$ is another form of n^2+n, so it follows that $(n \cdot n)+n=(2k+1)(2k+1)+2k+1=(4k^2+6k+2)=2(2k^2+3k+1)$, which is 2 times a number. Thus, we can conclude that $n^2+n=2k$ which we can conclude n^2+n is even. ♠ ✓

4. Formulate a conjecture concerning the product of an even integer and an odd integer. Then try to prove your conjecture.

Conjecture 6C-4.1: For any $n,m \in \mathbf{Z}$. If n is even and m is odd, then $n \cdot m$ is even.

Conjecture 6C-4.1: For any $n,m \in \mathbf{Z}$. If n is even and m is odd, then $n \cdot m$ is even.

Proof of Conjecture 6C-4.1: Assume that n is an even integer, and m is an odd integer. Our definition tells us that $n=2h$ and $m=2l+1$, for some $h,k,l \in \mathbf{N}$. Since integers are closed under multiplication, $n \cdot m=2h \cdot (2l+1)=(4hl+2h)=2(2hl+h)=2k$, so we can conclude that $n \cdot m$ is even. ♠

where $k = 2hl+h$ is an integer because $\mathbb{Z}$ is closed under both addition and multiplication

5. Use Factoring and your conclusion in (4) to construct another proof, one that doesn't use induction and which is different from the one you developed in (3), that n^2+n is even for every integer n.

Conjecture 6C-5.1: For any $n,m \in \mathbf{Z}$. If n is even, then n^2+n is even.

Conjecture 6C-5.1: For any $n,m \in \mathbf{Z}$. If n is even, then n^2+n is even.

Proof of Conjecture 6C-5.1: Assume n is an even integer. Our definition tells us that $n=2h$, for some $h,k \in \mathbf{N}$. Now, we can factor an n out of n^2+n, which equals $n(n+n)$. So it follows that $n(n+n)=2h(2h+2h)=(4h^2+4h)=2(2h^2+2h)=2k$, which is 2 times a number. Thus, $n^2+n=2k$ which we can conclude n^2+n is even for any even integer n.

$\to n(n+1)$

♠ What happens if n is odd?

In conclusion to this assignment, Conjectures 6C-2.1,2.2,2.3,2.4,3.1,4.1, and 5.1 are now claims.

See attached solutions for alternative approaches

6M. Counting Problems

7. Twelve runners compete in a race for which prizes are awarded for first place, second place, and third place finishers. In how many different ways could the prizes be assigned?

In addition to solving this problem, do both of the following:

a. Solving this problem may be viewed as determining the numerical value of $P(n, k)$ for specific numerical values of n and k. Explain why and identify the relevant values of n and k.

b. Most graphing calculators can compute $P(n, k)$ directly, as can a computer algebra system such as *Maple.* Figure out how either your graphing calculator or the *Maple* computer algebra system could be used to directly compute $P(n, k)$ for given values of n and k. Write a short paragraph describing the process as accurately and clearly as possible.

6P. Pascal's Triangle

4. A copy of David Burton's book, *The History of mathematics: An Introduction*, has been put on reserve in the library (there is a 3-hour limit to the time for which the book can be checked out and it cannot be removed from the library). Read the passage, "Pascal's Arithmetic Triangle" (pp. 429-440) and then answer the following questions.

a. Where and when did Blaise Pascal live?

b. Discuss two appearances of the triangular arrangement of binomial coefficients within the mathematics of non-European cultures before the time of Pascal.

c. Though Pascal cannot be credited with inventing the triangle that bears his name, what did he accomplish in his work Triangle Arithmetique that makes it reasonable to refer to the triangle as Pascal's Triangle? (You don't need to go into a lot of detail or specifics here; just give the gist of Pascal's accomplishment in this paper.)

Geoff Benson
Graded Assignment
12-01-2008

6M. Counting Problems

7. Twelve runners compete in a race for which prizes are awarded for first place, second place, and third place finishers. In how many different ways could the prizes be assigned?

To answer this, since there are 3 prizes and 12 racers, I have made three steps.
Step 1:1^{st} place has 12 options
Step 2:2^{nd} place has 11 options
Step 3:3^{rd} place has 10 options

$(1^{st}, 2^{nd}, 3^{rd}) = |\, 1^{st} \times 2^{nd} \times 3^{rd} \,| = |\, 12 \,| \times |\, 11 \,| \times |\, 10 \,| = 1320$ different assignments.

a. The race is a permutation of a finite set. A permutation is an arrangement of the members of the set in a particular order, where there are n-elements and k-permutations. Our number of k-permutations of an n-element set is denoted by $P(n,k)$.

Theorem 6.73 states that;

There are $n!$ permutations of an n-element set.

Theorem 6.75 states that;

There are k-permutations of an n-element set is

$$P(n,k) = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet (n-k+1) = \frac{n!}{(n-k)!}$$. So,

$$P(n,k) = \frac{n!}{(n-k)!}$$

As the $k = 3$ and $n = 12$

$$P(12,3) = \frac{12!}{(12-3)!}$$

$$= \frac{12!}{9!} = \frac{12 \bullet 11 \bullet 10 \bullet 9 \bullet 8 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1} = 12 \bullet 11 \bullet 10 = 1320.$$

b. To do a permutation on my *TI-83 Plus* I start by inserting my n value, which in this case is 12. Then I press the third button down on the left, MATH and use the left arrow once to get to PRB, which is an abbreviation of probability. Under PRB, I press the number 2 (if I wanted to compute a combination I would use the number 3). Now on my screen I have 12 $n\mathrm{P}r$. The r represents my k-permutation. Now I insert my k -value which is 3. Now, I press the bottom right button, ENTER, and my calculator computes the answer to be 1320.

Geoff Benson
Graded Assignment
12-01-2008

6P. Pascal's Triangle

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
\vdots
\end{array}
$$

4. This problem asks you to delve into history of Pascal's Triangle. One possible resource for answering the following questions is David Burton's book, *The History of Mathematics: An Introduction.*

a. Where and when did Blaise Pascal live?

b. Discuss two appearances of the triangle arrangement of binomial coefficients within the mathematics of non-European cultures before the time of Pascal.

c. Though Pascal cannot be credited with inventing the triangle that bears his name, what did he accomplish in his work *Triangle Arithmétique* that makes it reasonable to refer to the triangle as Pascal's Triangle?

Blaise Pascal was born Clermont, France. His mother passed away when he was three, and his father, Etienne, moved the family to Paris. An interesting part of Blaise's education was that he never attended a school or university. His father had an planned course of education which entailed that the youngest could not study mathematics until they reached the age of 15, which was later revised to 12 years old.

The *Triangle Arithmétique* works the same as the European ones that had been formatted by Omar Khayyam (circa 1050-1130) and Al-Tusi (circa 1200-1275). The way

Pascal's triangle looked was later rotated 45 degrees to what we see it as now. As we add the perpendicular rank cells (vertical column) with the parallel rank cells (horizontal row), we get the next perpendicular rank cell.

Pascal was not credited as the originator of the *Triangle Arithmétique*, but he was the last of the "discoverers", so his name is linked to it for his systematic study of relations. *Triangle Arithmétique* was finished at the end of 1654, but was not distributed until 1665. Pascal stray's from others by applying a recursion formula for the coefficients. Consequence XII is

$$\frac{\binom{n}{r+1}}{\binom{n}{r}} = \frac{n-r}{r+1}$$

In his work he used proof by induction repeatedly. Consequence XII is most significant because he used proof by induction. Pascal had formed 19 propositions or consequences.

Algebra and Almucabola to a lost work in which he dealt with the arithmetic triangle. Another Persian mathematician, Al-Tusi (circa 1200–1275), in a work called *Collection on Arithmetic by Means of Board and Dust*, approximated the value of the square root $\sqrt[n]{a^n + r}$ by $a + r/[(a+1)^n - a^n]$, where the denominator was calculated by the binomial expansion. For this purpose, Al-Tusi furnished a table of binomial coefficients in triangular form up to the twelfth power. Thus the so-called Pascal triangle, like the so-called Pythagorean theorem, is in reality the product of a much earlier Eastern culture.

The first triangular arrangement of the binomial coefficients to be printed in European books appeared on the title page of the *Rechnung* (1527) of Peter Apian (1495–1552). Apian, a professor of astronomy at the University of Ingolstadt, is interesting if only because he taught in the Germanic tongue at a time when the prevailing custom was to use Latin. While the arithmetic triangle had not previously been described in the West until depicted in Apian's text, it seems to have been discovered almost simultaneously by several authors of the 1500s. Michael Stifel (1486–1567) in his *Arithmetica Integra* (1544) calculated the device as far as the seventeenth line, though as he pointed out, there was no reason to stop there. Stifel's diagram took the form

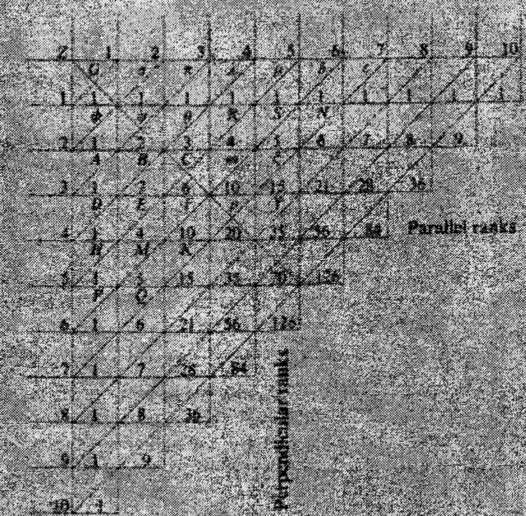
1				
2				
3	3			
4	6			
5	10	10		
6	15	20		
7	21	35	35	
8	28	56	70	
9	36	84	126	126
10	45	120	210	252

where each column after the first starts two places lower than the preceding one. Other schematic arrangements for the binomial coefficients were given by the old rivals Tartaglia and Cardan. Tartaglia in the *Trattato Generale* (1556) gave the numbers in the triangle through the eighth line, claiming the idea as his own invention. Cardan, in the work *Opus Novum de Proportionibus* (1570), presented a table to 17 lines, citing Stifel as the original discoverer.

Although Pascal was not the originator of the arithmetic triangle, being nearly the last of a long line of "discoverers," his name is forever linked with the triangle because he was the first to make any sort of systematic study of the relations it exhibited. The merits of Pascal's work in this regard are enough to justify the use of his name. The printing of Pascal's *Traité du Triangle Arithmétique* was finished towards the end of 1654 (Fermat received his copy sometime before September of 1654), but as Pascal had withdrawn from worldly matters, it was not distributed before 1665. The work is an exposition of the properties and relations between the binomial coefficients, and it includes a few general probability principles. In

particular, in a section entitled *Utilization of the Arithmetic Triangle to Determine the Number of Games Required Between Two Players Who Play a Large Number of Games*, Pascal applied the arithmetic triangle to the problem of stakes in games of chance.

In the *Triangle Arithmétique*, Pascal did not write the arithmetic triangle from the top down as we now do, but instead expressed the table as shown:



The numbers on the nth upward-sloping diagonal in this arrangement give the coefficients in the expansion of the binomial $(x + y)^n$. Thus, the figure that modern texts call Pascal's triangle differs from the triangle examined by Pascal himself by a rotation through 45°. In his scheme, Pascal called positions in the same vertical column cells of the same perpendicular rank, and those in the same horizontal row cells of the same parallel rank. Cells in the same northeast-running diagonal were said to be cells of the same base.

Pascal observed that his table could readily be enlarged by affixing further numbers, obtained without recourse to the binomial theorem. Once the 1s in the top line and left-hand column have been set down, any other entry is the sum of the number directly above the entry and the number immediately to the left of the entry. As he put it: "The number of each cell is equal to that of the cell which precedes it in its perpendicular rank, added to that of the cell which precedes it in its parallel rank." For instance, underneath the 5 in the second row and alongside the 10 of the third, a new number can be placed; this new entry is 15, the sum of 5 and 10. Pascal had no claim to originality here, for this generating rule had been accurately